INFLUENCE OF TRANSVERSE BEAM DIMENSIONS ON BEAM POSITION MONITOR SIGNALS

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Abstract

In this paper we will evaluate the influence of transverse beam dimensions on the signal functions of a beam position monitor (BPM) with capacitive pick-up electrodes. The error which occurs in the determination of the beam position when disregarding these effects is calculated as an example for the DELTA[1] BPM.

The possibility to use this effect for the measurement of the beam size / emittance is discussed.

1 CALCULATION OF THE SIGNAL FUNCTIONS

Fig.1 shows a typical BPM with 4 capacitive pick-up electrodes. When the beam passes through the BPM, the electric field, accompanying the beam, induces a charge pulse on the pick-up electrode, which depends on the beam charge and the position of the beam in the cross section of the BPM. To determine the beam position it is necessary to know the signal function S_i for each pick-up i. These $S_i(x,y)$ represents the response of the pick-ups for a normalised point charge (q=1) at the position (x,y).

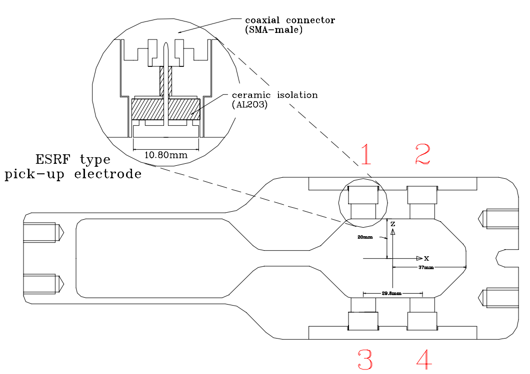

Fig.1 Sketch of the DELTA BPM.

In the case of relativistic beams ($\gamma >> 1$) the electric and magnetic fields are nearly transversal and therefore the determination of the signal functions can be treated as a 2-d problem.

[1] **D**ortmund **EL**ectron **T**est **A**ccelerator (1.5 GeV Synchrotron Radiation Source)

1.1 Signal Function of a Point Charge

The obvious solution to calculate the S_i is to solve for the electric field E of a point charge (pencil beam) at position (x,y) and to integrate E over the surface of pick-up i to get the induced charge which is proportional to the signal function. Because these calculations must be repeated for each position (x,y) a more clever way is to make use of the reciprocity theorem [1].

The potential $\phi_{pick-up}$ is allocated to pick-up i and the Laplace equation $\Delta\phi_i\,(x,y)=0$ with the vacuum chamber on zero potential is solved. The solution ϕ_i is proportional to the signal function. These calculation can easily done by using programs like MAFIA or Poisson (see. Fig. 3).

1.2 Signal Function of a Gaussian Charge Distribution

In most cases a good representation for the transverse charge distribution ρ(x',y') of a particle beam at position (x,y) is a 2-d Gaussian distribution.

$$\rho(x',y') = \frac{1}{2\pi\cdot\sigma_x\cdot\sigma_y}\cdot\exp\left[-\frac{1}{2}\left(\left(\frac{x-x'}{\sigma_x}\right)^2+\left(\frac{y-y'}{\sigma_y}\right)^2\right)\right]$$

The signal function $\tilde{S}_i(x,y)$ of a beam with beam size σ_x and σ_y can be described in the following way:

$$\tilde{S}(x,y) = \iint \rho(x',y')\cdot S(x',y')\,dxdy \quad (1)$$

To study the effect on the determination of the beam position usually the calculation will be done numerically. To get a better understanding of the influence of the beam size we will give a analytical solution.

In the following we expand, at a fixed position, the signal function S_i in a Taylor series and use a Cartesian co-ordinate system with the origin at the centre of the beam. This gives (after evaluating the double integral and using identities concerning integration of Gaussian distributions [2]) for a fixed beam centre

$$\tilde{S} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{2i,2j}\cdot m_{2i,2j}\cdot\sigma_x^{2i}\cdot\sigma_y^{2j} \quad (2)$$

where the $a_{2i,2j}$ are coefficients of the taylor series and the $m_{2i,2j}$ can be written as

$$m_{2i,2j} = \prod_{m=0}^{i} |2m-1| \cdot \prod_{n=0}^{j} |2n-1| \quad (3)$$

Because the signal function S is a solution of the Laplace equation

$$\frac{\partial^2}{\partial x^2} S(x,y) + \frac{\partial^2}{\partial y^2} S(x,y) = 0 \ ,$$

