USE OF OPTICAL TRANSITION RADIATION INTERFEROMETRY FOR ENERGY SPREAD AND DIVERGENCE MEASUREMENTS

R. B. Fiorito and A.G. Shkvarunets, Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, MD 20742, USA

Abstract

OTR interferometry (OTRI) has been shown to be an excellent diagnostic for measuring the rms divergence and emittance of relativistic electron beams when the fractional energy spread $\Delta\gamma/\gamma$ is less than the normalized rms divergence $\sigma = \gamma\theta_{rms}$. This is the case for most beams previously diagnosed with OTRI. To extend this diagnostic capability to beams with larger energy spreads, we have calculated the effects of all the parameters effecting the visibility of OTR interferences, V; i.e. energy spread, angular divergence, the ratio of foil separation to wavelength ratio, d/λ and filter band pass. We have shown that: 1) for a given $\Delta\gamma/\gamma$, the sensitivity of V to σ is proportional to the observation angle θ_0, the fringe order *n* and the ratio d/λ ; 2) the sensitivity of V to $\Delta\gamma/\gamma$ is independent of θ_0 and n but is proportional to d/λ. Thus, by adjusting d/λ, and choosing the appropriate fringe order, one can separate out and measure *both* the energy spread and divergence. However, the filter band pass must decrease with θ_0 and *n*. Results of our calculations will be given for various beams of interest.

INTRODUCTION

A conventional optical transition radiation interferometer [1] consists of two parallel thin foils, oriented at 45 degrees with respect to the electron beam. A charged particle or particle beam produces forward directed OTR from the first foil and backward directed OTR from the second foil, which is usually a mirror. Interferences between the two radiations will be seen near the direction of specular reflection when the distance between the foils, *d* is comparable to the vacuum coherence length, L_V, the distance in which phase (ϕ) of field of the electron and the OTR photon differ by π.

The component of intensity of OTR interferences parallel to the plane of incidence can be written as

$$\frac{d^2I_{\parallel}(\gamma,\lambda,\theta)}{d\omega d\Omega} = \frac{4\alpha}{\pi^2\omega}\frac{{\theta_x}^2}{(\gamma^{-2}+\theta^2)}\left|1-e^{i\phi}\right|, \qquad (1)$$

where $\phi = d/L_V$ and $L_V(\theta,\gamma,\lambda) = (\lambda/\pi)((\gamma^{-2}+\theta^2)^{-1}$,

α is the fine structure constant, γ is the Lorentz factor of the beam, λ is the observed wavelength, $\theta = (\theta_x^2+\theta_y^2)^{1/2}$ is the observation angle measured with respect to the direction of specular reflection, θ_x is the x component of θ projected onto a plane perpendicular to the direction of specular reflection and ω is the angular frequency of the observed photon.

We have developed a useful rms beam divergence diagnostic using OTR interferences [2]. In this method one focuses the beam to a waist. Then the visibility of the interference pattern is primarily a function of the rms beam divergence, θ_{rms} provided that the energy spread of the beam is less than the normalized divergence, $\sigma = \gamma\,\theta_{rms}$. This is the usual case observed for high energy beams. Also, if one observes perpendicular and parallel polarized OTR interferences, i.e. by inserting a variable polarizer into the optical path, the corresponding components of the divergence can be measured.

Since the visibility is also affected by the optical bandwidth of the observation, one must take care to make this narrow enough so that the change in visibility due to the divergence can be seen. The inter foil distance and the filter band pass must be designed to produce the required number and spacing of the fringes for a given range of divergence. This is typically done with the aid of a computer code, which calculates the OTR interferences, for example, Eq. (1) convolved with a Gaussian distribution of particle trajectory angles and a given filter transmission function. The fit of measured to calculated interferences give the beam energy via the position of the fringes to about 1% precision and the rms divergence to about 10 % precision [2].

