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ELECTRON COOLER INTRODUCED PERTURBATIONS ON ION BEAM

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Abstract

The influence of electron cooler on stored ion beams in heavy ion synchrotron has been studied for many years. Usually, only the influence on lattice from the magnetic field of solenoid and toroids were considered, in addition to the cooling effect from Coulomb Scattering. However, electron cooling experiments show that there exist limits on intensity of electron beam and the stored ion beams. Meanwhile, experiments of cooling with pulsed e-beam show structure instability that should be explained. The influence of electron beam induced electromagnetic field on ion beams was studied in this paper by means of Lie Algebraic method. The combined transport matrix of solenoid and e-beam field focusing is given. Application of the matrix may help to understand the above phenomena.

INTRODUCTION

As is well known, the general requirements for an e-cooler includes:

1. Parallel and similar velocity of e-beam and ion-beam;
2. Lower electron beam temperature;
3. Adequate cooling force (electron density n_e and length L_e);
4. Compensable or Neglectable influence on ion beam.

In order to satisfy these requirements, solenoid with high uniformity field is introduced to constrain the electron beam and keep its temperature, toroids and/or electrostatic inflectors are introduced to guide the e-beam orbit. The magnetic field of solenoid and toroids will affect the ions passing through it, and usually these effects will be compensated for. The coupling effects of solenoid field can be compensated by a pair of additional solenoids or skew quadrupoles, but the focusing effects are usually neglected. The bending effects of toroid field can be compensated by dipole correctors.

Another source may come from the space charge and current focusing of electron beam, which was usually neglected in practice.

In this paper, we will focus on the study of the e-cooler introduced focusing effects of solenoid and the e-beam field. The combined transport matrix of solenoid and e-beam field will be deduced by means of Lie Algebraic method [1].

Application of the matrix may help to understand why high current e-beam not be used to cool low energy ion beam, why in pulsed e-beam cooling the electron bunches should have the same pulse frequency of ion bunch pulse and cover the ion bunches, and how to get higher deceleration efficiency.

EFFECTS OF ELECTRON BEAM

In addition to the cooling effect from Coulomb Scattering, the electron beam also affects the ion beam by its self-field.

For a round uniformly distributed electron beam, its space charge electric field is given by the following formula:

$$\bar{E}(r) = -\frac{n_e e}{2\varepsilon_0} r \qquad (1)$$

where n_e is the electron density, e is the electron charge. From Eq.(1), the focusing strength can be given as:

$$K_E = \frac{n_e e}{2\varepsilon_0\,\beta c\,B\rho} \qquad (2)$$

where βc is the velocity of ion/electron, $B\rho$ is the magnetic rigidity of the ion beam.

For a round uniformly distributed electron beam, its current induced magnetic field is given by the following formula:

$$\bar{B}(r) = \frac{\mu_0}{2\,\pi\,r^2} I \times r = \frac{\mu_0 n_e\,e\,c}{2\,r} \beta \times r \qquad (3)$$

From Eq.(3), the focusing strength can be given as:

$$K_B = -\frac{\mu_0 n_e\,e\,\beta c}{2\,B\rho} = -\beta^2 K_E \qquad (4)$$

The total focusing strength of electromagnetic field of electron beam can be given as:

$$k = K_E + K_B = (1-\beta^2)\frac{n_e\,e}{2\varepsilon_0\,\beta c\,B\rho} \qquad (5)$$

For the stand-alone electron beam, its transport matrix can be written as:

$$R_{ef} = \begin{bmatrix} \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}L) & 0 & 0 \\ -\sqrt{k}\sin(\sqrt{k}L) & \cos(\sqrt{k}L) & 0 & 0 \\ 0 & 0 & \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}L) \\ 0 & 0 & -\sqrt{k}\cdot\sin(\sqrt{k}L) & \cos(\sqrt{k}L) \end{bmatrix} \qquad (6)$$

From Eq.(5) we noticed that the focusing strength of electron beam induced field will decrease along with the beam energy.

