

# TRANSVERSE BUNCH-BY-BUNCH FEEDBACK FOR THE VEPP-4M ELECTRON-POSITRON COLLIDER

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## Abstract

Transverse mode coupling instability (TMCI or fast head-tail) is the principal beam current limitation of the VEPP-4M electron-positron collider. For the high-energy physics experiments at the 5.5 GeV energy, the VEPP-4M bunch current should exceed much the TMCI threshold. To suppress transverse beam instabilities, a broadband bunch-by-bunch digital feedback system is developed. The feedback concept is described, the system layout and first beam measurements are presented.

## HEAD-TAIL EFFECTS

For high-energy physics experiments in the 5.2-5.5 GeV energy range, design value of the VEPP-4M beam current is 40 mA per bunch in  $2e^- \times 2e^+$ -bunch operation mode. At the injection energy of  $E = 1.8$  GeV, the beam current is limited by the vertical transverse mode coupling instability (TMCI or fast head-tail) [1]. There is an approximate formula for the TMCI threshold current derived using a two-particle model [2]:

$$I_{tmci} = \frac{\sigma_z}{\sqrt{2\pi R}} \frac{8\pi \frac{E}{e} \nu_s}{\sum_k \Im Z_{\perp k} \beta_k}, \quad (1)$$

where  $\sigma_z$  is the r.m.s. bunch length,  $R$  is the average machine radius,  $\nu_s$  is the synchrotron tune, and  $\sum_k \Im Z_{\perp k} \beta_k$  is the beta-weighted broad-band reactive impedance of the ring. For the VEPP-4M at the injection energy, the threshold current is 10-12 mA.

If a machine chromaticity  $\xi = \frac{\delta \nu_\beta}{\delta p/p}$  is non-zero, the chromatic head-tail effect appears, and some oscillation modes become unstable for any beam current, and the current threshold can result from radiation damping only. For the chromatic head-tail, an increment/decrement of the coherent oscillation mode ( $1/\tau_+$ ) and incoherent one ( $1/\tau_-$ ) is expressed as [2]:

$$\frac{1}{\tau_{\pm}} = \mp \frac{I_b c R}{16\pi \nu_\beta \frac{E}{e} \sigma_z} \Im [Z_{\perp} f(2\chi)], \quad (2)$$

where  $\nu_\beta$  is the betatron tune,  $f(2\chi)$  is the complex function  $f(u) = \int_0^\pi e^{iu \sin x} dx$  of the head-tail phase

$$\chi = \frac{\xi \sigma_z}{\alpha R}, \quad (3)$$

which is a betatron phase advance caused by the chromaticity during a half-period of synchrotron oscillation (from head to tail). The coherent mode (center of mass oscillation) is damped if  $\xi > 0$  and anti-damped if  $\xi < 0$  (for a

positive momentum compaction  $\alpha$ ), whereas for the incoherent modes (beam size) the effect is vice versa.

## FEEDBACK THEORY

As it follows from (2), positive chromaticity suppress the coherent oscillation mode, i.e. makes bunch center of mass stable, but other oscillation modes are unstable, negative chromaticity has the inverse effect.

A detailed analysis of a feedback applicability is given in [3]. The main idea is to suppress the coherent oscillation mode using a resistive feedback, while to keep other modes stable due to a negative chromaticity.

Because of the beam-environment interaction, each particle in a bunch is perturbed by electro-magnetic fields induced by all other particles. For the model bunch of  $N$  macro-particles uniformly distributed over synchrotron phases, a system of differential equations can be written:

$$\frac{dy_k}{dz} + \frac{1}{N} \frac{\omega_0 I_b \langle \beta \rangle}{4\pi \frac{E}{e}} \sum_{j=0}^{N-1} \left( y_j \sum_{m=0}^{\infty} W_{kjm} \right) = 0, \quad (4)$$

where

$$W_{kjm} = Z_{\perp} [-i\omega_0(m - \nu_s)] \exp \left[ i\omega_0(m - \xi) \frac{z_k - z_j}{c} \right],$$

$y_k$  is the complex betatron oscillation amplitude of  $k$ -th particle,  $\omega_0$  is the revolution frequency,  $I_b$  is the bunch current,  $\langle \beta \rangle$  is the average beta-function.  $Z_{\perp}$  is the broad-band transverse coupling impedance, characterizing the short-range beam-environment interaction. The VEPP-4M vertical broad-band coupling impedance estimated from the coherent tune shift measurement [1] is about 2 M $\Omega$ /m.

