INVESTIGATIONS OF THE LONGITUDINAL CHARGE DISTRIBUTION IN VERY SHORT ELECTRON-BUNCHES

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Abstract

Electro-optical-sampling is a powerful technique to measure the longitudinal charge distribution of very short electron bunches. The electrical field moving with the bunch induces an optical anisitropy in a ZnTe crystal which is probed by a polarized laser pulse. Two measurement principles are possible. In the first one a short laser pulse of lengths < 50 fs is used directly to scan the time varying optical properties of the crystal. In the second method the laser pulse is frequency chirped and the temporal information is encoded into the time ordered frequency spectrum, which can be recovered by an optical grating and a CCD camera.

A resolution in the 100 fs regime can also be achieved with longitudinal phase space tomography. Acceleration on the slope of the rf wave at different phases and measurements of the energy profiles are sufficient for a reconstruction algorithm based on maximum entropy methods. The longitudinal phase space distribution can be obtained without artefacts due to the limited angular range of the projections.

1 INTRODUCTION

Linear electron positron colliders or free electron lasers (FEL) require electron bunches of subpicosecond bunch length. Measurement techniques have been developed in recent years to provide diagnostics in this parameter regime.

One possibility is to measure the coherent radiation emitted by the bunches under certain circumstances: Coherent transition radiation (CTR), diffraction radiation, or synchrotron radiation (CSR). By analysis of the radiation spectrum one can determine the longitudinal bunch profile. In most cases the phase information is lost and has to be reconstructed for example with the Kramers-Kronig-Relation. This kind of analysis can be considered as well established [3][4][5] and will be used as a reference in this paper.

The development of Ti:sapphire lasers with ultra-short pulses of FWHM < 50 fs led to the concept of electro-optic sampling and imaging in THz-spectroscopy [17][20]. At the FEL Laboratory for Infrared Experiments (FELIX) in Rijnhuizen near Utrecht, NL electro-optic sampling (EOS) has been successfully applied to measure bunch lengths [19][7]. Similar measurements are being prepared at several accelerators and as well at the TESLA Test Facility (TTF)[2][8].

Many experiments have been carried out to measure longitudinal beam profiles by means of the rf acceleration in the linac themself [3][16]. Using off-crest acceleration in the rf cavities an energy deviation is induced depending on the longitudinal position of the electrons in the bunch. This can be measured with a spectrometer dipole. One of the problems with this kind of measurement is the entanglement with the initial energy spread, which often is in the same order of magnitude. Using magnetic chicanes followed by an acceleration section, it is possible to rotate the phase space by $90°$ so that the longitudinal position is projected onto the energy [13][4]. In general tomographic methods can be used to get a reconstruction of the full longitudinal phase space.

2 ELECTRO-OPTIC METHODS

Exposed to a strong electric field some optical crystals exhibit the Pockels effect: The electric field distorts the lattice of the crystal and the material becomes birefringent. Linearly polarized light with its polarisation oriented $45°$ to the optical axis is transformed into elliptically polarised light with the fraction of circularly polarized light proportional to the strength of the electrical field.

Compared to the oszillation of the laser even THz-fields can be considered as slowly varying and so the Pockels Effect can be applied to measure the electric field strength in THz pulses with a resolution governed by the width of the laser pulse used to sample. There are Ti:Sapphire lasers available which deliver pulses shorter than 20 fs FWHM. For the EOS often ZnTe is used because it provides good sensitivity and good optical properties for THz frequencies as well as for the light from Ti:sapphire lasers. The Phonon resonances with the lowest frequency in ZnTe can be found at 5.6 THz, but their influence on the measurement can be modelled precisely. Electro-optical Sampling has been demonstrated with ZnTe up to frequencies of 37 THz [18].

