LINEAR COUPLING COMPENSATION FOR THE LHC VERSION 6.1

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Abstract

Random and systematic skew-quadrupole errors in the main dipole and quadrupoles magnets and orbit errors in the arc sextupole magnets give rise to a linear coupling of the LHC machine. Operating the machine near the coupling resonance requires an efficient compensation of the global coupling in the machine. The following report summarises the relative contributions from the various sources for linear coupling and presents different options for a compensation scheme for version 6 of the LHC optics.

1 SKEW QUADRUPOLE COMPONENTS IN MAIN DIPOLE MAGNETS

The overall skew quadrupole error of a main dipole magnet is given by

$$\begin{aligned} a2 &= a2gM + a2pM + a2tM + \\ & \frac{\xi_1}{1.5} \times \sqrt{a2gU^2 + a2pU^2 + a2tU^2} + \\ & \xi_2 \times \sqrt{a2gR^2 + a2pR^2 + a2tR^2}, \end{aligned} \tag{1}$$

where ξ_1 and ξ_2 are random numbers with a Gaussian distribution cut at $1.5\ \sigma$ and $3.0\ \sigma$ respectively, a2g, a2p, a2t refer to the geometric, persistent and ramp induced errors respectively. Each error class is again divided into systematic ('mean' $\rightarrow$ a2gM), systematic per arc/manufacturer ('uncertainty' $\rightarrow$ a2gU) and random errors ('random' $\rightarrow$ a2gR). Table 1.1 lists the skew quadrupole components in the LHC dipole magnets.

Table 1: a_2 components in the LHC main dipole magnets in units of 10^{-4} at $R_{ref} = 17$ mm.

a_2 component in units of 10^{-4} at $R = 0.017$ m								
geometric (a2g)			persistent (a2p)			dynamic (a2t)		
M	U	R	M	U	R	M	U	R
0.0	0.5	1.7	0.0	0.0	0.77	0.0	0.75	1.7

The normalised skew quadrupole gradient k_s is given by

$$k_s(dipole) = \frac{1}{R_r \rho} \cdot a2 \tag{2}$$

where R_r is the reference radius for the field expansion (R_r = 17 mm) and ρ the bending radius inside the dipole magnets ($\rho = 2778$ m for the main bending magnets).

The integrated skew quadrupole strength per main dipole is given by ($l = 14.3$ m)

$$l \cdot k_{s,uncertainty}(MB) \approx 2.75 \cdot 10^{-5}\mathrm{m}^{-1} \tag{3}$$

$$l \cdot k_{s,random}(MB) \approx 7.6 \cdot 10^{-5}\mathrm{m}^{-1}. \tag{4}$$

1.1 Quadrupole magnets

The skew quadrupole field of a tilted quadrupole is given by

$$k_s(quadrupole) = -k_2 \cdot \sin{(2 \cdot \triangle\phi)}, \tag{5}$$

Assuming an rms tilt angle of $\triangle\phi = 0.3$ mrad and a normalised quadrupole gradient of $k_2 = 0.0085\ m^{-2}$ one obtains (l = 3.4 meter)

$$l \cdot k_{s,random}(MQ) \approx 1.75 \cdot 10^{-5}\mathrm{m}^{-1} \tag{6}$$

1.2 Sextupole magnets

A vertical orbit offset in the sextupole magnets leads to a skew quadrupole component

$$k_s(sextupole) = -k_{sext} \cdot \triangle y, \tag{7}$$

where k_{sext} is the normalised sextupole strength and $\triangle y$ the vertical orbit error inside the sextupole magnet. Assuming an rms orbit error of $\triangle y = 1$ mm and a normalised sextupole gradient of $k_{sext} = 0.047\ m^{-3}$ one gets (l = 0.32 meter)

$$l \cdot k_{s,random}(sextupole) \approx 1.5 \cdot 10^{-5}\mathrm{m}^{-1}. \tag{8}$$

Weighted by the number of elements in the machine (ca. 350 quadrupole and sextupole and 1100 main dipole magnets) the contributions of the quadrupole and sextupole magnets amounts to only 20 % of the total random coupling coefficient.