Recent theoretical studies have considered use of OTR from a stack of multiple dielectric foils as a beam energy distribution diagnostic [3]. In this study the effect of divergence is neglected. We are currently planning to measure divergence and transverse emittance of a low energy beam (8 MeV) where nonlinear space charge forces create a large energy spread on the beam (1- 10 %), and the normalized divergence $\sigma=\gamma\theta_{rms} \sim 0.03$, which is comparable to the energy spread. For this application a careful analysis of the effects of all beam and measurement parameters and a strategy for separating out the effects of divergence and energy spread is required.

APPROACH

Our approach to this problem is twofold:

1. Calculate of the effect of variations of beam angle (divergence), energy spread and wavelength (filter band pass) on the difference in phase between OTR photons produce at each foil. It is this phase that determines the fringe visibility and corresponding capability of OTR interferometry (OTRI) to diagnose beam parameters with good sensitivity and precision.
2. Develop estimates and computer codes which convolve variation effects with OTRI patterns to determine and define parameter ranges needed to separate out effects of energy spread and beam divergence.

Calculation of the phase and its variation

To illustrate our method we will calculate the phase of forward directed photons produced at each foil of an OTR

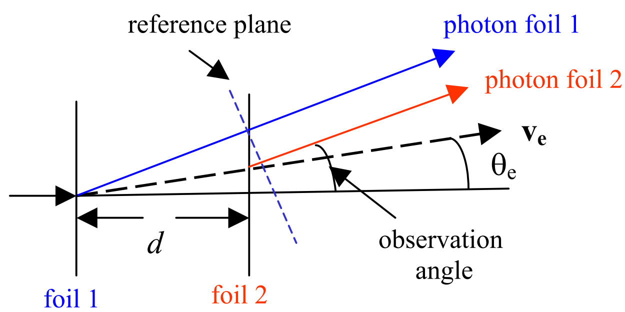

Figure 1: Diagram of an ideal forward directed OTR interferometer.

interferometer observed *in the plane of incidence*, i. e. the plane formed by the average velocity vector of the beam and the observation direction as shown by the arrows in Figure 1. A similar analysis pertains to forward-backward OTRI, and can be extended out of the plane of incidence.

The phases of the OTR photons generated at foils 1 and 2 are given respectively by:

$$\begin{aligned}\psi_1 &= kd/\cos\theta_x \\ \psi_2 &= kd/(\beta\cos\theta_e) + kd(\tan\theta_x - \tan\theta_{ex})\sin\theta_x\end{aligned} \qquad (2)$$

where k is the wave number of the photon, d is distance between the foils, θ_x is the x component of the observation angle in the plane of incidence, θ_e is the trajectory angle of the electron, and θ_{ex} is the x component of this angle. In the limit of small angles and relativistic energies, the phase difference at the interference maxima is given by $\Delta\psi = 2n\pi$ or:

$$(\gamma^2 - 1)^{1/2}/\gamma + \theta_x^{\,2} - \theta_x \cdot \theta_e - 1 = n\lambda/d \qquad (3)$$

where n is the fringe order and λ is the observed wave-length. The total variation of Eq. (3) gives:

$$\Delta\gamma/\gamma^3 + \theta_x\Delta\theta_{xe} - (2\theta_x - \theta_{xe})\Delta\theta_x + \Delta\lambda n/d + \lambda\Delta n/d = 0 \qquad (4)$$

where the term $\Delta\xi$ refers to a finite variation or spread in the associated variable ξ.

We choose $\Delta\theta_x = 0$, i.e. a fixed angle of observation; and $\theta_e = 0$, i.e. the beam direction along the axis. Note that if $\Delta n = 0$, the fringes are 100% visible and if $\Delta n \sim 1$, they are washed out (0% visibility). We estimate that $\Delta n = 0.5$ as the value of highest sensitivity to a given variable variation. Then Eq. (4) becomes:

$$\Delta n = 0.5 = (d/\lambda)[\Delta\gamma/\gamma^3 + \theta_x\Delta\theta_e] + n\Delta\lambda/\lambda \qquad (5)$$

From this fundamental equation we can estimate how variations in energy spread $\Delta\gamma$, angular divergence, $\Delta\theta_e$ and filter bandwidth, $\Delta\lambda$ affect the visibility of OTR interference fringes.