It is not conveniently to analyze such complicated motion using the system (4), because the number of equations is equal to the number of particles  $N$ , which should be big enough to obtain reasonable results. Moreover, since the longitudinal coordinates of the particles  $z_k, z_j$  are explicitly time-dependent, the system (4) is a system of differential equations with variable coefficients.

As it is shown in [3], an analysis using symmetric mode expansion is much more efficient because only a few of lowest oscillation modes are significant. In addition, this approach allows us to avoid the variable coefficients. Using the continuous medium model and Vlasov equation, the problem of stability can be reduced to a system of algebraic equation:

$$(i\lambda + im)a_{nk} + \frac{I_b \langle \beta \rangle}{4\pi \nu_s \frac{E}{e}} \sum_{n'=-\infty}^{\infty} \sum_{k'=0}^{\infty} A_{nk}^{n'k'} a_{n'k'} = 0, \quad (5)$$

where  $a_{nk}$  are the complex amplitudes of oscillation modes,  $n, n'$  are the indices of transverse modes,  $k, k'$  are the indices of longitudinal modes. The matrix elements  $A_{nk n' k'}$  are functions of the broad-band coupling impedance  $Z_{\perp}(\omega)$ , and of the head-tail phase (3).

A feedback can be introduced in (5) as an equivalent transverse impedance  $Z_{FB}$ . In terms of oscillation modes, the system of equations with a feedback takes on form:

$$i\lambda a_{nk} + \sum_{n'=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} [I(A_{nk n' k'} + \delta_{n0}\delta_{n'0}fB_{nk}^{n' k'}) + in\delta_{nn'}\delta_{kk'}] a_{n' k'} = 0, \quad (6)$$

where  $B_{nk}^{n' k'}$  are the feedback matrix elements dependent on the feedback impedance  $Z_{FB}$ . A real feedback system including beam position monitors and kickers can effect on a beam center of mass only, this is expressed in (6) by the Kronecker delta product  $\delta_{n0}\delta_{n'0}$ .  $I$  is the dimensionless bunch current normalized in such a way that the factor before the sums in (6) is equal to unity:

$$I = \frac{I_b}{I_{b0}}, \quad I_{b0} = \frac{4\pi\nu_s \frac{E}{e} \sigma_z}{Z_{\perp} \langle \beta \rangle R}. \quad (7)$$

The feedback parameter  $f$  is a complex normalization factor of the feedback matrix elements  $B_{nk}^{n' k'}$ . If only  $B_{00}^{00} = 1$  and all other  $B_{nk}^{n' k'} = 0$ , the feedback contribution to the complex betatron frequency shift, related to the synchrotron frequency, is  $ifI$ . For a rigid bunch of the  $I_b = I \cdot I_{b0}$  current, a decrement of the resistive feedback with the parameter  $f$  is  $\tau_{FB}^{-1} = fI\omega_s \text{ s}^{-1}$ .

Stability analysis is done in the following way: the system (5) is an algebraic system of equations with zero right-hand part, therefore it has nontrivial solutions only if  $-i\lambda$  values coincide with eigenvalues of the matrix related to the  $a_{nk}$ . Because the system (5) is infinite-dimensional, it should be truncated to the required number of modes. For each oscillation mode,  $\lambda$  is the complex dimensionless frequency shift,  $\Re\lambda = \frac{\Delta\omega_{\beta}}{\omega_s}$  is the relative betatron frequency shift,  $\Im\lambda = \frac{2\pi}{\tau\omega_s}$  is the increment (if  $\Im\lambda < 0$ ) or decrement (if  $\Im\lambda > 0$ ), normalized by the synchrotron frequency. So, on the basis of the system (5) eigenvalues, it is possible to make a conclusion about stability of the motion: if  $\Im\lambda < 0$  for at least one mode, the motion is unstable.

Numerical solutions of the eigenvalue problem (6) for 10 lower oscillation modes are shown in figure 1. There are  $\Re\lambda$  (upper plots) and  $\Im\lambda$  (lower plots) in dependence of the bunch current  $I_b$ . The left pair of plots corresponds to zero chromaticity  $\xi = 0$  without a feedback  $f = 0$ , the right pair corresponds to a negative chromaticity  $\xi = -8$  with a resistive feedback  $f = 5$ . For the  $\xi = 0, f = 0$  case, when the bunch current exceeds the threshold (about 11 mA), one can see first two-mode coupling. At the same time a negative  $\Im\lambda$  values corresponding to an increment appear. However for the  $\xi = -8, f = 5$  case, a negative  $\Im\lambda$  corresponding to an increment appears when the bunch current exceeds the TMCI threshold about 4 times ( $I_b > 40$  mA).

