DIAGNOSTICS OF THE WAVEFORM OF PICOSECOND ELECTRON BUNCHES USING THE ANGULAR DISTRIBUTION OF COHERENT SUB-MM TRANSITION AND DIFFRACTION RADIATION*

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Abstract

The spectra of sub-mm wavelength coherent transition radiation (TR) and diffraction radiation (DR) have previously been used to measure the bunch length of picosecond electron beam pulses. However, both the spectral and angular distributions of the radiation from a finite target with dimension r, are strong functions of the wavelength, when $\lambda \approx 2\pi r/\gamma$ where γ is the relativistic factor of the beam. This dependence must be taken into account to determine the bunch form factor and bunch shape. Also the spectral density of the bunch is a strong function of wavelength when $\lambda \approx d$, the characteristic length of the bunch. When both the above conditions are fulfilled, i.e. $\lambda \approx 2\pi r/\gamma \approx d$, the angular distribution (AD) of the radiation is very sensitive to the longitudinal distribution of the bunch. We are investigating the use of the AD, rather than the spectrum of TR or DR, to diagnose the bunch length and shape. Here we present a comparison of measured and calculated angular distributions (projected on the plane of observation) from two targets: a solid rectangle and a rectangular slit, which we have used to determine the bunch length of PSI's SLS pre-injector LINAC for two different beam tunes.

THEORETICAL BACKGROUND

TR from a radiator whose dimensions are close to the radiation impact parameter $\gamma\lambda$ is closely related to DR via Babinet's principle [1]. We have calculated the spectral angular intensities of coherent TR from a finite size solid rectangular radiator and coherent DR from a rectangular radiator with a rectangular slit.

The TR and DR radiation fields can be calculated by assuming that, the source of these fields are the fields induced by the electron on the solid surface of the radiator. In cylindrical coordinates the longitudinal Fourier components of the electric field of a relativistic electron in free space can be written as:

$$E_r(r,\varphi,z,\omega) = E(r,\omega)\exp(i\omega z/V) \qquad (1)$$

where $E(r,\omega) = e\alpha K_1(\alpha r)/\pi V$, $K_1(\alpha r)$ is the first order MacDonald function, e is the charge of the electron $\alpha = \omega/V\gamma$ and the velocity of the electron V is parallel to the axes z [2].

We will assume that the induced surface fields radiate to the vacuum and that the field radiated into free space can be found using the Huygens-Fresnel principle. Thus, the components of the electric field parallel and perpendicular to the (x,z) or horizontal plane at the observation point p are given by:

$$E_{P,\perp}(p) = \frac{k}{2\pi i}\int_{S'} \frac{a_{P,\perp}\cos\nu\cdot\exp(ikR'))}{R'}dS' \qquad (2)$$

Where $a_P = E(r',\omega)\cdot\exp(ikz'/\beta)\cdot cos\varphi'$ and $a_\perp = E(r',\omega)\cdot\exp(ikz'/\beta)\cdot\sin\varphi'$ are the complex amplitudes of the components of the electron electric field on the tilted surface S', R' is the distance from surface element dS' to the observation point p and r',φ',z' are the coordinates of the surface element $dS' = r'dr'd\varphi'/\cos\nu$, ν is a tilt angle of the surface, $k=\omega/c$ is the modulus of the wave vector, ω is the frequency of the radiation and c is the speed of light in vacuum [3].

We assume that at moderate distance from the radiator where $R' >> \sqrt{S'}$ and $kR' >> 1$ the radiation does not differ from a spherical wave and the field can be found using Eq. (2). In this work we calculate the integral in Eq.(2) numerically. The spectral energy density at the point p on a flat observation plane, averaged over the period of oscillation is given by:

$$J_{P,\perp}(\omega,p) \propto \left|E_{P,\perp}(\omega,p)\right|^2\cos\vartheta \qquad (3)$$

where ϑ is the angle of incidence of the spherical wave on the observation plane. For each target the code calculates a set of 100 distributions in the relevant frequency band (25 - 2525 GHz), where coherent emission of radiation is expected from a short electron bunch. Some of these are shown in Fig. 1 for CDR.

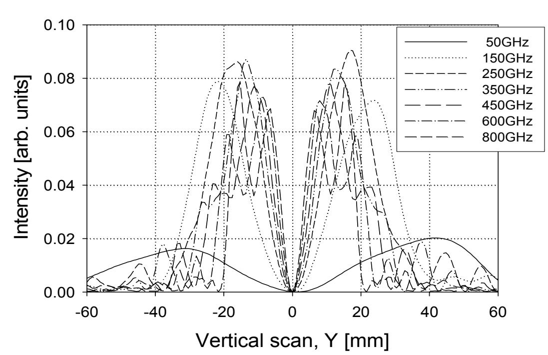

Figure 1: Spectral energy density (projected angular distribution) of CDR for various frequencies.