In contrast to many applications in THz-Spectroscopy where the same laser pulse is utilized for generation of the THz-pulse as well as for probing the electrical field, in a linac the bunches and the ultra-short laser pulses are generated by different sources. A demanding task in the adaption of EOS to accelerator diagnostics is therefore the synchronisation of the Ti:Sapphire laser to the master clock of the accelerator. This can be accomplished by building a phase locked loop which synchronizes a harmonic of a photodiode signal from the laser with the rf-reference from the master clock. In this way phase noise can be reduced by a factor of 10. The residual noise translates into a timing

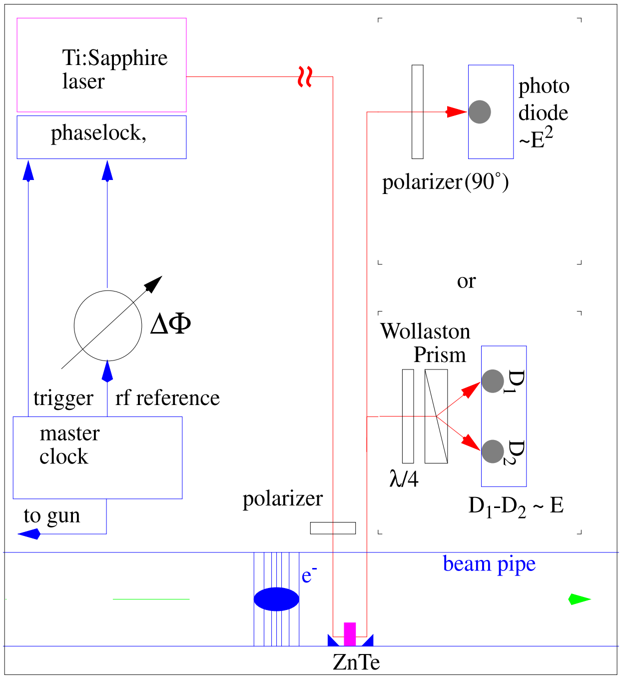

Figure 1: Possible setup for the Electro Optic Sampling

jitter of 100 fs within 1 ms respectively 1 ps over 1 second. This dominates the achievable resolution in a direct sampling of the electron bunches.

The ZnTe crystal can be mounted directly inside the vacuum chamber to probe the electrical field travelling together with the bunch. Due to the Lorentz contraction of the field the electrical field lines of each electron are concentrated in a disk with opening $1/\gamma$ orthogonal on the trajectory of the electrons. Thus the longitudinal charge distribution can directly be measured by scanning the copropagating fields. The design of the vacuum system has to be done carefully to minimize distortions of the measurement by wakefields.

2.1 Setting up the Timing

There are several ways to resolve the time information. The first attempt one would do to start this kind of measurement is to scan the laser pulse across the electron bunches. With a variable delay each laser pulse overlaps with a different part of the electron bunches. As can be seen from the previous section the time jitter and thereby the achievable resolution depends on the velocity of scanning. With an electronic phaseshifter a full scan can be performed within milliseconds to achieve the best resolution of

$$\triangle T = \sqrt{T_0^2 + (T_{jitter})^2}\,. \qquad (1)$$

At FELIX this kind of measurement has been established. Results can be seen in figure 2. Instead of shifting the laser pulse with respect to the beam at FELIX the frequency of the whole accelerator was shifted. The resolution achieved in these experiments is reported to be 440 fs, a full discussion of the experiment can be found in [7][19].

It is possible to take advantage of the timing jitter by using a technique called differential optical gating. In this method the THz-pulses are sampled by two laser pulses with a fixed time delay between them. The pair of pulses is allowed to jitter with respect to the THz-pulses so that statistically they cover them totally. With each measurement one obtains a value for the instantaneous intensity I and its time derivative $\dot{I}$. The resulting distribution can be represented by a function

$$F(I) = \frac{dI}{dt}. \qquad (2)$$

The time information can be obtained by integration, yielding

$$t(I_1) = t(I_0) + \int_{I_0}^{I_1} \frac{dI}{F(I)}. \qquad (3)$$

The integral can only be performed where $F(I) \neq 0$, which creates problems at the maximum of the pulse where the derivative is zero. Using only equation 3, one can reconstruct the rising and the falling edge separately. This leaves some uncertainty about the length of the plateau at maximum. One can reconstruct this information from the density of data points where $F(I) = 0$ in comparison to other parts of the curve. A more detailed description can be found in [12].