Beam Instrumentation and Feedback

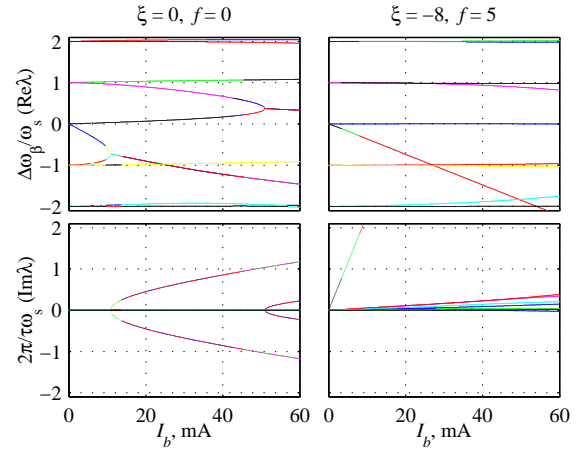


Figure 1: Eigenvalue problem solution.

Thus, using numerical solution of the eigenvalue problem (6) with various bunch current  $I$ , we can find a range of the feedback parameter where all the oscillation modes are stable. Figure 2 shows the maximum possible current  $I_{max}$  of a stable bunch in dependence of the complex feedback parameter  $f$  (presented as the module  $|f|$  and argument  $\arg f$ ) for the chromaticity of  $\xi = -8$ . One can see an area of  $f$  when the bunch of current exceeding 40 mA is stable, the black arrow indicates the maximal current of 42 mA achievable with  $f = 2.65 + 0.38i$ .

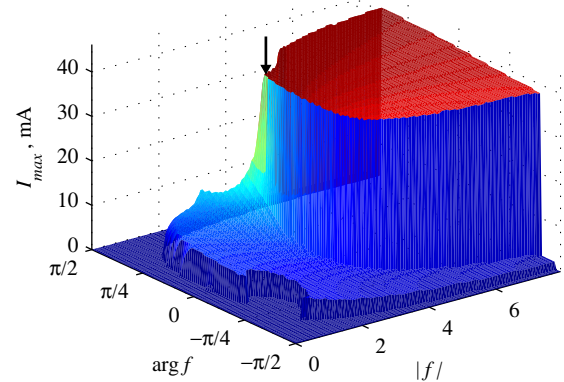


Figure 2: Feedback parameter optimization,  $\xi = -8$ .

Let's calculate the kicker voltage required. If the feedback works in the presence of coherent betatron oscillation, a beam is deflected turn-by-turn by the feedback kicker. The feedback kick  $\Delta y'_{FB}$  is related to the beam position  $y$  and angle  $y'$  at the feedback beam position monitor (BPM) through the transition matrix dependent on the  $\beta$  and  $\alpha$  lattice functions at the BPM and at the kicker, and on the betatron phase advance between the BPM and the kicker. To obtain the  $y'$  value which can not be measured directly, two BPMs are used. For rough estimation, we can assume  $\Delta y'_{FB} \propto \frac{y}{\langle \beta_y \rangle}$ , where  $\langle \beta_y \rangle$  is the average beta-function.

As it was mentioned above, the feedback parameter  $f$

Feedbacks

is an  $\omega_s$ -normalized decrement introduced by the feedback into a motion of a rigid bunch with a unit normalized current  $I = 1$ . Expressing the feedback kick  $\Delta y'_{FB}$  through the feedback parameter  $f$ , we can write:

$$\Delta y'_{FB} = 4\pi\nu_s |f| I \frac{y}{\langle \beta_y \rangle}. \quad (8)$$

For the kicker formed by two matched strip-lines, the voltage amplitude required for the  $\Delta y'_{FB}$  kick, is:

$$V_{FB} = \frac{E}{e} \frac{d}{L} \Delta y'_{FB}, \quad (9)$$

where  $L$  is the kicker length and  $d$  is the gap between the strip-lines.

