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# VERTICAL FOCUSSING WITH A FIELD GRADIENT SPIRAL INFLECTOR

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*Abstract*

Traditional spiral inflectors suffer from vertical defocussing, leading to beam loss. In this study the electrode shape of an inflector is modified to intentionally produce transverse electric field gradients along the beam path, which have a significant influence on the optics. This is done by placing the traditionally parallel electrodes at an angle relative to each other in the transverse plane, creating a quadrupole field on the central path. Varying the electrode angle along the path length creates an alternating-gradient effect. The electrode entrance and exit faces are also shaped to create quadrupoles inside the fringe field. By numerical optimisation a design with good vertical focussing is obtained. Experiments show a roughly 100% improvement in transmission in cases where the buncher is turned off. However, high losses at extraction are observed with the buncher turned on, due to RF-phase spread introduced by longitudinal defocussing in the inflector. This results in an improvement of only 20% during normal cyclotron operation, and shows that an inflector should ideally focus vertically and longitudinally at the same time. Ongoing work to achieve such combined focussing is briefly described.

## INTRODUCTION

Spiral inflectors based on the Belmont-Pabot [1] design are known to have very good transmission, but suffer from vertical defocussing, which can lead to beam loss in the inner region of the cyclotron [2]. This undesired vertical behaviour is illustrated in Fig. 1, where the beam passing through the C-inflector of the Solid-Pole-Cyclotron 2 (SPC2) [3] at iThemba LABS is modelled in TOSCA [4]. A substantial portion of the beam strikes the vertical slits downstream from the inflector, or is lost vertically on the puller electrode.

Solutions to the vertical defocussing problem implemented in the past include the addition of an electrostatic or magnetic quadrupole behind the inflector, but this requires additional space in the cramped inner region [5], and does not prevent emittance blow-up in the inflector itself. Another solution proposed at Dubna, is to give the electrodes a V-shape to create focussing electric fields, similar to the vertical direction of a spherical electrostatic bend [2]. In this article a design is introduced that involves shaping the inflector electrodes to create quadrupole electric fields in the transverse plane, and varying their strengths along the path length, to create an effect similar to strong focussing.

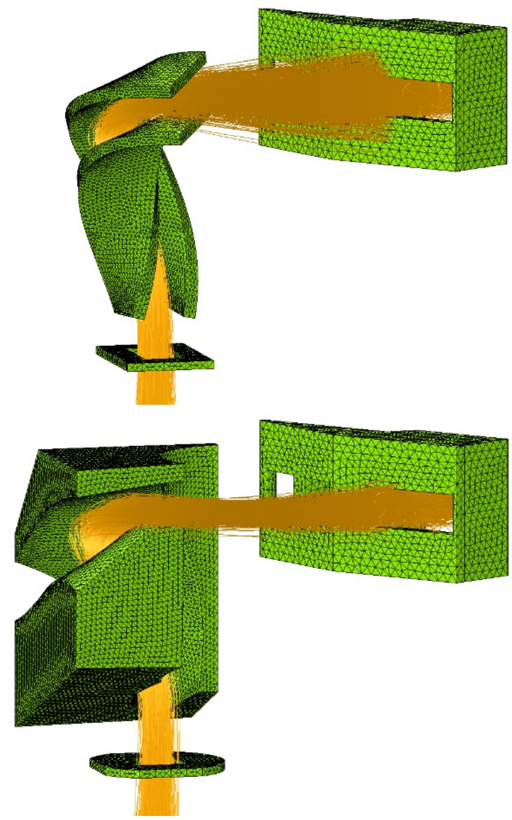

Figure 1: Beam passing through the inflector and striking the vertical slit in an ordinary spiral inflector (top), and in a field gradient inflector with vertical focussing (bottom).

## ELECTRIC FIELD GRADIENTS

The traditional spiral inflector design by Belmont and Pabot specifies the electric field on the central trajectory, but places no constraints on the field gradients. Since the first order optics of the device depends on these gradients, it might be possible to control the focussing of an inflector by selecting appropriate electric field gradients.

