## TOWARDS A ROBUST PHASE LOCKED LOOP TUNE FEEDBACK SYSTEM

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#### Abstract

Attempts to introduce a reliable tune feedback loop at RHIC (BNL) [1] have been thwarted by two main problems, namely transition crossing and betatron coupling. The problem of transition crossing is a dynamic range problem, resulting from the increase in the revolution content of the observed signal as the bunch length becomes short and from the fast orbit changes that occur during transition. The dynamic range issue is being addressed by the development of a baseband tune measurement system [2] as part of the US LHC Accelerator Research Program (US-LARP). This paper will focus on the second problem, showing how a phase locked loop (PLL) tune measurement system can be used to continuously measure global betatron coupling, and in so doing allow for robust tune measurement and feedback in the presence of coupling.

#### **INTRODUCTION**

There are two main difficulties associated with utilizing a PLL tune measurement and feedback system in the presence of coupling. The first arises from the fact that in a coupled machine the excitation from one plane shows up in the other. A PLL therefore has the possibility to become confused regarding which signal is associated with a given measurement plane, which causes problems for both PLL measurement and tune feedback.

The second difficulty arises when a tune feedback system tries to maintain the tunes at their 'set' tune values in the presence of coupling. When the coupling amplitude becomes larger than the difference in the unperturbed tunes (the value of the tunes for a completely decoupled machine), then no amount of quadrupole adjustment can diminish this minimum tune split and restore the tunes to their desired 'set' values. This again leads to a breakdown in the tune feedback loop.

As these obstacles to tune measurement and feedback became evident at RHIC, the need for improved coupling measurement became clear, and the PLL was reconfigured to permit measurement of the projections of both eigenmodes in both planes. The excellent quality of the data obtained by this method motivated the development of a proper formalism [3, 4] for its interpretation.

## MEASUREMENT OF COUPLING PARAMETERS USING A PLL TUNE TRACKER

This section will discuss the use of a phase locked loop tune tracker to measure the betatron coupling amplitude and phase.



Figure 1: Schematics showing the two eigenmodes rotated with respect to the horizontal and vertical planes due to coupling.

### Equations of Interest

For a linearly coupled circular accelerator the observed displacement on turn n in the horizontal (x) and vertical (y) planes are a combination of the projections of two eigenmodes (see e.g. [3]). This is illustrated in Fig. 1 and can be expressed as

$$\begin{cases} x(n) = A_{1,x} \cos(2\pi Q_1 n + \phi_{1,x}) + A_{2,x} \cos(2\pi Q_2 n + \phi_{2,x}) \\ y(n) = A_{1,y} \cos(2\pi Q_1 n + \phi_{1,y}) + A_{2,y} \cos(2\pi Q_2 n + \phi_{2,y}) \end{cases}$$
(1)

The eigenmode frequency of Mode 1 is denoted by Q1, while  $A_{1,x}$  and  $A_{1,y}$  represent the amplitudes of this mode in the horizontal and vertical plane respectively. Similarly  $\phi_{1,x}$  and  $\phi_{1,y}$  represent the phases of this mode in the horizontal and vertical plane respectively. The same notation applies for Mode 2.

Using Hamiltonian perturbation theory in the absence of intentionally strong local couplers, it is possible to define the following relationships [3]

$$r_{1} = \frac{A_{1,y}}{A_{1,x}} = \sqrt{\frac{\beta_{y}}{\beta_{x}}} \cdot \frac{|C^{-}|}{2\nu + \Delta}, \begin{cases} \Delta \phi_{1} = \phi_{1,y} - \phi_{1,x} = \chi \\ \Delta \phi_{2} = \phi_{2,x} - \phi_{2,y} = \pm \pi - \chi \end{cases}$$
(2)  
$$r_{2} = \frac{A_{2,x}}{A_{2,y}} = \sqrt{\frac{\beta_{x}}{\beta_{y}}} \cdot \frac{|C^{-}|}{2\nu + \Delta}, \end{cases}$$

Here  $\Delta$  is the difference between the fractional part of the unperturbed tunes,  $\beta_{x,y}$  the beta functions at the

observation location,  $|C^-|$  and  $\chi$  the coupling amplitude and phase respectively and

$$\nu = \frac{1}{2}\sqrt{\Delta^2 + \left|C^-\right|}$$

It is also possible to write the following relations for the eigenmode frequencies,  $Q_1$  and  $Q_2$ 

$$\begin{cases} Q_1 = Q_{x,0} - \frac{1}{2}\Delta + \frac{1}{2}\sqrt{\Delta^2 + |C^-|^2} \\ Q_2 = Q_{y,0} + \frac{1}{2}\Delta - \frac{1}{2}\sqrt{\Delta^2 + |C^-|^2} \end{cases}$$
(3)

Solving for  $\Delta$  and |C| one obtains

$$\left|C^{-}\right| = \frac{2\sqrt{r_{1}r_{2}}\left|Q_{1}-Q_{2}\right|}{\left(1+r_{1}r_{2}\right)} \quad , \qquad \Delta = \frac{\left|Q_{1}-Q_{2}\right|\left(1-r_{1}r_{2}\right)}{\left(1+r_{1}r_{2}\right)} \quad (4)$$

which are independent of the beta functions at the observation location.

# Configuring a PLL tune tracker to measure betatron coupling parameters

In the classical PLL tune tracker implementation, the horizontal plane is configured to track one eigenmode (assumed to be eigenmode 1), while the vertical tracks the other (eigenmode 2). If, instead, the vertical detection frequency is forced to be the same as that of the horizontal, then the vertical acquisition chain will observe the projection of eigenmode 1 in the vertical plane (see Fig. 2). By duplicating the number of channels the same principle can be applied to eigenmode 2.