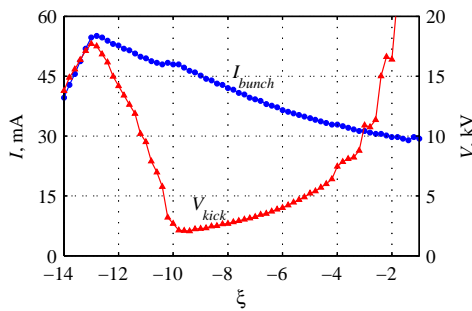


Figure 3: Maximal beam current and kicker voltage.

For the VEPP-4M, taking  $L = 1.8$  m,  $d = 25$  mm,  $\nu_s = 0.02$ , and  $\langle \beta_y \rangle = 13.6$  m, the maximal achievable beam current together with the required kicker voltage have been calculated with various chromaticity and with initial betatron oscillation amplitude of  $y = 1$  mm. The calculation result is presented in figure 3. As one can see, the optimal chromaticity is about  $-10$ , where the bunch current exceeds 40 mA with a reasonable kicker voltage of 1.8 kV. Since four kickers are planned to use, and the initial betatron oscillation amplitude can be reduced down to 0.5 mm by fine adjustment of the injection, the kicker voltage of 200-250 V seems to be enough.

## FEEDBACK SYSTEM LAYOUT

Figure 4 shows a block diagram of the VEPP-4M transverse feedback system. Beam-induced signals of two 45 m-distanced strip-line BPMs pass through a sum-difference circuit to the pickup station, which forms an analogous signal of 30 MHz frequency band, proportional to the beam position. These signals come to the signal processing board to be digitized by the 12-bit 50 MHz ADC. Then the beam position data are processed by the digital signal processor (DSP) TMS320C6713, which calculates kick parameters required for the oscillation damping. The kick signals formed by the DSP are converted in the 12-bit digital-to-analog converter, and after amplification by the broad-band power amplifiers come to the kickers. The same BPMs and kickers are used both for the electron and positron bunches, but with separated signal processing electronics.

Beam Instrumentation and Feedback

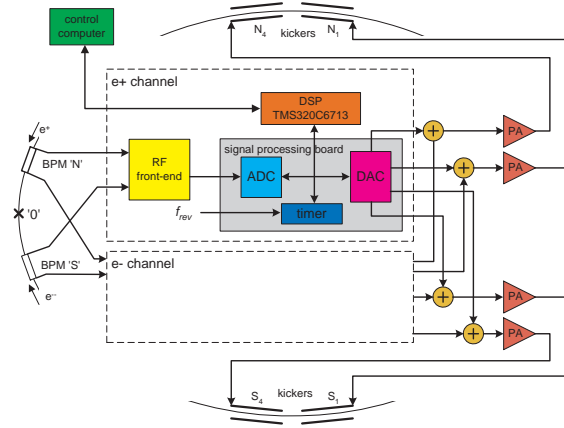


Figure 4: Feedback system block diagram.

## BEAM MEASUREMENTS

At present, all the electronics for one feedback channel is designed, produced and installed at the VEPP-4M. A test beam measurements have been done to evaluate the system sensitivity and spatial resolution. Turn-by-turn r.m.s. resolution of the strip-line BPM, measured with a beam of  $10^{10}$  particles, is about  $80 \mu\text{m}$ .

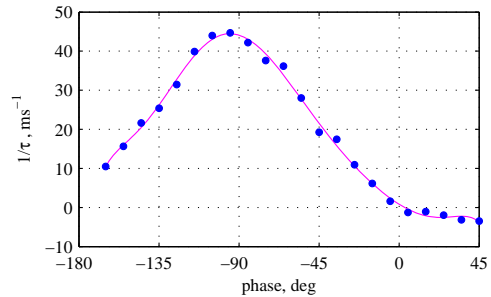


Figure 5: Feedback decrement vs phase.

One more set of beam measurements have been done to find an optimal range of the feedback phase. The feedback decrement in dependence of the phase is presented in figure 5. A range of the feedback phase where the decrement is higher than  $35 \text{ ms}^{-1}$  (i.e.  $\tau_{FB} < 2\pi/\omega_s$ ) is about  $45^\circ$ . This suggests that the feedback will be able to work stably during the VEPP-4M energy ramp, when the betatron tune can vary with time.

## REFERENCES

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- [2] A. Chao, Physics of Collective Beam Instabilities, Wiley, New York, 1993
- [3] M. Karliner, K. Popov, Theory of a feedback to cure transverse mode coupling instability, Nuclear Instruments and Methods in Physics Research A 537 (2005) 481-500

Feedbacks