TRANSITION RADIATION IN BICYLINDRICAL CAVITY

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Abstract

The problem of electron bunches transition radiation (TR) in the cavity the cross section of which is composed by two overlapped circles (bicylindrical cavity) is discussed. The solution is created with the help of bicylindrical modes series, calculation method of which was developed in [1]. The properties of TR field in the bicylindrical cavity discussed in the case of bunches are passing the cavity as well as at bunch presence. Such a cavity is proposed as a basic module in two beam acceleration (TBA) scheme [2].

1 INTRODUCTION

In the proposed TBA scheme [2] the driver and accelerated bunches periodical sequences are passing through the front wall of the bicylindrical cavity (BCC) at the circles centers vicinity (Fig.1).

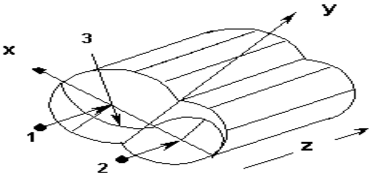

Figure 1. Bicylindrical cavity; driver (1) and accelerated (2) bunches. Second symmetrical bicylindrical E-mode (ES_2 - mode [1]) E_z component distribution (3).

The transversal sizes of the cavity (cylinders radii $r_{1,2}$ and distance d between their axes) are conditioned by the bunches repetition rate frequency: so only the ES_2 mode is generated in the cavity due to the TR of the bunches infinite periodical sequence. Thus the bunches insertion points are combined with the extremes $\left(y_{1,2}=0,\, x_{1,2}=r'_{1,2}\right)$ of this mode E_z component distribution (Fig.1) and placed at some distance $\Delta r'_{1,2}$ from the centers of circles. For the single bunch (or terminate bunches sequence) TR field calculation it is necessary to obtain the bicylindrical modes sequence.

2 BICYLINDRICAL MODES

The bicylindrical modes calculation method is presented in [1]. It permits to calculate only symmetrical ($E_z(x,y)=E_z(x,-y\}$) E-modes ($H_z \equiv 0$) if the bunches are passing the cavity parallel to its sidewalls and are placed on its symmetry plane. At $r_1=r_2=r$ modes have additional symmetry $(E_z(x,y)=\pm E_z(-x,y))$ in connection with the slit between the quasi-cylinders and are closing pairwise to the same circular mode (***CM***) in each of cylinders at $d \rightarrow 2r$. For this case are calculated (see Tab.1) 42 modes eigenvalues (k) and the normed values of these E_z components at the driver and accelerated bunches trajectories. Calculation results for the first 12 of them are presented in Table 1. The sizes of the cavity correspond to the bunches repetition frequency of YerPhI accelerator: $\nu = 2.7972GHz$: $r=4.1524cm$, $d=7.4743cm$, $\Delta r' = 0.062cm$.

Table 1. First 12 symmetrical E modes

N	k	$E_z(r',0)$	$E_z(-r',0)$	CM
1	0.5397	-0.167	-0.167	***E$_{01}$***
2	0.5858	-0.187	0.187	
3	0.8213	$8X10^{-2}$	$8X10^{-2}$	***E$_{11}$***
4	0.9420	$2X10^{-3}$	$-2X10^{-3}$	
5	1.1336	$8X10^{-2}$	$8X10^{-2}$	***E$_{21}$***
6	1.2565	$-5X10^{-2}$	$5X10^{-2}$	
7	1.3009	0.234	0.234	***E$_{02}$***
8	1.3454	0.278	-0.278	
9	1.4689	0.113	0.113	***E$_{31}$***
10	1.5574	$-3X10^{-2}$	$3X10^{-2}$	
11	1.6245	$-8X10^{-2}$	$-8X10^{-2}$	***E$_{12}$***
12	1.7215	$-8X10^{-3}$	$8X10^{-3}$	

The field in the cavity with the height $a=30cm$ is calculated by the help of mode matching method, using the results, obtained in [3] for the circular cavity and generalized them for the cavity with the arbitrary cross section. Driver bunch's TR fields E_z component distributions were calculated at the driver and accelerated bunches trajectories at the different time moments. At $t=0$ the bunch is entering into the cavity. The bunch was taken as a linear one with the charge longitudinal distribution $F(\xi)=2\sin^2(\pi\xi/L)/L$. The bunch length $L=4cm$ and Lorentz factor $\gamma=1000$.

3 BUNCH IN THE CAVITY

At the TR field calculation on the driver bunch trajectory for the bicylindrical cavity it is convenient to compare it with the same field distribution in the circular cavity with the same radius. Particularly at $vt<2r$ it can be a complete coincidence: for both cases there is