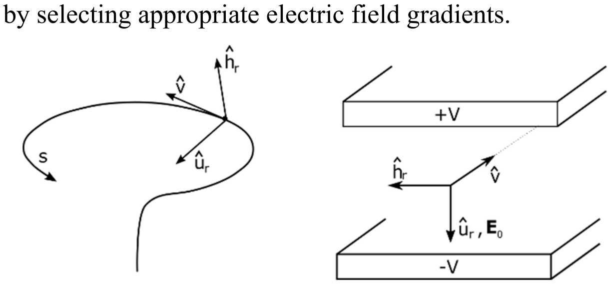


Figure 2: Standard inflector coordinate system (left) and the positioning of the electrodes to create the central field $\boldsymbol{E_0}$ (right).

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Standard inflector coordinates are used here, as illustrated in Fig. 2: The coordinates $(u_r, h_r, s)$ form a right handed system, where s is the path length and the transverse $(u_r, h_r)$ plane is Cartesian so that $\hat{u}_r$ points in the direction of the central electric field $\boldsymbol{E_0}$.

First order electric field gradients on the central path can intentionally be created by tilting the electrodes relative to one another, as shown in Fig 3.

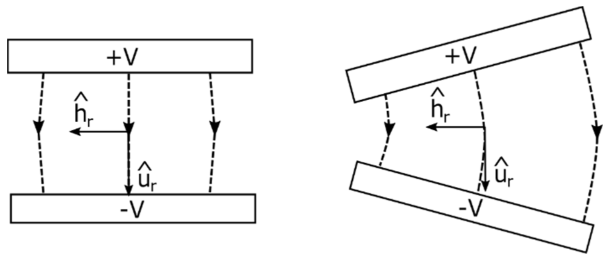


Figure 3: Electrode cross sections in the transverse plane. Traditional parallel electrodes (left) keep the electric field as uniform as possible, while tilting the electrodes (right) intentionally creates electric field gradients.

For simplicity, when ignoring the 3D bending and rotation of the central path, the electric field can be approximated by a 2D field in the $(u_r, h_r)$ transverse plane. The tilting of the electrode surface then produces a quadrupole field:

$$\frac{\partial E_{h_r}}{\partial u_r} = \frac{\partial E_{u_r}}{\partial h_r} = -QE_0$$

Where the titling parameter Q is given by the relative slope of the internal electrode surface in the $(u_r, h_r)$ plane:

$$Q = \frac{1}{u_r}\frac{\partial u_r}{\partial h_r}$$

The strength of the tilting, expressed by $Q(s)$, can be varied along the path length, to create a strong focussing effect similar to a doublet or triplet lens. Additionally, the entrance and exit faces of the electrodes can be cut at an angle in the $(h_r, v)$ plane, as shown in Fig. 4, producing electric field gradients in the fringe field region.

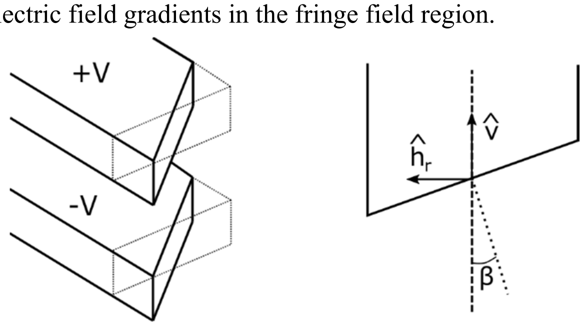


Figure 4: Modifying the electrode entrance and exit angles to produce field gradients in the fringe field region. The main field changes in strength along $h_r$, producing a quadrupole with $\frac{\partial E_{h_r}}{\partial u_r} = \frac{\partial E_{u_r}}{\partial h_r}$.

The entrance and exit edge angles $\beta_1, \beta_2$ can then be selected to enhance the gradients produced by $Q(s)$, in the internal part of the inflector.

