ROLE OF PRE-WAVE ZONE EFFECTS IN TR-BASED BEAM DIAGNOSTICS

V.A.Verzilov, INFN-LNF, CP 13, 00044 Frascati, Italy

Abstract

Transition radiation (TR) is nowadays intensively exploited by a number of techniques to characterize different beam parameters. These methods are based, sometimes implicitly, on standard formulae, and used often without paying due attention to their applicability. In particular, standard expressions are only first-order asymptotic, i.e., strictly speaking, valid at infinity. In this paper TR is examined in a spatial domain where conventional results are no more exact and variations in radiation properties are observed. Under certain conditions, for example, at long wavelengths or very high energies the effect is so considerable that should be taken into account in accurate beam measurements.

1 INTRODUCTION

Transition radiation is nowadays intensively exploited by a number of techniques to characterize different beam parameters. These methods are based, sometimes implicitly, on the standard theory of TR, whereas, often it is not fully applicable under conditions of measurements. Therefore, refinements of the theory become essential for both the design of experiments and interpretation of results.

There is a class of phenomena appearing because the electromagnetic field of a relativistic particle has quite macroscopic dimensions. In fact, while the particle itself can certainly be considered a point, its electromagnetic field occupies a finite space, outer border of which scales in the transverse (to the particle trajectory) plane roughly as $\lambda\gamma$, where λ is the radiation wavelength and γ is the relativistic factor. Since, eventually, the source of TR is the particle field interacting with the interface between two media, its size is that of the field. Strong variations in radiation properties are expected when $\lambda\gamma$ exceeds the dimension of a screen used to produce the radiation. Another relevant effect is that, the transverse extension of the TR source appears to be responsible for that the radiation needs to propagate over a substantial distance before acquiring all the well-known properties. Both effects may occur at the same time interfering with each other. In this paper an outline of the second problem is given along with the results of calculations related to applications in beam diagnostics.

2 THEORETICAL BACKGROUND

It is widely accepted that forward TR is formed over the so-called *formation length*, whereas, backward TR can be collected very close to the source. Meanwhile, as shown bellow, it is not always the case. Even backward TR evolves over a distance of the same order as the formation length of forward TR. This fact has to be taken into account in beam diagnostics, since backward TR is typically used in measurements and a space available for the experimental instrumentation is often limited by practical reasons.

Only in the *wave zone* the standard formulae can be used. The wave zone is treated in this paper as a spatial domain where the radiation field at any arbitrary point can be considered a plane wave. In other words, in the wave zone the source is seen as a quasi-point one. Therefore, the "border" of the wave zone is determined by the dimension of the source.

To make clear physical arguments we consider an extended coherent source of a radiation. Let O and S be

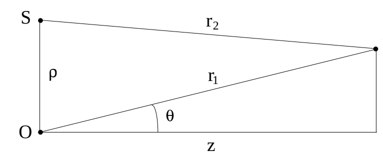

Figure 1: Waves emitted by two different points O and S of the source reach an arbitrary point P with a phase difference $\Delta\varphi = k(r_2 - r_1)$, where k is the wave vector.

two points on the source surface separated by a distance ρ (Fig. 1). Generally, waves emitted by these points at the same phase will arrive at an arbitrary observation point P with a phase difference $\Delta\varphi$. For all the source points between O and S to contribute at P fully constructively, the phase difference must be $|\Delta\varphi| \ll \pi$. Assuming the observation point to be far from the source, so that $z \gg \rho$, the above condition becomes

$$\left|\frac{\rho^2}{z} - 2\rho\theta\right| \ll \lambda\,. \tag{1}$$

Thus, for the given distance z only a source region of the size ρ satisfying Eq. (1) forms mainly the field at P. On the other hand, to obtain a constructive interference from all the points of the source, the radiation must be collected far enough. It should be noted that in our definition, with ρ being the size of the source, Eq. (1) specifies the border of the wave zone. The obvious conclusion is the larger the dimension of the source the farther the wave zone from it.

For TR the size of the source is of the order of $\lambda\gamma$ and the characteristic angle of emission is $\theta \sim 1/\gamma$. This gives for the wave zone

$$z \gg \lambda\gamma^2\,. \tag{2}$$

Now we will touch upon the mathematical aspect of the problem. Let's consider backward TR emerging when a normally incident particle with a charge q and a velocity $v \to c$ hits a perfectly conducting infinite screen. In this case only transverse components of the field are essential and the spatial-spectral distribution of TR, that is the radiation power per unit of the frequency and per unit of the transversal area, valid at any distance (except, perhaps, very short ones), can be written in the form [1]:

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega \mathrm{d}\mathbf{u}} = \frac{q^2}{\pi^2 c} |\Phi(u, w, \gamma)|^2 , \tag{3}$$

where dimensionless variables $u = k\rho$ and $w = kz$ are used and

$$\Phi(u, w, \gamma) = \int_0^\infty \frac{t^2 \mathrm{d}t}{t^2 + \gamma^{-2}} J_1(ut) \mathrm{e}^{\mathrm{i}w\sqrt{1-t^2}} . \tag{4}$$

At large distances $w \gg 1$ the integral in Eq. (4) can be approximated by the contribution from the vicinity of a single point where the derivative of the phase in the exponential vanishes. The size of the domain around this so-called "stationary" point t_s, giving the main contribution to the integral, is of the order of $1/\sqrt{w}$. When this quantity is much smaller then the range $0 \leq t \leq 1/\gamma$, within which the fractional part undergoes a maximum variation, the latter may be approximated by its value at t_s. Then Eq. (3), in turn, approximates the classical formula for TR. Therefore, for the standard theory to hold true a condition $1/\sqrt{w} \ll 1/\gamma$, that is fully equivalent to Eq. (2), must be fulfilled. It is worthwhile to note that, even in the wave zone, the standard expression is asymptotic, i.e. strictly speaking, valid at infinity.

At shorter distances, i.e. in the pre-wave zone, the solution of Eq. (3) differs from the standard one. The reason is that, for $w \leq \gamma^2$, the poles $t = \pm\mathrm{i}/\gamma$ of the fractional factor turn out to interfere with the contribution from the stationary point. Therefore, a proper account of these singularities should be taken in the complex plane. However, a detailed analysis of the problem goes beyond the scope of this paper. Instead, in the following, we give the results of numerical calculations based on Eq. (4), aiming to demonstrate the role of pre-wave zone effects in beam diagnostics.

3 EFFECT ON THE TR ANGULAR DISTRIBUTION AND RELATED BEAM DIAGNOSTICS

The angular distribution of TR can be used to obtain an information about the beam energy and beam angular divergence [2, 3]. This technique seems to be attractive, since, it does not require dispersive sections and can be performed at any position along an accelerator. Characterizing capabilities of the method, we note that in [3], when measuring the variation of the beam energy along the macropulse at TTF, a relative accuracy of the order of 1.5 % was achieved though conditions were not optimized for the measurements.

The beam energy measurements are based, essentially, on such a property of TR as the position between the central depth and the peak that is $1/\gamma$ according to the standard theory. Meanwhile, if measurements are performed in the pre-wave zone, this is no longer true.

To obtain an angular distribution from the spatial one, in Eqs. (3) a new variable $x = u/w\gamma$ is introduced. In relativistic regime $x \approx \theta\gamma$. An advantage of use of this variable is that all the results given below are independent of the beam energy.

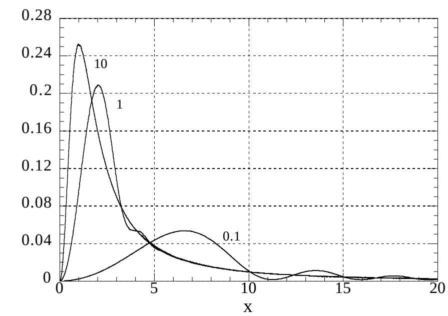

Figure 2: TR angular distribution $\frac{\pi^2 c}{q^2} \frac{\mathrm{d}^2 W}{\mathrm{d}\omega \mathrm{d}\mathbf{x}}$. Numbers by the curves are distances w from the screen in units of γ^2

Fig. 2 shows TR angular distributions at different distances from the emitting screen. While at $w = 10\gamma^2$ the angular distribution is quite consistent with the classical form, at $w \leq \gamma^2$ the difference is significant. The distribution changes its form and becomes wider. Fig. 3 shows

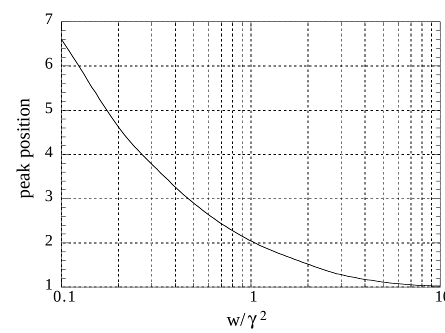

Figure 3: Peak position versus the parameter w/γ^2.

the the angular peak position x_{peak} with respect to the center $x = 0$ as a function of the distance from the source. As seen, the distance between the peak and the center increases very rapidly with decreasing w in the region $w \leq \gamma^2$.

Thus, in the pre-wave zone the TR angular distribution differs from the classical one and can affect beam energy measurements. Since such kind of measurements are normally performed for visible light, the effect should be measurable from $\gamma \sim 1000$ and higher.

4 EFFECT ON TR SPECTRA IN BUNCH LENGTH MEASUREMENTS

Recently, a capability of methods based on coherent transition radiation (CTR) to measure the length of ultra-short bunches has been demonstrated. In this technique [4] the bunch longitudinal dimension can be extracted from the measured CTR spectrum if that of incoherent TR is precisely known. In the case of the classical flat incoherent spectrum, the spectrum of CTR is directly proportional to the bunch form-factor.