Another method to become independent from timing jitter is a technique called electro-optical imaging. This method also offers the possibility for single pulse measurements. Before interacting with the THz-radiation the laser pulse is chirped, i.e. stretched by sorting its wavelengths in time. In this way the time axis is marked by the corresponding wavelength inside the pulse. For single shot measurements the resulting pulse length has to be longer than the expected bunch length plus some margin for the jitter, typically 10 ps. Each part of the laser pulse interacts with the corresponding part of the THz-pulse. The longitudinal profile can then be obtained using a monochromator.
The achievable time resolution is given by

$$\tau = \sqrt{T_0 \cdot T_c}\,, \qquad (4)$$

with T_0 being the initial length of the laser pulse and T_c the length of the chirped pulse [17]. A measurement of this kind has been successfully performed at FELIX [11].

2.2 Sensitivity

The sensitivity of the setup increases with the thickness of the non-linear crystal. On the other hand dispersion effects inside the material lead to divergence of the pulses in the crystal. So one has to find a trade-off between sensitivity and time resolution. For picosecond THz-pulses generated from biased semiconductors a signal-to-noise ratio of $S/N = 60$ has been demonstrated for single THz-pulses with $E = 1$ kV/cm [1].

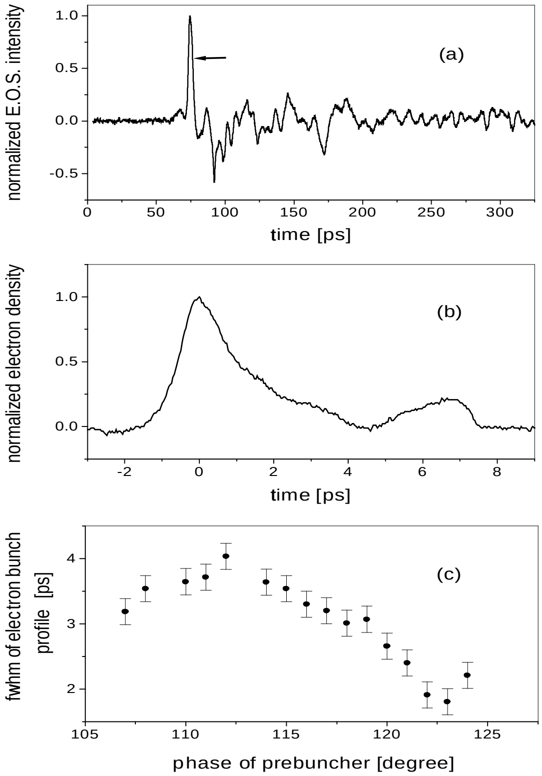

Figure 2: Logitudinal charge profile in the FELIX accelerator measured with the method of electro-optical sampling. The head of the bunch is shown left. The upper part (a) shows the bunch together with wakefields travelling along the beamline. The second part (b) shows the shortest bunch achieved, the third (c) the dependence of the RMS bunch-length on the phase of some buncher cavity in the injector.

At FELIX the sensitivity in the measurement shown in figure 2 was estimated to be $E = 1$ kV/cm with a signal to noise ratio of $S/N = 1$. The maximum field strength was estimated to be 12 kV/cm at the crystal.

3 TOMOGRAPHY

By means of computerized tomography it is possible to reconstruct a multi-dimensional distribution from a series of projections. In case of phase-space tomography a 2-dimensional distribution is reconstructed from 1-dimensional projections. Most of the reconstruction algorithms have been developed under the assumption that the different projections were obtained by rotation of the object. The most popular algorithms require a set of projections that cover 180° for the projection angles. If the projection data do not fulfill this requirement, the reconstruction produces severe artefacts [10], peaks are broadened and streaks are produced. In this way the achievable resolution is dilluted and sometimes the whole reconstruction becomes meaningless.

In a linear accelerator in most cases the only possibility to create different projections of the longitudinal phase space is off-crest acceleration, i.e. acceleration at different phases between bunch and rf wave. But this method does not induce a rotation of the phase space but only a shearing, so that the required rotation of 180° can never be accomplished.