The amplitude and phase measured by each of these channels are then the same amplitudes and phases described in Eq. (1). Since the PLL kicker is the same for a given eigenmode, its gain and the beta-function at its location do not have to be taken into account when computing the ratios  $r_1 \& r_2$  of Eq. 2. This is not true for the receiver gains, which can be different for each channel.



Figure 2: Schematic representation of the RHIC PLL system as modified for coupling measurement

When the PLL is locked, to first order  $\phi_{1,x} = \phi_{2,y} = 0$  and hence the coupling phase,  $\chi$ , is simply given by  $\phi_{1,y}$  (or  $\pm \pi$ -  $\phi_{2,y}$ ). The coupling amplitude,  $|C^-|$ , and unperturbed tune difference,  $\Delta$ , can be obtained using Eqs. (4). In addition the unperturbed tune values  $Q_{x,0}$  and  $Q_{y,0}$  can be calculated using Eqs. (3). Hence by reconfiguring the PLL as shown in Fig. 2 it is possible to provide all the necessary information to measure coupling [4]

## RESULTS OF COUPLING MEASUREMENTS USING A PLL TUNE TRACKER AT RHIC

To perform continuous coupling measurements during the acceleration ramp, the RHIC PLL tune tracker was configured as shown in Fig. 2. Since the measurement required doubling the number of acquisition channels it could only be performed for one ring. The yellow ring PLL was therefore used in its standard configuration to acquire eigenmode 1 (horizontal) and eigenmode 2 (vertical), while the blue PLL system was used to



Figure 3: Continuous coupling amplitude measurement using the PLL tune tracker during a RHIC ramp.

acquire the projection of yellow eigenmode 1 in the vertical and that of yellow eigenmode 2 in the horizontal.

Fig. 3 shows the results of one such measurement taken during Cu run number 6280. The coupling is seen to be well adjusted during injection, but becomes large near transition and again towards the end of the ramp. When considering only the usual PLL tune data (eigenmodes 1 and 2) the tune would seem to be well adjusted during the early part of the ramp. Looking at the unperturbed tunes  $(Q_{x,0} \text{ and } Q_{y,0})$  as calculated from |C| and  $\Delta$  using Eq. (3), one can clearly see that this is not the case. The unpertubed tunes actually cross during this time, something which was confirmed by the kicked tune measurement system (ARTUS) from a similar, earlier ramp (the kicked tune system was left off during ramp 6280 to minimize beam loss due to emittance blowup). This is a good example of where the PLL will continue to track a given eigenmode, even though its major projection is now in the other plane. Incidentally, this leads to a negative  $\Delta$  at these locations, since the amplitude ratios r<sub>1</sub> and  $r_2$  become larger than 1 due to the fact that the PLL is now tracking the eigenmode with the smallest projection amplitude in both planes. In addition, if the tune feedback loop were closed, it would become unstable in this coupling condition.

## MAKING A PLL TUNE TRACKER ROBUST IN THE PRESENCE OF BETATRON COUPLING

By tracking a single eigenmode in each plane, as is typically done by most PLL tune measurement systems, it is very easy for the PLL to become confused when the unperturbed tunes are close together and the coupling amplitude becomes large. This can lead to the PLL in both planes tracking the same eigenmode, or with one of the planes losing lock altogether.

The addition of the information related to the projection of the excitation in the other plane allows the PLL to know the state of its current eigenmode with respect to that being tracked by the other plane. This means that if the unperturbed tunes cross, then the horizontal and vertical PLLs can be forced to change from one eigenmode to the other. In this way they keep track of the eigenmode with the largest amplitude projection in their respective planes, reducing the chance of losing lock should the coupling amplitude decrease.

As was shown in the previous section, if a robust tune tracker can be successfully implemented during the ramp, then coupling is automatically measured. Taking this one step further one could imagine performing direct coupling feedback using the parameters measured by the PLL.

## MAKING PLL TUNE FEEDBACK ROBUST IN THE PRESENCE OF BETATRON COUPLING

It has been shown how it is possible to configure a PLL tune tracker to continuously measure the coupling parameters. By knowing  $|C^{-}|$  and  $\Delta$  and the two eigenmode frequencies  $Q_1$  and  $Q_2$  it is possible to determine the unperturbed tunes,  $Q_{x,0}$  and  $Q_{y,0}$ , that would be measured in the absence of betatron coupling. The nice thing about the unperturbed tunes is that they remain constant for any value of the coupling amplitude. This means that these values could be used in a tune feedback loop without giving rise to the problems encountered when feeding back on the eigenmode frequencies.

In practice this would imply that the set tunes requested by the tune feedback loop would be the unperturbed tunes rather than the actual oscillation frequencies (eigenmodes) undertaken by the beam. The feedback loop would therefore be stable in the presence of coupling, but would not prevent the beam from oscillating on or near resonances were coupling to become large. By performing tune feedback in this manner, however, the problems of coupling and tune correction become two separate issues rather than being interdependent.

### CONCLUSIONS

Results from RHIC have shown that continuous betatron coupling measurements can be obtained throughout the acceleration ramp by appropriately configuring a PLL tune tracker. Not only does this provide the coupling amplitude, but also the coupling phase, both of which are required for successful coupling correction. The ability to measure the coupling parameters gives a PLL tune tracker two added advantages: the ability to track tunes in the presence of betatron coupling, and the ability to provide the necessary quantities for an 'unperturbed' tune feedback system in the presence of betatron coupling. All of this greatly enhances the power of a PLL tune measurement system, and points the way to a robust tune feedback system for RHIC and future hadron machines.

So far the effect of local coupling on the measurements has not been studied, but future beam experiments will attempt to verify that the observed quantities do indeed allow for global betatron coupling correction or eventual feedback.

#### REFERENCES

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