DEDUCING OF COMBINED TRANSPORT MATRIX

To deduce the combined transport matrix of solenoid and electron beam, Lie Algebraic method is adopted [1]. The simplified combined Hamiltonian for transversal movement is:

$$H = \frac{1}{2}\left(\left(x' + \frac{1}{2}k_s y\right)^2 + \left(y' - \frac{1}{2}k_s x\right)^2\right) + \frac{k}{2}\cdot(x^2+y^2) \quad (7)$$

where $k_s = \frac{B_s}{2B\rho}$ is the strength of solenoid field, B_s is the magnetic field of solenoid, and k is the strength of e-beam field.

As well known, the Lie transformation associated with $f(x,x',y,y') = -L\cdot H$ will give out the polynomial representation of the elements of transport matrix. It's not convenient and not precise to use the truncated polynomial in

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beam transport calculation. We tried to guess out the representations of the elements using elementary functions, which will be easily used in 1st order transport calculations.

And finally, we found the combined transport matrix of solenoid and e-beam field as:

$$R = \begin{array}{cccc} \cos(KL)cos(k_sL) & \sin(KL)\cos(k_sL)/K & \cos(KL)\sin(k_sL) & \sin(Kz)\sin(k_sz)/K \\ -K\sin(KL)\cos(k_sL) & \cos(KL)cos(k_sL) & -K\sin(KL)\sin(k_sL) & \cos(KL)\sin(k_sL) \\ -\cos(KL)\sin(k_sL) & \sin(Kz)\sin(k_sz)/K & \cos(KL)cos(k_sL) & \sin(KL)\cos(k_sL)/K \\ \lfloor K\sin(KL)\sin(k_sL) & -\cos(KL)\sin(k_sL) & -K\sin(KL)\cos(k_sL) & \cos(KL)cos(k_sL)\rfloor \end{array} \tag{8}$$

$$= \begin{pmatrix} \begin{matrix} \cos(KL) & \sin(KL)/K \\ -K\sin(KL) & \cos(KL) \end{matrix} & 0 \\ 0 & \begin{matrix} \cos(KL) & \sin(KL)/K \\ -K\sin(KL) & \cos(KL) \end{matrix} \end{pmatrix} \begin{pmatrix} \cos(k_sL) & 0 & \sin(k_sL) & 0 \\ 0 & \cos(k_sL) & 0 & \sin(k_sL) \\ -\sin(k_sL) & 0 & \cos(k_sL) & 0 \\ 0 & -\sin(k_sL) & 0) & \cos(k_sL) \end{pmatrix}$$

where $K = \sqrt{k + {k_s}^2}$ is the combined focusing strength, L is the length of the solenoid and the electron beam. As can be seen, when $k = 0$, R is transformed into the transmission matrix of the solenoid; when $k_s = 0$, it returns to that of e-beam field, R_{ef}. The transmission matrix R can be rewritten as two parts as in Eq.(8), the left part is the focusing matrix in both transversal plane, and the right part is cyclotron rotation matrix.

APPLICATIONS OF THE MATRIX

Usually the rotation effect of R is easily compensated by introducing of additional solenoids or skew quadrupoles, but meanwhile additional focusing is introduced. The focusing strength of electron cooler and the compensation solenoids/skew quadrupoles usually should be considered in ring lattice design.

As an example to show the combined influence of solenoid and e-beam field to the HIRFL-CSRm[2,3]. In CSRm, the solenoid of e-cooler usually working at a field of 395 G, which is much smaller than the designed 1500 G. Using the parameters of e-cooler: length L=2.56 m, radius of e-beam r_e =3 cm, mean betatron amplitude function of $\langle\beta\rangle$ =10 m, and a supposed electron current of I_e=330 mA, the focusing strength of e-cooler on ion beam can be calculated as shown in Table 1.

Table 1: Example of Electron Cooler Focusing Effects

Ion	E(MeV/u)	$k(\mathrm{m}^{-2})$	$k_s^2(\mathrm{m}^{-2})$	$\Delta Q_{x,y}$
$^{238}\mathrm{U}^{30+}$	1.0	0.03	0.0003	0.061
$^{12}\mathrm{C}^{6+}$	7.0	0.0064	0.0007	0.013

As shown in Table 1, the focusing strength varies in large range for different conditions, and the contribution of electron beam field domains the contribution. This may help to understand why only tens of mA electron beam current were usually used in practice.