2 COUPLING COEFFICIENTS

The coupling coefficient per dipole magnet can be written as

$$\triangle c^i_{\pm} = \frac{1}{2\pi} \cdot \int_{MB} ds \sqrt{\beta_x \beta_y} \cdot k_s \cdot e^{i(\mu_x \pm \mu_y)} \tag{9}$$

For the LHC version 6.1 we have $\triangle(\mu_x - \mu_y)_{cell} = 2\pi \cdot 0.02$, $\triangle(\mu_x + \mu_y)_{cell} \approx 2\pi \cdot 0.5$ and the contribution of an individual LHC cell can be written

$$\begin{aligned} \triangle c^i_- &\approx \frac{6 \cdot l_{MB}}{2\pi} \cdot \sqrt{\beta_x \beta_y} \cdot k_s \\ \triangle c^i_+ &\approx \frac{2}{2\pi} \cdot \sqrt{\beta_x \beta_y} \cdot k_s \end{aligned} \tag{10}$$

where l_{MB} is the dipole length ($l_{MB} \approx 14.3$ m).

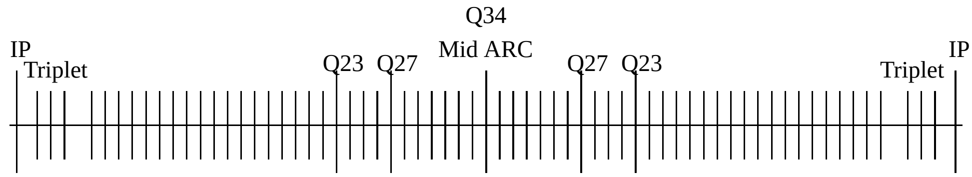

Figure 1: Each short vertical line indicates one quadrupole. The skew quadrupole magnets are located next to Q23 and Q27 left and right from the mid arc.

2.1 Systematic Coupling Coefficient per arc

Summing the contributions of all dipole magnets in one arc one gets

$$\triangle c_{-}^{k} = f_{-} \cdot \frac{6}{2\pi} \cdot \sqrt{\beta_x \beta_y} \cdot l_{MB} \cdot k_s \qquad (11)$$
$$\triangle c_{+}^{k} = \frac{2}{2\pi} \cdot \sqrt{\beta_x \beta_y} \cdot k_s$$

with

$$f_{-} = \frac{\sin\left(N_{cell} \cdot [\mu_x - \mu_y]_{cell}/2\right)}{\sin\left([\mu_x - \mu_y]_{cell}/2\right)} \qquad (12)$$

Fig. 2 shows f_{-} as a function of the tune split per cell. For a tune split of 5 we have $f_{-} = 15.9$. Inserting $\sqrt{\beta_x \cdot \beta_y} = 78$ m and $l_{MB} \cdot k_{s,uncertainty}$ from Equation 4 we get $\triangle c_{-}^{k} = 0.042$. Summing the contributions from all arcs we get for the total coupling coefficient

$$\triangle c_{-}^{k} = 0.27. \qquad (13)$$

2.2 Random Coupling Coefficients

Taking the incoherent sum over all dipole magnets yields for the total coupling coefficient

$$\|\triangle c_{\pm,tot}\| = \sqrt{2 \cdot 1100} \cdot \frac{1}{2\pi} \cdot \sqrt{\beta_x \beta_y} \cdot l_{MB} \cdot k_{s,r} \qquad (14)$$

Inserting $\sqrt{\beta_x \cdot \beta_y} = 78$ m and $l_{MB} \cdot k_{s,r}$ from Equation 4 we get

$$\|\triangle c_{\pm,tot}\| = 0.044 \qquad (15)$$

3 SKEW QUADRUPOLE CORRECTOR LAYOUT

Each arc of the LHC lattice is equipped with two pairs of skew quadrupoles. Each magnet is 0.32 meter long and has a maximum field of 125 T/m. The two skew quadrupole magnets of each pair are separated by a phase advance of $\mu_y = \pi$ in order to minimise the excitation of vertical dispersion. The two pairs are separated by a phase advance of $\mu_x + \mu_y = n \cdot \pi$ in order to minimise the β-beat generated by the skew quadrupole magnets. The corresponding corrector layout is illustrated in Fig. 1.