By setting the variation of two of the three variables on the right hand side of Eq. (5) equal to zero, we can determine the dependence of the remaining variation on θ_x, γ, λ and d. For example, if we set the variations $\Delta\theta_e = \Delta\lambda = 0$, and fix d/λ, we conclude that the effect of energy spread, $\Delta\gamma$ on the fringe visibility decreases as γ^{-3} but is independent of θ_x. Similarly if $\Delta\gamma$ and $\Delta\lambda$ are neglected, the effect of $\Delta\theta_e$ on the visibility is proportional to θ_x but is independent of the energy, γ. Also, the effect of bandwidth is proportional to the fringe order n but is independent of the beam energy.

These dependences can be used to advantage to diagnose either the energy spread or the divergence. However, control of the bandwidth is necessary for both diagnostics. We can experimentally adjust d and λ to optimize the number of fringes for a given range of divergence or energy spread.

Table I. Design Parameters for OTR Interferometer

E MeV	$\Delta E/E$	d mm	$\Delta\theta_e$ mrad	$\Delta\lambda/\lambda$	Δn	$\gamma\theta_x$
95	0	25.4	0.6	0	0.25 0.75	1 3
95	0	25.4	0	0.11	0.13 1.17	1 3
95	< 0.1	25.4	0	0	<0.1	
50	0.003	1000	0	0	0.5	
50	0	162	0.2	0	0.5	1
50	0	54	0.2	0	0.5	3
50	0	54	0	0.007	0.5	3
8	0.01	9.5	0	0	0.5	1
8	0	9.5	0	0.006	0.5	1
8	0	1.5	1.8	0	0.5	0.5
8	0	1.5	0	0.04	0.5	0.5

RESULTS OF VARIATIONAL ANALYSIS

Consider a measurement of the energy spread $\Delta\gamma$. The analysis given above shows that lower order fringes, which are minimally affected by $\Delta\lambda$ and $\Delta\theta$, are most useful to measure $\Delta\gamma$. On the other hand for maximal sensitivity to divergence it is useful to employ higher order fringes.

Estimated design parameters for an OTR interferometer are presented in Table I. for various beam energies, energy spreads and divergences. We include parameters for 95 MeV for which the divergence was actually measured.

COMPUTER CALCULATIONS

We have developed a series of computer codes, which convolve the OTR intensity, e.g. Eq. (1) with energy, angle and filter distribution functions. The results are shown in Figures 2-5 for a fixed beam energy, E= 8 MeV. Parameters d and λ are chosen to optimize the fringe pattern for a measurement of either energy spread or divergence.

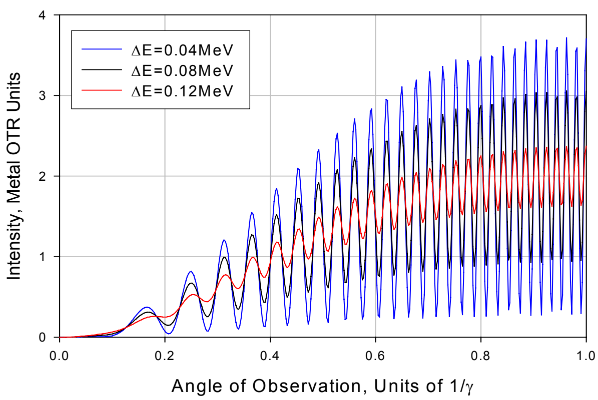

Figure 2: Effect of energy spread on OTR interferences; λ = 650nm, $\Delta\lambda$ = 0, $\Delta\theta_e$ = 0, d = 10 mm.