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Calculation of CDR intensity for a bunch

Assume that a longitudinally modulated electron beam hits the target and radiation of each particle has the same distribution of the Fourier component of the field $E(\omega, p)$ with the same initial phase at the moment of impact. In this case the total radiation field of the bunch of electrons is

$$E_\rho(\omega, p) = E(\omega, p)\int \rho(z)\exp(ikz/\beta)dz \qquad (4)$$

where $\rho(z)$ is longitudinal distribution of the bunch. The spectral density is then given by

$$J_\rho(\omega, p) = J(\omega, p)K(\omega) \qquad (5)$$

$$K(\omega) = \left|\int_{z1}^{z2} \rho(z)\exp(i\omega z/V)dz\right|^2 \qquad (6)$$

where $K(\omega)$ is the longitudinal form factor of the beam. The energy density in the spectral interval (ω_1, ω_2) is given by

$$W(p) = \int_{\omega 1}^{\omega 2} J(\omega, p)K(\omega)d\omega \qquad (7)$$

In the case of a "finite" target, and/or within the "near field" of the radiation [3], the intensity *J* is a function of frequency. As the result, the energy density Eq. (7) is a function of the form factor of the beam. By comparing the measured and calculated distributions one can determine how close the model predicts the real waveform of the beam. In this work we use a single Gaussian distribution to model the waveform of the bunch, i.e. $\rho(z) = \rho_0 \exp(-z^2/2\sigma^2)$, where $\sigma = 0.425L$ and *L* is the full width at half maximum; the pulse duration $T = LV$. For each *L* or *T* the code calculates the form factor and the energy density and compares the measured (7) and calculated energy distributions. Typical distributions for the CDR target integrated over the frequency band 25 – 2525 GHz are shown in Fig.2.

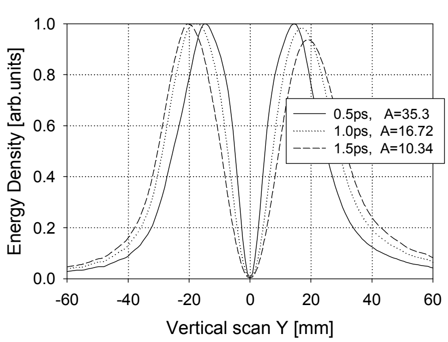

Figure 2: Energy density distribution of CDR for various pulse widths; A is the amplitude for each case.

One can see that the shorter is the pulse, the narrower and more intensive is the distribution.

EXPERIMENTAL METHOD

Measurements of the transverse emission distribution of CDR and CTR were performed at the ALIDI-SM-5 optical monitor station behind the 100 MeV pre-injector LINAC of the Swiss Light Source (SLS). While the LINAC is usually running in the top-up operation mode of the SLS, it was set-up to a "short bunch mode" for the CDR-CTR measurements. Single-shot electro-optical bunch length measurements at the same optical monitor station have resulted in bunch lengths between σ_{PBU-0} = 0.75 ps and σ_{PBU+3} = 1 ps, depending on the setting of the pre-buncher phase [4]. Here, PBU-0 corresponds to the optimum phase setting for shortest bunches and PBU+3 corresponds to a 3° phase offset from this optimum setting. Results of the CDR-CTR measurements shown in the final paragraph of this paper have been obtained with comparable LINAC settings.

Optical Monitor and Measurement Set-Up

A specific radiator has been designed for the measurements at the ALIDI-SM-5 optical monitor station providing well defined emission characteristics for CDR and CTR. At its upper position, the radiator houses a YAG:Ce scintillation screen, which is typically used for beam profile and emittance measurements. The CTR radiator is a plane and polished Al surface (λ/10 at 630 nm) and the CDR radiator consists of a 10 mm rectangular slit within the polished Al surface. All edges are chamfered to avoid diffraction. A mechanical drawing of the radiator is shown on the right side of Fig. 3.

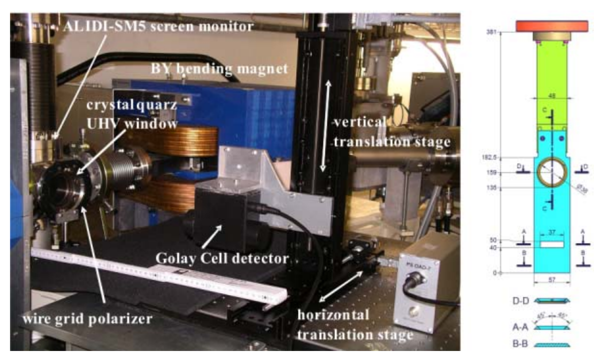

Figure 3: Experimental set-up at ALIDI-SM-5 optical monitor station (left), CTR-CDR radiator (right)

The radiator was mounted in the UHV chamber under 45° so that the backward CDR and CTR is reflected out of the UHV system through z-cut crystal quartz window, which provides good transmission above 100 μm (3 THz) wavelength. The 100 MeV electron beam has been focused on the radiator and the vertical radiator positions could be adjusted by a motorized linear vacuum feedthrough with a precision of better than 10 μm. At a distance of 430 mm from the radiator, a Golay cell detector was scanned horizontally and vertically by two motorized linear stages. The polarization of the radiation was selected by a wire grid polarizer, which was placed directly behind the UHV window. A photograph of the set-up is shown on the left side of Fig. 3.