## EFFECTIVENESS OF THIS METHOD

To see how effective these gradients can be, consider a simplified situation where the inflector has been "straightened out" so that its central trajectory is a straight line. If the first half of the inflector is then given a positive Q, and the second half a negative Q, a doublet lens is formed, as shown in Fig. 5. Note that the maximum electrode gradient that can easily be obtained in an inflector is around Q ≈ 60 $m^{-1}$. For a typical inflector with a path length of 10 cm, electric bending radius 6 cm and electrode tilt Q = ±60 $m^{-1}$, the point-to-point focussing distance obtained by such a doublet is 17 cm. This is very similar to the distance between the collimator in front of the inflector and the slit behind the inflector, so clearly the field gradients can have a substantial effect on the optics.

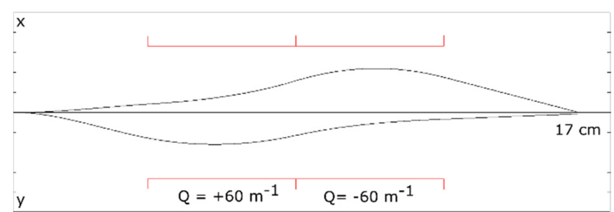


Figure 5: TRANSPORT calculation of a doublet equivalent to a "straightened out" inflector with alternating electric field gradients.

## DESIGN PROCEDURE

The design of a field gradient spiral inflector consists of specifying a Belmont-Pabot type central path, the electrode gradient $Q(s)$ along the length of the path, as well as the entrance and exit angles $\beta_1, \beta_2$. The transfer matrix for the inflector, written as $R(Q, \beta_1, \beta_2)$, is very complicated and must be obtained numerically. A Matlab program was created to quickly compute the transfer matrix:

$$R = TransferMatrixProgram(Q, \beta_1, \beta_2)$$

To do this the function $Q(s)$ was discretized and represented by in a piecewise linear fashion. The discretization only consisted of 3 points along the path length (inflector entrance, middle and exit), which is the minimum needed to represent a triplet-like shape, and these values are indicated by $(Q_1, Q_2, Q_3)$. The *TransferMatrixProgram* had to be able to compute *R* within a few seconds. This was done by ray-tracing through the electric and magnetic fields. The same magnetic field was always used, and obtained from a TOSCA-based numerical model of the SPC2 cyclotron [6]. Using an accurate magnetic field is important due to the strong solenoid focussing and rotation occurring as the beam enters the main magnetic field through the axial entrance hole in the magnet yoke.

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When performing ray tracing, the electric field has to be constantly adapted, depending on the gradient parameters $Q_1, Q_2$ ... etc. Since a full TOSCA simulation of the electric field is too time consuming, a simplified method was used. This method only requires performing a small number of TOSCA simulations, and then using their pre-calculated results to estimate the electric field during run time. To do this, it was assumed that the electric field can, in all of space, be approximated by the first derivatives of the field with respect to the electrode parameters:

$$\boldsymbol{E}(Q_1, Q_2, \dots) = \boldsymbol{E}|_0 + \frac{\partial \boldsymbol{E}}{\partial Q_1} Q_1 + \frac{\partial \boldsymbol{E}}{\partial Q_2} Q_2 + \cdots$$

Where $\boldsymbol{E}|_0$ refers to the 3D TOSCA electric field when all the parameters are zero, while the derivatives with respect to the gradient parameters were estimated by:

$$\frac{\partial \boldsymbol{E}}{\partial Q_1} \approx \frac{1}{\Delta Q}\left(\boldsymbol{E}|_{Q_1=\Delta Q} - \boldsymbol{E}|_0\right)$$

Where $\boldsymbol{E}|_{Q_1=\Delta Q}$ refers to the 3D TOSCA field calculated when $Q_1 = \Delta Q$ and all the other parameters zero. The value of $\Delta Q = 20$ was chosen since it lies roughly in the middle of the range of obtainable gradients. In this way, only 6 pre-calculated TOSCA models were required. The complete *TransferMatrixProgram* was validated by comparing it to the full TOSCA simulation in a number of cases.