Still for CSRm, we calculated the tune shifts and maximum betatron amplitudes of lattice at different e-beam field focusing strength, as shown in Table 2. As shown in the table, not only the tune shifts are large enough (Fig.1) to be considered seriously, but also the significant change in betatron oscillation amplitude function will significantly change the transverse acceptance of the storage ring and must be fully considered.

Table 2: Influence of e-cooler to CSRm Lattice

k	Q_x	Q_y	Max β_x (m)	Max β_y (m)
0	3.630	2.624	14.99	30.16
0.01	3.679	2.695	16.95	53.89
0.02	3.709	2.736	19.53	66.81
0.03	3.735	2.773	22.15	81.35

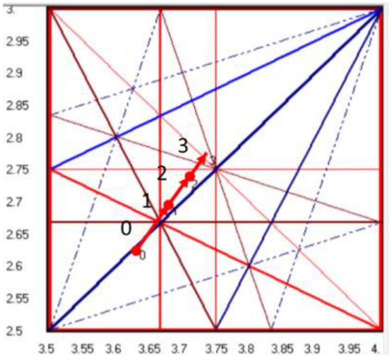

Figure 1: Tune shifts induced by solenoid and e-beam field at CSRm

Recently, electron cooling with pulsed electron beam was studied as HIRFL-CSR[3-5]. The dependence of lifetime on a synchronization between ion and electron pulses (see Figure 2) was found in experiments.

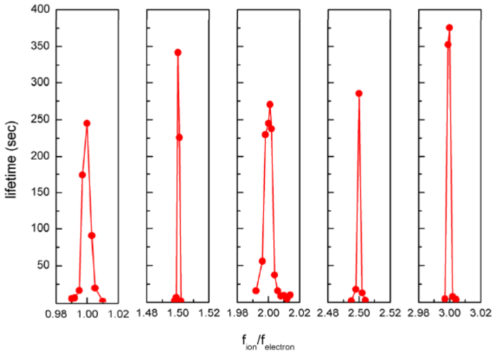

Figure 2: The dependence of lifetime on synchronization between ion and electron pulses[4].

One explanation of this phenomena is the barrier-like electric field at both ends of the e-beam bunch, but it's not sufficient. From the point view of transverse motion, considering the fast varying focusing strength of e-cooler on the ion beam when the bunches of two beams are not synchronized, it’s easy to “naturally” explain the experiment results. The ring lattice will change turn-by-turn according to the coverage condition of ions by electrons and it introduces variation of acceptance and super-period instabilities.

From Eq.(5,6,8), it’s clear that the focusing strength will change during the de/acceleration, if the magnet field of solenoid and the current and energy of electron beam are fixed.

For a normal cooling accumulation and acceleration period of the synchrotron, the cooler parameters were usually kept for the injection and cooling accumulation. The tune shift varies smoothly during acceleration. A suitable condition can be found to get high acceleration efficiency if the e-cooler focusing strength is relatively low, as the transverse geometric emittance of ion beam will decrease during acceleration. But if the initial strength is high, crossing of low order resonant lines during acceleration will create significant beam loss.

Deceleration of ion beam is more complicated, as the transverse geometric emittance of ion beam will increase during deceleration. As an example, for the deceleration of Ni^{28+} ion beam from 400 MeV/u to 4 MeV/u at ESR/GSI[6], the efficiency is only 1/6 of the theoretic value. A better consideration of the e-cooler introduced influence on ion beams may help to improve this condition.

CONCLUSION

The combined transport matrix of solenoid and eletron beam field was deduced by Lie Algebraic method for the 1st order study convenience. The transverse perturbation of e-beam field of electron cooler on ion beam is ***not*** neglectable, especially for low energy ion beam cooling, pulsed e-beam cooling and especially when pulsed e-beam used before de/accelerations.

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