DATA FITTING PROCEDURE

The goal of our fitting procedure is to achieve the best value of *L* which provides a 'best fit' of theory and experiment. The essence of this procedure is to scale the calculated scan *T(y)* by a constant *A* until the best fit of the theoretical scan to the data scan *E(y)* is obtained. To do this we have written a code to compare the similarity of the two functions $A \cdot T(y) > 0$ and $E(y) > 0$ in the interval (y_1, y_2) defined in terms of integral deviation defined as:

$$D(A) = \left[(y_2 - y_1)^{-1} \int_{y_1}^{y_2} \left(\frac{A \cdot T(y) - E(y)}{A \cdot T(y) + E(y)} \right)^2 dy \right]^{1/2} \quad (8)$$

The closer the functions $A \cdot T(y)$ and $E(y)$ are to each other, the smaller is the value of *D*. We define the maximum similarity (*RMS*) of the two functions when *D(A)* is at its minimum value, as $RMS = D(A_{\min})$. For each *L* the code calculates value of $RMS(L)$. Minimum value of $RMS(L)$ corresponds to the best fit of theoretical and experimental distributions and corresponding value of *L* is the best fit of the model to the width of the bunch.

RESULTS AND ANALYSIS

The result of our fitting procedure for the case of beam tune PBU-0 for CTR and CDR are shown in Fig. 4 and Fig. 5. Here the finite aperture of detector (~10mm in diameter) is taken into account. Fig. 4 shows the energy distributions for the best fit of theory to the experimental data. Calculations using Eq. 8 shows that the minimum RMS integral deviation (5% and 8.6%) occurs at a value of bunch widths T=0.69 and 0.78 ps for the CTR and CDR data respectively (Figs 4 and 5).

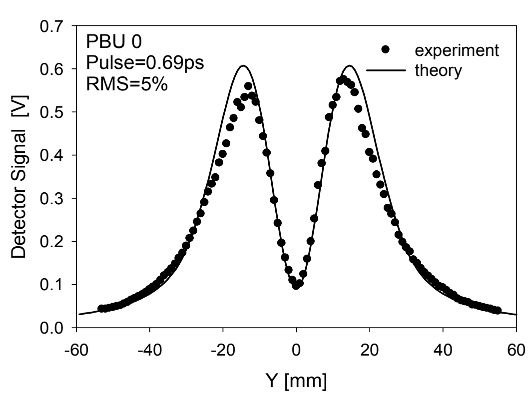

Figure.4: Energy distribution of CTR for tune PBU-0.

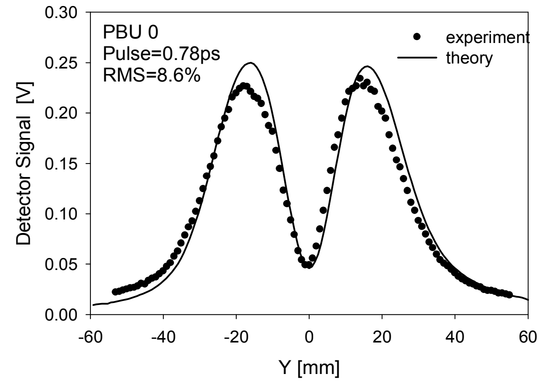

Figure 5: Energy distribution of CDR for tune PBU0.

Table I: Fitted Parameters

Target	Tune	T(ps)	RMS,%
CTR	PBU-0	0.69	5
CTR	PBU+3	1.1	9.1
CDR	PBU-0	0.78	8.6
CDR	PBU+3	1.07	10.7

Table I. shows the fitted bunch lengths *T*, for CTR and CDR targets for both beam tunes PBU-0 and PBU+3. There is excellent agreement for bunch sizes determined using the two radiation sources.

CONCLUSIONS

We have used the angular distributions of CTR from a finite rectangular solid and CDR from a rectangular slit to determine the beam bunch length of the SLS pre-injector LINAC for two different beam tunes. Both types of radiation produce the same results for each tune and show that the angular distribution alone can provide the bunch size without the need for spectral or interferometric measurements. If a linear or 2D array is used to obtain the data, a single shot determination of bunch size may be possible with this method providing the potential for an online diagnostics of bunch length and bunching along an X-FEL LINAC.

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