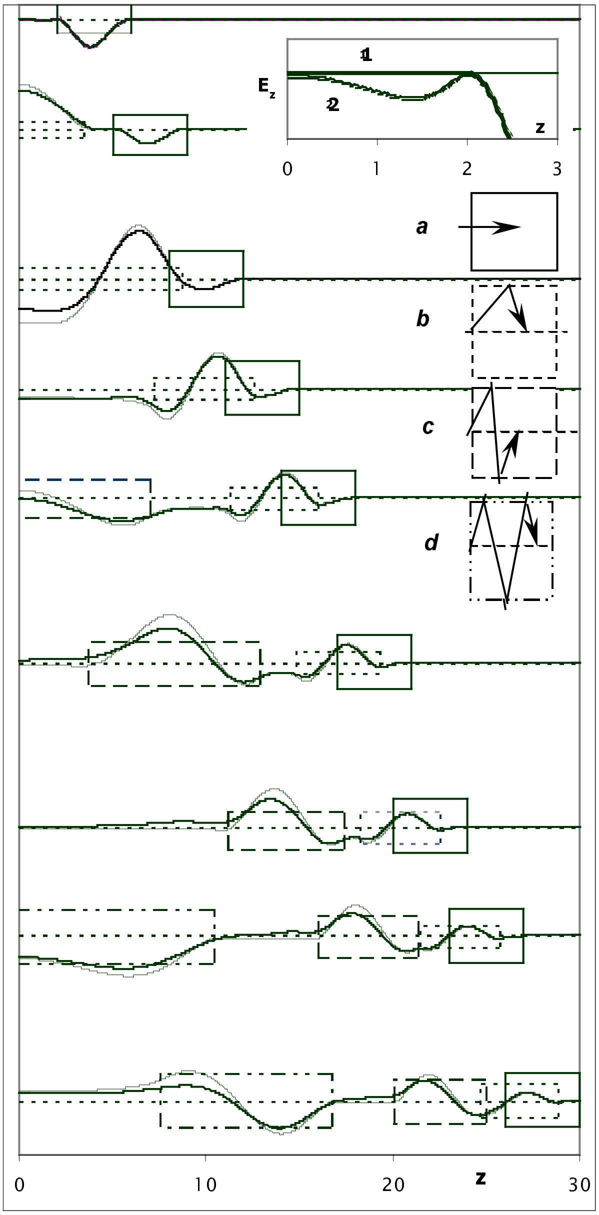

Figure 2. TR field E_z component distributions on the bunch trajectory at the bunch presence in the bicylindrical (thick) and in the circular (thin) cavities. Top-down: $t = 0.2a/v$, $0.3a/v$, $0.4a/v$, $\ldots, a/v$. For the driver bunch trajectory: a - direct radiation region, b, c, d - once, twice and thrice reflected rays regions.

only direct radiation outgoing from the bunches intersection point with the front wall of the cavity. At $t = 0.2a/v$ (Fig.1) non zero equal field can be distributed in $2 < z < 6$ region. Such correct result can be obtained for the circular cavity by help of too many modes summation and presented by curve 1 on the top right corner of Fig.2. Curve 2 at the same picture presents coincided results obtained with the help of 40 bicylindrical and 20 (4X5) transverse cylindrical modes. For $vt > 2r$ there is complete coincidence for the direct radiation, coinciding with the bunch location, for both of cases. The difference between the curves, arises due to single, twice and thrice reflection explained by help of slit presence. The regions of the reflected radiation are marked by the rectangles and are including the rays outgoing from the head up to the tail of the bunch.

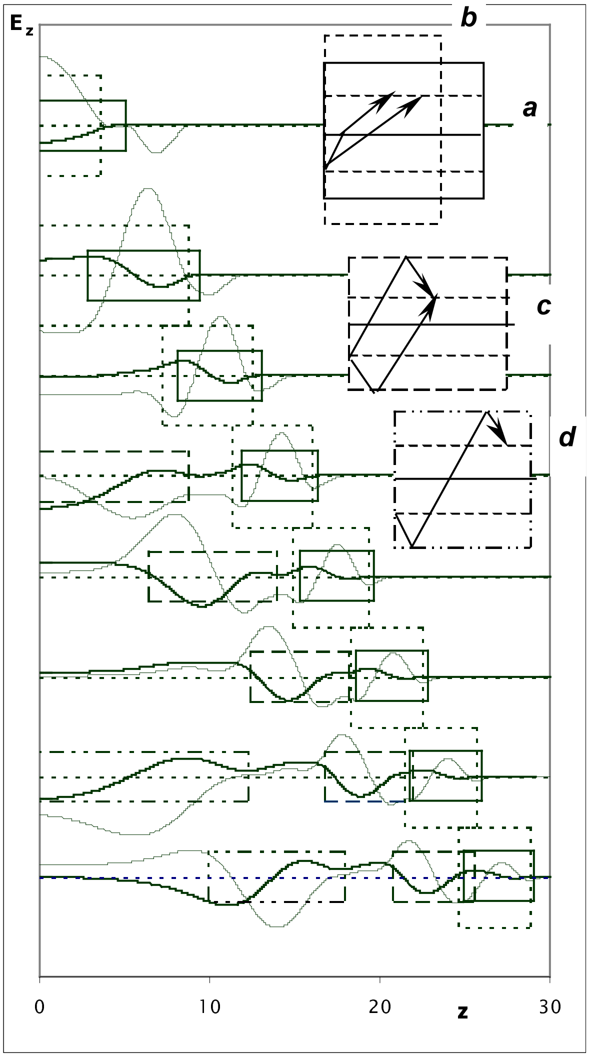

Figure 3. Bunhes TR field E_z component distributions on the driver (thick) and accelerated (thin) bunch trajectories at bunch presence in the cavity. Top-down: $t = 0.3a/v$, $0.4a/v$, $0.5a/v, \ldots, a/v$. For the accelerated bunch trajectory: a - direct, b - single difracred on the slit, c, d - once and twice reflected radiation regions.