The transfer matrix $R(Q_1, Q_2, Q_3, \beta_1, \beta_2)$ was numerically optimised so that the resulting beam height and vertical divergence at the first acceleration gap were kept to a minimum. This optimisation was performed using a combined method of random sampling, to obtain promising starting positions, followed by steepest descent, to reach the optimal values.

## DESIGN OF A NEW INFLECTOR FOR SPC2

The SPC2 injector cyclotron at iThemba LABS uses a spiral inflector, called inflector C1, to accelerate heavy ions. Inflector C1 is equivalent to a standard spiral inflector with properties shown in Table 1. A new vertically focussing inflector, called C2, was designed to replace C1.

Table 1: Specifications of Inflector C

| Description | Parameter | Value |
|---|---|---|
| Magnetic bending radius | $R_m$ | 4.9 cm |
| Electric bending radius | $A$ | 6.0 cm |
| Tilt parameter | $k'$ | 0.38 |

In the design of the C2 inflector, a beam with a 3 mm x 30 mrad elliptical transverse profile and a 0.5% momentum spread was used as the input, and the beam profile at the first acceleration gap was optimised. The final design is shown in Fig. 6. It was found that a doublet-type design, where Q goes from negative to positive along the path length, worked well. The entrance and exit angles $\beta_1, \beta_2$ were set at 20° and oriented to enhance the gradients produced by Q.

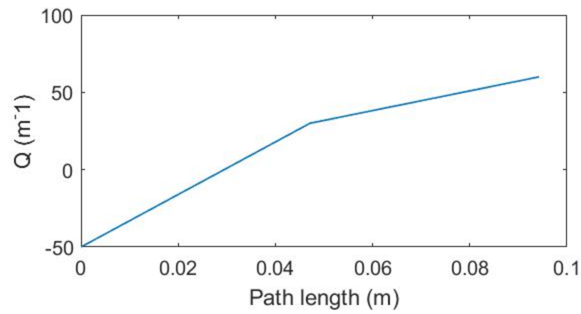


Figure 6: Design of the C2 inflector, showing how the tilting parameter Q changes along the path length.

The simulated vertical performance of C2, shown in Table 2, is very good. In particular, there was a 65% decrease in the beam spot size compared to C1. The horizontal behaviour did not change much. The longitudinal spread introduced by C2 is however substantially more than C1, by a factor of around 2. The phase space plots of the final beam, as determined by ray tracing in the full 3D inflector model in TOSCA, are shown in Fig. 7.

Table 2: Comparison of the C1 and C2 Beam Parameters at the First Acceleration Gap

| Parameter | C1 | C2 |
|---|---|---|
| Vertical emittance growth (%) | +45 | +5 |
| Vertical half-width (mm) | 8.9 | 2.9 |
| Horizontal emittance growth (%) | +45 | +30 |
| Horizontal half-width (mm) | 3.0 | 1.5 |
| Longitudinal half-width (mm) | 5.5 | 12.9 |
| RF phase spread (degrees) | ±39 | ±90 |

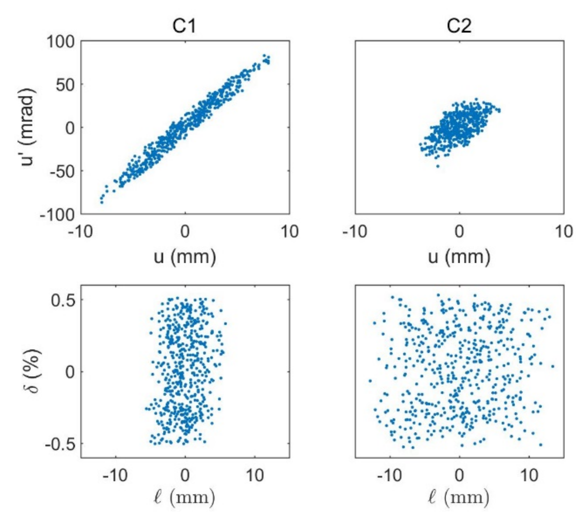


Figure 7: Vertical (top) and longitudinal (bottom) emittance plots at the first acceleration gap.