3.1 Correction of the coupling coefficient due to systematic a_2 errors

Correction of the coupling coefficient due to systematic a_2 errors per arc (uncertainty) requires a powering of the four skew quadrupole magnets in series. In this case the coupling coefficient of the corrector magnets can be written as:

$$c_{cor-s} = \frac{0.32}{2\pi} \sqrt{\beta_x \beta_y} \cdot k_{corr} \cdot \sum_j \cos\left(\mu_{x,j} - \mu_{y,j}\right) \qquad (16)$$

where $\mu_{x,j}$ and $\mu_{y,j}$ are the horizontal and vertical phases with respect to the mid-arc. The top line in Fig. 3 shows the phase advance depended terms in Equation 16 for different tune splits.

3.2 Correction of the coupling coefficient due to random a_2 errors

The compensation of the coupling coefficient due to random a_2 errors can be done in two ways:

- powering of two orthogonal arcs
- left-right independent powering of the two skew quadrupole pairs in one arc

The first option depends on the phase advance between two arcs and thus, on the phase advance of the insertions. While this option is viable for the LHC optics version 6.1 it is possible to imagine a scenario for which the phase advance $\mu_x - \mu_y$ differs by multiples of $180°$ from arc to arc. In this case, the arcs are not independent and can not be used for compensating the coupling coefficient due to random a_2 errors.

4 LEFT-RIGHT POWERING OF THE TWO SKEW QUADRUPOLE PAIRS

A robust solution for the compensation of the coupling coefficient due to random a_2 errors is to power the skew quadrupole corrector magnets independently on the left and right-hand side of the arcs.

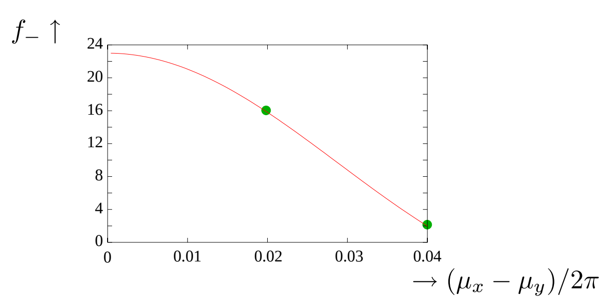

Figure 2: f_{-} as a function of the tune split per cell. The points indicate the values for the LHC V6.1 with tune split 5 and 9 (resonance free lattice [1])

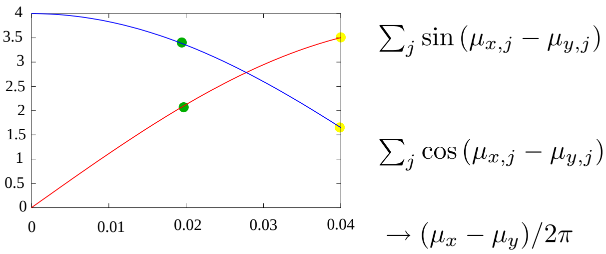

Figure 3: $\sum_j \cos(\mu_{x,j} - \mu_{y,j})$ *and* $\sum_j \sin(\mu_{x,j} - \mu_{y,j})$ as a function of the the tune split per cell. The points on the left indicate the values for the LHC V6.1 with tune split 5 and the points on the right the values for the resonance free lattice (tune split 9)

4.1 *Coupling coefficients for the corrector magnets*

The coupling coefficient of the corrector magnets with antisymmetric powering can be written as:

$$c_{cor-r} = \frac{0.32}{2\pi}\sqrt{\beta_x\beta_y} \cdot k_{corr} \cdot \sum_j \sin(\mu_{x,j} - \mu_{y,j}) \quad (17)$$

For the LHC version 6.1 every other arc has an independent powering of the corrector magnets and the arcs with independent powering of the two rings are interleaved such that each arc only provides left-right powering for one of the two rings. This implementation presents a good compromise between distributing the asymmetric powering of the arcs over the machine and minimising the number of required cables in the arc. The left-right powering spoils the compensation of the β-beat generated by the skew quadrupole magnets. However, we will see later that the left-right powering amounts only to 20 % of the nominal corrector strength and the resulting β-beat remains below 1 %. Fig.3 shows the phase advance depended terms in Equation 17 for different tune splits.