As one can see from Fig.3, obtained bicylindrical modes sequence also describes the field distribution on the accelerated bunches trajectory: the extremes of the obtained curves coincides with the geometrically calculated direct radiation and once and twice reflected rays pass through the slit. In this case direct radiation formed by the directly radiated rays having passed through the slit as well as by the help of diffracted rays on the fissure.

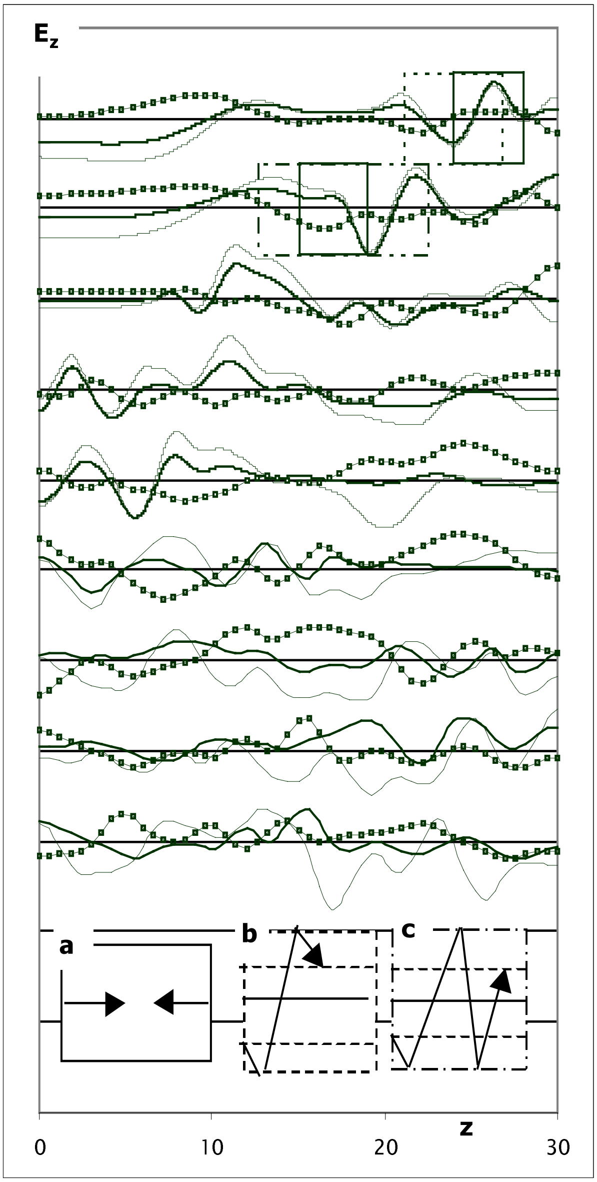

Figure 4. TR field E_z component distributions for the bicylindrical (thick) and cylindrical (thin) cavities on the driver bunch trajectory and on the accelerated bunch trajectory (marked) after bunches passing through the cavity Top-down: $t = 12a/v$, $15a/v$, $18a/v$, $21a/v$, $24a/v$, $27a/v$.$31a/v$,$100a/v$,$1000a/v$. For driver bunch trajectory: a - direct and back wall reflected radiation region: For accelerated bunch trajectory: b, c - twice and thrice reflected ray regions.

4 BUNCH OUTSIDE OF CAVITY

One can observe a superposition between the direct field outgoing from the back wall of the cavity and the field being reflected from this wall after the bunch has crossed it (Fig.4). This combined wave package location corresponds to the mirror reflected bunch's image. It is possible to calculate the location of the corresponding reflected rays at this case also by the help of geometrical optics laws, but owing to the ray picture complexity and spatial superposition of the different reflection it is too difficult to chose the contribution of the determined reflection. The field distributions is coinciding mainly for the bicylindrical and cylindrical cavities at $t = 12a/v, 15a/v$ and $t = 18a/v$ (Fig.4). It is significant that the counter influence of the another cylinder (accelerated bunch cylinder) is weak at the $t < 2a/v$. The field distributions difference is conditioned, as in the previous case (Fig.2) by the reflection absence from the slit region in bicylindrical cavity.

The interaction between the cylinders is growing insensibly by the time increasing and brings to the statistically identical distributions in both of cylinders.

5 CONCLUSIONS

The obtained results are the first application of the bicylindrical function series and show the possibility to use them successfully for the bunches TR field investigation in BCC. TR field in BCC is investigates by the help of comparison with the TR field in the cylindrical cavity and by the help of geometrical rays reflection regions obtained as well. The bicylindrical modes series were used for the bunches sequence TR field calculation in the bicylindrical cavity with the finite Q-factor [4] as well as Cherenckov radiation in the bicylindrical wavequide, filled with the dispersion medium [5,6].

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