we find the following relation for the coefficient $a_{i,j}$

$$a_{i,j} = -a_{i-2,j+2} \cdot \frac{(j+1)(j+2)}{(i-1)i} \quad (4)$$

Using equations (3) and (4) and rearranging equation (2) leads to the following exact representation of the signal function of a beam with Gaussian charge distribution:

$$\tilde{S}(\sigma_x,\sigma_y) = \sum_{i=0}^{\infty} c_i \cdot (\sigma_x^2 - \sigma_y^2)^i \quad (5)$$

$$c_i = a_{2i,0} \cdot m_{2i,0}$$

This result shows that the variation of the signal function with the beam size depends only on the difference between the squares of the transverse sigmas. A round beam especially leaves the signal function unchanged. A polynomial fit with MAFIA calculated values for the signal functions for different beam sizes to σ_x^2-σ_y^2 shows, that the summands with i>1 in eq. (5) are nearly vanishing (see. Fig. 4). It should also be mentioned that the constant summand $a_{0,0}$ in (5) is the value of the signal function for a pencil beam.

2 POSITION ERRORS DUE TO TRANSVERSE GAUSSIAN CHARGE DISTRIBUTION OF THE PARTICLE BEAM

In most accelerator control systems the signal functions (calculated numerically or measured on a test bench) of a pencil beam are used to calculate the beam position from the measured signals of the BPMs. In Fig.2 we have calculated the position error due to disregarding the transverse beam size in the case of DELTA, a 3rd generation light source, as a worst case estimation for the BPM with the biggest beam size.

To simplify the representation we have calculated the distance between the given centre of the beam and the calculated beam position.

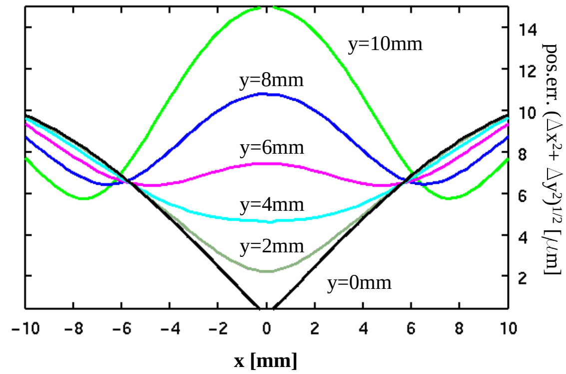

Fig.2 Absolute position error for a beam with σ_x=500µm and σ_y=50µm.

A 3rd generation light source has an emittance in the order of some nmrad and operates often with a emittance coupling in the order of some %. Therefore the horizontal beam size is in the order of 100 µm and the vertical of 10 µm. In the case of DELTA at 1.5GeV we have maximum values of 500 µm horizontal and 50-100 µm vertical. The resulting error is for most cases smaller than 10 µm. Therefore no influence on routine operation is expected.

On the other hand we should keep in mind that modern closed-orbit measuring and orbit-feedback systems have a resolution in the order of some µm. All coherent movements of the beam, maybe induced by instabilities, power supply ripple or the tune measuring system, with time constants smaller than the integration time of the measuring system, can also be seen as a beam with changing size, resulting in an virtual orbit drift. At facilities with much greater beam sizes, a significant influence can be expected because eq. 5 shows a quadratic dependency of the σ.

3 ELECTROSTATIC EMITTANCE MONITOR

In chapter 1 we have shown, that the signal functions of a beam with transverse Gaussian charge distribution depends on σ_x^2-σ_y^2. Therefore it should be possible to extract information on the beam sizes by measuring normalized pick-up signals for know positions of the particle beam in comparison with the signals for pencil beams. These reference signals must be calculated or measured in the laboratory.

Fig.3. shows a sketch of a pick-up monitor which can be used to determine the beam size.