5 REQUIRED CORRECTOR STRENGTH

5.1 *Correction of the systematic a_2 per arc*

Requiring that Equation (16) compensates the expression in Equation (11) we get

$$k_s(corr) = -\frac{6}{0.32} \cdot l_{MB} \cdot k_{s,sys}(MB) \cdot \frac{f_-}{f_{corr+}} \quad (18)$$

with $f_{corr+} = \sum_j \cos(\mu_{x,j} - \mu_{y,j})$. Inserting f_- from Fig. 2 and f_{corr+} from Fig. 3 we get

$$k_s(corr, sys) = 0.0024 \rightarrow B_{QS,sys} < 60 \text{ T/m}$$
$$k_s(corr, sys) = 0.0007 \rightarrow B_{QS,sys} < 16 \text{ T/m}$$

for a tune split of 5 and 9 (resonance free lattice), respectively. Both values are comfortably below the maximum gradient of the skew quadrupole magnets (125 T/m).

5.2 *Correction of the random a_2*

Requiring that Equation (17) compensates 1/4 of the expression in Equation (14) (4 arcs with left-right powering of the skew quadrupole magnets) we get

$$k_s(corr) = -\frac{\sqrt{2 \cdot 1100}}{4 \cdot 0.32 \cdot f_{corr-}} \cdot l_{MB} \cdot k_{s,sys}(MB) \quad (19)$$

with $f_{corr-} = \sum_j \sin(\mu_{x,j} - \mu_{y,j})$. Inserting $l_{MB} \cdot k_{s,sys}(MB)$ from Equation 15 and f_{corr+} from Fig. 3 we get

$$k_s(corr, ran) = 0.0014 \rightarrow B_{QS,sys} < 33 \text{ T/m}$$
$$k_s(corr, ran) = 0.0008 \rightarrow B_{QS,sys} < 18 \text{ T/m}$$

for a tune split of 5 and 9 (resonance free lattice), respectively. The total corrector strength (systematic per arc plus random $\rightarrow B_{QS,tot} < 100$ T/m) is still comfortably below the maximum gradient (125 T/m).

6 RESULTS FOR LHC OPTICS 6.1

The following section gives some numerical results obtained with MAD for a tune split of 5.

6.1 *The LHC without correction*

Choosing a distribution of the a_2 uncertainty where the coupling coefficient contributions of the different arcs add up (worst case) yields the following perturbations:

- closest tune approach (a_2 uncertainty): $\Delta Q \approx 0.2$
- closest tune approach (a_2 uncertainty plus random): $\Delta Q \approx 0.213$
- average tilt-angle of the particle distribution: $\psi \approx 41°$ (rms $\approx 5°$)
- vertical dispersion: $D_{y,rms} \approx 0.09$ m, $\rightarrow D_{y,peak} \approx$ 0.4 m
- β_y-beat: $\Delta\beta/\beta_0 < 2.5$ %

6.2 *The LHC after correction of systematic and random a_2*

Correcting the systematic and random a_2 yields:

- $\Delta Q < 10^{-5}$
- average tilt-angle of the particle distribution: $\psi \approx$ 1.5° (rms $\approx 0.5°$)
- vertical dispersion: $D_{y,rms} \approx 0.07$ m, $\rightarrow D_{y,peak} \approx$ 0.35 m
- β-beat: $\Delta\beta/\beta_0 < 0.4$ %

REFERENCES

[1] F. Schmidt and A. Verdier, "Optimisation of the LHC Dynamic Aperture via the Phase Advance of the Arc Cells", PAC'99, New York, June 1999.