To circumvent this problem at the TESLA Test Facility the reconstruction is done in a different way [6]. By defining the entropy of the phase space distribution f one finds a criterion to minimize artefacts

$$\eta(f) = -\iint_{\mathcal{D}} \mathrm{d}s\mathrm{d}t\, f(s,t)\ln\left(f(s,t)A\right), \qquad (5)$$

where A is the area of $\mathcal{D}$, s the energy and t the time. Every streak or broadening of peaks is additional information to the necessary minimum information. By maximizing the entropy the additional information is suppressed. The distribution with the least additional data that can still reproduce the projections is considered to be the most probable candidate for the real distribution. This can be seen

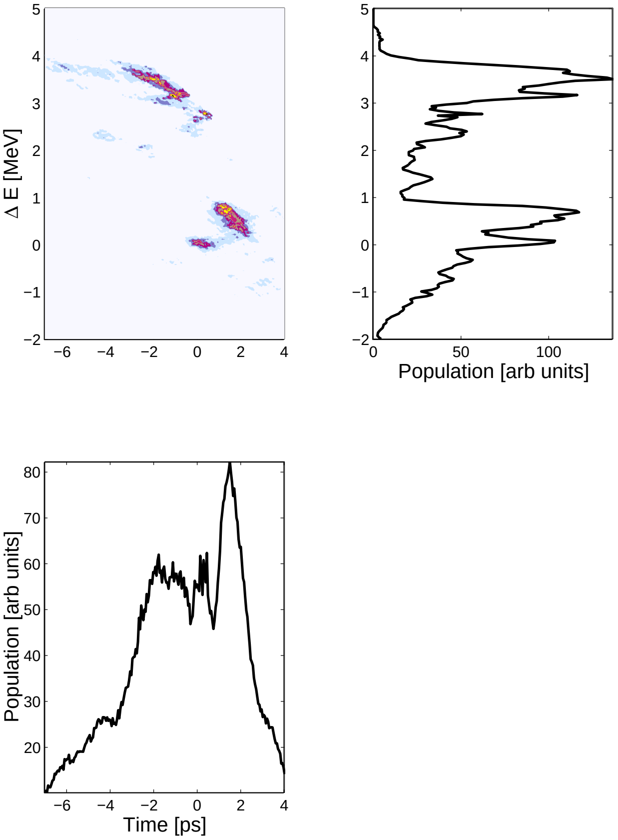

Figure 3: Result from longitudinal tomography in the TESLA Test Facility. In the TTF a splitting of the energy distribution has been observed and the tomography is one of the techniques to study this effect. The upper left part shows the 2-dimensional distribution whilst the upper right and lower left show the projection onto the energy axis respectively the time axis. Note: the head of the bunch is on the right.

in analogy to statistical thermodynamics where the distribution with the least order, i.e. the least information, is realized with highes probability. The corresponding algorithm was named by the author Maximum Entropy Algorithm (MENT) [9][14].

The shearing of the phase space can be written as

$$\triangle s = a_j + b_j \cdot t + c_j \cdot t^2, \tag{6}$$

where a_j is the energy shift of the centroid, b_j is the linear shearing, and c_j takes care for the nonlinear curvature of the rf. Each projection may have its own binning s_{jm}. Let G_{jm}, $j = 1 \ldots J$, $m = 1 \ldots M$ be the contents of the m^{th} bin of the j^{th} projection. Then this data can be calculated from the phase space distribution via

$$G_{jm} = \iint \mathrm{d}s\mathrm{d}t f(s,t)\chi_{jm}(s + a_j + b_j t + c_j t^2), \tag{7}$$

with χ_{jm} being the characteristic function of the interval $[s_{jm}, sjm+1)$

$$\chi_{jm} = \begin{cases} 1, & s_{jm} \le s < s_{jm+1} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The task is now to find a distribution $f(s,t)$ which satisfies all boundary conditions (7) and maximises the entropy (5). This problem can be solved by introducing Lagrange multipliers. After some manipulations the solution is

$$f(s,t) = A^{-1} \prod_j \sum_m H_{jm} \chi_{jm}(s + a_j + b_j t + c_j t^2). \tag{9}$$

The value of H_{jm} is found in an iterative process, the nonlinear Gauss-Seidel method

$$\begin{aligned} H_{jm}^{i+1} &= \frac{A G_{jm}}{\iint \prod_k \sum_n H_{kn}^i \chi_{kn}(s + a_{jk} + b_{jk} t + c_{jk} t^2)}, \\ & \qquad m = 1 \ldots M, \quad j = i \bmod J + 1, \\ H_{jm}^{i+1} &= H_{jm}^i, \quad m = 1 \ldots M, \quad j \neq i \bmod J + 1, \qquad (10) \\ a_{jk} &= a_j - a_k, \quad b_{jk},\, c_{jk} \text{ similar.} \end{aligned}$$

Note that it is not necessary to calculate any logarithm or exponential. Reasonable results can be expected after 3 turns of iteration.