It has been also recognized that there are practical factors as the detector bandwidth, diffraction, etc., that cause losts in the low-frequency part of spectra thus leading to a considerable uncertainty in the bunch length determination. The corresponding analysis for effects of diffraction and the size of the emitting screen on bunch length measurements was given [5].

As shown below, spectra of TR, collected in the prewave zone, are distorted at low frequencies and, thereby, become a limiting factor in bunch length diagnostics. Fig. 4 presents spectra of TR calculated for "detectors" of different apertures located at 1 meter from the screen for a wavelength range typical in bunch length measurements. Spectra are normalized to corresponding classical flat spectra. All the spectra exhibit a reduction in the intensity for long wavelengths. The effect is weaker for larger detector apertures and becomes clearly marked for higher beam energies. The energy dependence of the spectra is due to the

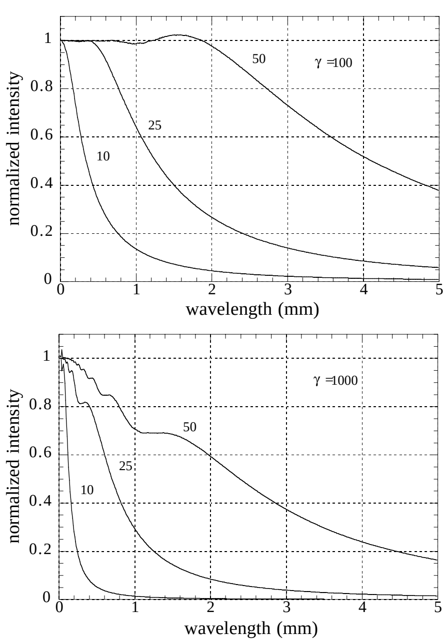

Figure 4: Spectra of TR at a distance of 1 m from the screen integrated over the "detector" apertures given in millimeters next to the curves.

fact that the border of the wave zone moves very rapidly (quadratically) away from the screen with increasing the beam energy. At the same time, the dependence on the aperture shows that low frequencies are not fully lost; a redistribution of the frequency contents, as a result of interference, rather takes place, namely, the central part of the angular (or spatial) distribution is depleted with low frequencies. For the sufficiently large aperture all frequency components may be, in principle, collected.

In Fig. 5 spectra are given for the detector aperture of 25 mm, as it is placed at different distances from the screen. The distortion of the spectra becomes stronger with increasing either the energy or the distance. The unexpected,

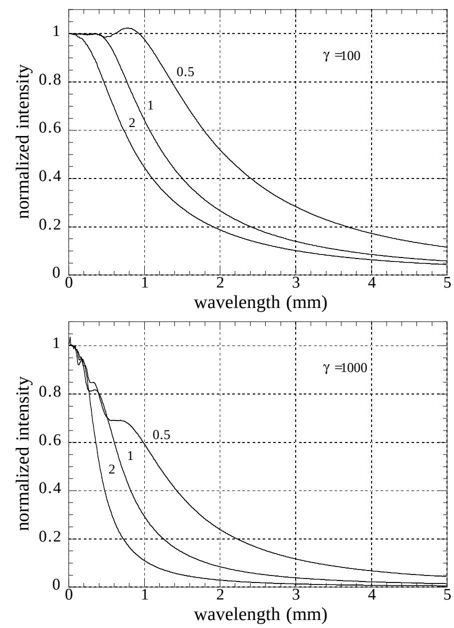

Figure 5: Spectra of TR at different distances (given in meters next to the curves) integrated over the "detector" aperture of 25 mm.

for the first view, effect of the distance is a simple consequence of a variation in the detector angular acceptance. If the radiation is collected in a fixed cone the situation changes to the opposite one, namely, the distortion of the spectra becomes smaller with the distance increase.

5 CONCLUSIONS

Backward TR evolves over the distance comparable with the formation length of forward TR and acquires all the well-known properties only in the wave zone. In the prewave zone TR characteristics are quite different from the classical ones. This fact must be taken into account in TR-based beam diagnostics.

6 REFERENCES

[1] V.L.Ginzburg and V.N.Tsytovich, *Transition radiation and transition scattering* (Adam Higler, Bristol, 1990).

[2] D.Giove et al.,*Optical Transition radiation measurements on the elettra linac*, Proc. of 1997 PAC.

[3] M.Castellano et al.,*Time resolved energy measurement of the TESLA test facility beam through the analysis of optical transition radiation angular distribution* , Proc. of 1999 PAC.

[4] W.Barry, AIP Conference Proceedings 320 (1997) 173.

[5] M.Castellano et al., Preprint LNF-99/005(P).