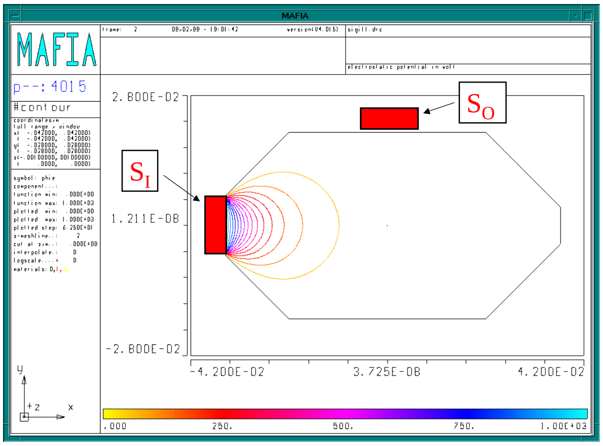

Fig.3 Simplified monitor design for an "electrostatic" emittance monitor.

The procedure is the following: The beam must be centred at a fixed position (how this can be done will be described later on). Then the signals S_I and S_O are measured and the current independent value

$$S_I^{norm} = \frac{S_I}{S_I + S_O}$$

is calculated. The deviation ΔS_I^{norm} from the calibrated one for the pencil beam at this position is a function of $\sigma_x^2 - \sigma_y^2$.

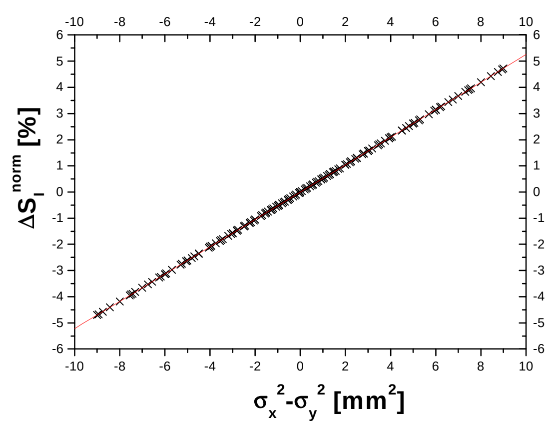

Fig.4 Relative variation of the normalised signal ΔS_I^{norm} as a function of $\sigma_x^2 - \sigma_y^2$ at a fixed beam position (x=0mm and y=0mm)

Fig.4 shows this relative variation ΔS_I^{norm} as a function of $\sigma_x^2 - \sigma_y^2$, which can also be written as

$$\sigma_x^2 - \sigma_y^2 = \varepsilon_x \cdot \beta_x - \varepsilon_y \cdot \beta_y \qquad (6)$$

$$= \sigma_x^2 \cdot \left(1 - k \frac{\beta_y}{\beta_x}\right) \qquad (7)$$

$$(\ \sigma_{x,y} = \sqrt{\beta_{x,y} \cdot \varepsilon_{x,y}} \quad , \quad \varepsilon_y = k \cdot \varepsilon_x\ )$$

Eq. 7 shows, that the horizontal beam size can be calculated from one measurement if $k << 1$ and $\beta_x << \beta_y$.

If it is possible to perform this measurement at 2 different monitors with well selected beta functions eq. 6 gives the possibility to calculate the horizontal and vertical emittance independently.

As mentioned earlier it is absolutely necessary to place the beam at a well known position during the data taking. This can be done on 2 different ways:

1. Especially the positioning of the beam in the centre of the BPM is possible by adding the 2 pick-ups from Fig.3 to a monitor as shown in Fig.1 (without the pumping channel). The determination of a centred beam with the 4 pick-ups, which are placed symmetrically around the centre, is possible independent of the beam size (if the bpm is calibrated and all transfer functions well adjusted). Even for a beam with small offsets (<0.25mm) the position error is smaller than 10 µm for a wide variation of beam sizes, which turns out to be sufficient. This method needs a very carefully calibrated BPM, with well adjusted electronics for position and emittance measurements.
2. By combining the 2 pick-up BPM with a dedicated, well aligned quadrupole magnet. Because the response of the beam to the quadrupole field is linear, it is possible to use methods based on beam based calibration techniques [3][4] to centre the beam independent of the beam sizes. This arrangement has also the great advantage that it allow for the measurement of the beta-functions at the same position where the beam size is measured and gives therefore directly the emittance. This solution needs a pick-up monitor which is well centred on the magnetic axis of the quadrupole and needs a absolute accuracy concerning the positioning of the beam at the axis of better than 10µm. At DELTA we have realised values of < 70 µm [5] and we expect to reach smaller values in the future.

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