3.1 *Resolution*

The resolution expected depends on the layout of the accelerator and its mode of operation. In the TTF there are two accelerating modules each delivering the same energy gain, by the time of the experiment ≈ 100 MeV. There is a magnetic chicane for bunch compression between the two modules. The goal was to measure the longitudinal phase space at the entrance of the second acceleration module. By various reasons the phase offset of the second module was limited to $\pm 45°$. Together with the operating frequency of 1.3 GHz and the energy gain this results in a shearing parameter

$$b(45°) \approx 570 \text{ keV/ps}. \tag{11}$$

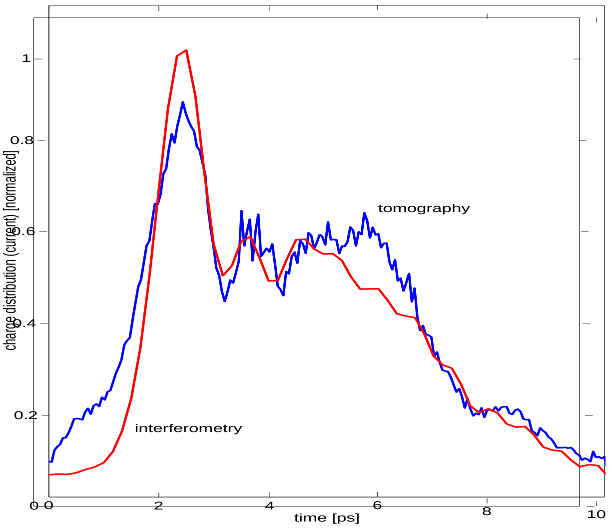

Figure 4: Longitudinal profile as reconstructed with tomography in comparison with a result from CTR interferometry.

The energy resolution of the spectrometer is estimated to be $\delta_E \approx 60$ keV. From these two numbers it is possible to calculate the minimum separation in time one can allow for two gaussian peaks. Due to the shearing the peaks are shifted against each other by $\triangle E = b \cdot \triangle t$ and their variance increases to

$$\sigma'_E = \sqrt{\sigma_E^2 + b^2 \sigma_t^2}. \tag{12}$$

This means a loss of resolution. This implies that two peaks have to be separated by 3σ in time to achieve a separation of 2σ in the projection onto the energy. Then the achievable resolution is

$$\delta_t = \frac{3}{\sqrt{5}b}\delta_E. \tag{13}$$

For the case of the TTF this yields a resolution of $\delta_t = 150$ fs. Tests with the reconstruction algorithm showed that there is no significant contribution from the reconstruction algorithm.

The longitudinal tomography has been implemented in the TESLA Test Facility to study effects in the longitudinal phase space. The figure 3 shows experimental results from a measurement of that kind. It is remarkable that the phase space distribution is broken up especially in energy. Investigations are in progress whether this is caused by short range wakefields or coherent synchrotron radiation. The timeprofile measured with tomography has been compared to the result of interferometry of coherent transition radiation (CTR) analysed with the help of the Kramers-Kronig Relation. The (CTR)-interferometry suffers from a cutoff for low frequencies which had to be extrapolated in the reconstruction. Except for that uncertainty the agreement is good as can be seen in figure 4.

4 CONCLUSION

Diagnostics for bunch length measurements are being developed aiming at resolutions in the 100 fs regime. In this paper two promising techniques are described. First the electro optic methods measuring the fields of the bunches directly. The potential of this method in terms of achievable time resolution has to be explored, the limit is not reached yet by far. The method has the opportunity for single bunch diagnostic with the results delivered online and noninterceptive. It is possible to measure wakefields inside the accelerator as well.

The longitudinal tomography can only be performed with dedicated machine operation but it only relies on standard diagnostics and delivers very high resolution depending on the setting of the accelerator. It can reconstruct the two dimensional longitudinal phase space and therefore offers the possibility to study wakefields, coherent synchrotron radiation, and similar effects directly on the bunches themselves.

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