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## EXPERIMENTAL TESTING

Inflectors C1 and C2 were experimentally tested by measuring their transmission throughout SPC2. The transmission was defined as the beam current measured at a certain point divided by the beam current on the last Faraday cup before the inflector. The results of all four experiments performed so far are shown in Fig. 8. It should be noted that performing experiments with SPC2 is difficult since it displays large differences in performance (at least 20%) between consecutive runs, so it is important to look at the trends in the data rather than the individual values.

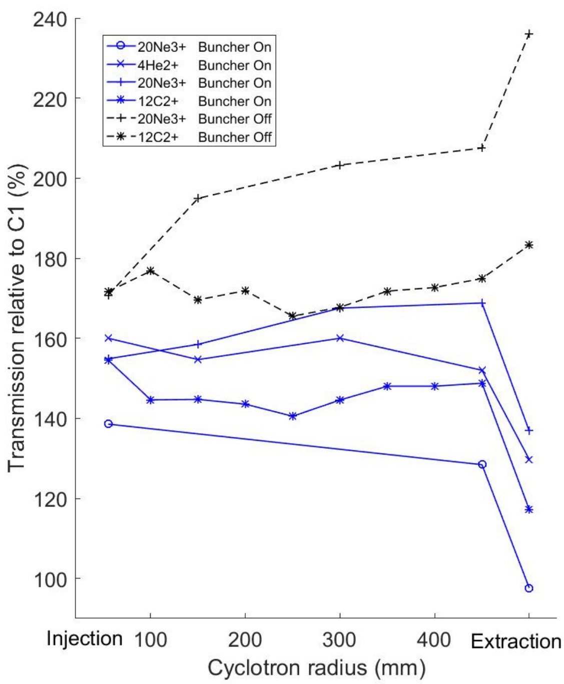


Figure 8: Experimental transmission through SPC2 when using the field gradient inflector C2, compared to the transmission with the ordinary spiral inflector C1.

When the buncher is turned off, so that the longitudinal behaviour of the inflector plays no role, the C2 inflector improved transmission by between 80% - 120%. With the buncher activated, C2 improves the injection efficiency by about 50% and it then maintains this improvement throughout the cyclotron, but suffers from bad extraction. This meant that the average total improvement in transmission was only about 20%.

## IMPORTANCE OF LONGITUDINAL BEHAVIOUR

The experimental results can be explained as follows: C2 has very good vertical focussing and loses relatively few particles on the puller slits, resulting in the good performance when the buncher is turned off. But C2 also has a large longitudinal spread (see Table 2 and Fig. 7), resulting in a large RF phase spread when the buncher is turned on. The inflector is in effect de-bunching the beam. This leads to decreased performance, with a larger energy spread probably resulting in a wider beam with poor extraction.

It was attempted to improve the longitudinal behaviour of C2, while still maintaining its vertical focussing, using the electrical gradient method described in this article. Unfortunately, this was unsuccessful. Instead, a new more general method for the control of all possible electric gradients has been developed, resulting in quadratic electrode profiles in the transverse plane. This doubles the degrees of freedom available to control the first order electric field gradients. The development of a new inflector, C3, capable of controlling both the vertical and longitudinal focussing is ongoing.

## CONCLUSION

By tilting the electrode surfaces of a spiral inflector it is possible to create electric field gradients along the central path that greatly influence the optics. Selecting an appropriate tilting profile Q(s) results in good vertical focussing and much improved transmission through the cyclotron. However, an increase in longitudinal spread reduces the effectiveness of this method, and a possible solution is being investigated.

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