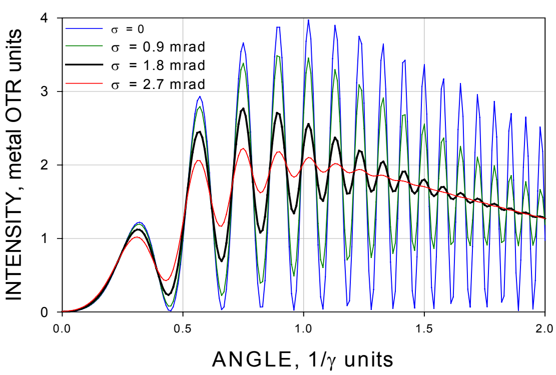

Figure 3: Effect of divergence on OTR interferences; λ = 650nm, $\Delta\lambda$ = 0, ΔE = 0, d = 1.5 mm.

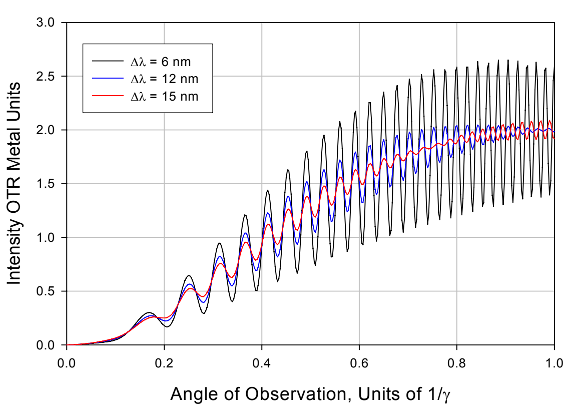

Figure 4: Effect of bandpass on energy spread measurement; λ=650nm, ΔE=0.08MeV, $\Delta\theta_e$=0, d= 10mm.

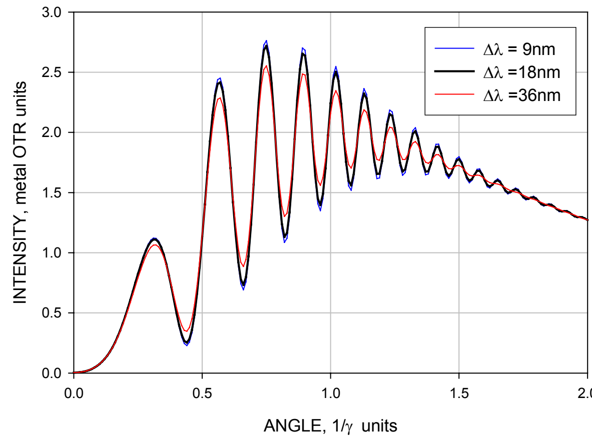

Figure 5: Effect of bandpass on divergence measurement; λ = 650nm, $\Delta\theta_e$ = 1.8 mrad, ΔE = 0 MeV, d= 1.5 mm.

Figure 2. e.g. shows that effect of the energy spread on fringe visibility is independent of angle. Comparatively, Figure 3. shows the angular dependence of the visibility for different beam divergences, i.e. that a larger divergence washes out the fringes at higher orders more rapidly than a smaller divergence. Figure 4. shows the effect of band pass on energy spread measurements and that lower fringe orders should be used to measure energy spread to avoid the effect of $\Delta\lambda$. Similarly Figure 5. shows the effect of band pass on divergence measurements.

CONCLUSIONS

Variational analysis has resulted in a number of simple conclusions affecting the ability of OTRI to measure beam divergence and energy spread: (a) the effect of energy spread on OTRI is independent of observation angle and the fringe order and falls off as γ^{-3} ; (b) the effect of divergence is proportional to the observation angle; (c) the effect of bandwidth is proportional to the fringe order; (d) the scaling of energy spread and divergence effects depends on d and λ, which can both be adjusted for diagnostic purposes.

The above results have been verified by more exact analysis using computer calculations and are further verified by experimental data.

We are using these results to design OTRI diagnostics to separate out and measure both energy spread and divergence.

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