AXISYMMETRIC RF FOCUSING DEVELOPMENT FOR ION LINAC

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Abstract

The ion beam dynamics in axisymmetric RF focusing structure for linear accelerator is considered. New equations of particles motion in polyharmonic RF field are devised by means of smooth approximation. A method of RF focusing analysis is suggested.

1 INTRODUCTION

It's known that simultaneous longitudinal and transverse stability can be ensured by external focusing elements or by accelerating RF field of special configuration (RF focusing). For small-size low energy linacs the second way is more effective. The next kinds of such focusing are known nowadays: alternating phase focusing (APF), radio frequency quadruples (RFQ) and undulator RF focusing (RFU). The main principles of APF in alone wave approach were described in [1]-[3]. Later the model with two traveling waves, one of which is synchronous with the beam, was taken as a base of APF description [4], [5]. It was shown in further analysis [6] that sometimes presence of two nonsynchronous field harmonics should be considered. A case of large number standing wave space harmonics in the short gaps approximation was discussed in [7]. Focusing and acceleration without synchronous harmonic by using of undulators was suggested in [8]. In all works pointed out the RF field spectrum and the phase advance were different. In this paper the extended method of RF focusing analysis in arbitrary polyharmonic fields is offered. The main principles of RF focusing for ion beams in low energy ion linacs are discussed. The standing wave structure is supposed.

2 PARTICLE MOTION EQUATIONS

The RF field in the periodical structure can be presented in the form:

$$E_z = \sum_{n=0}^{\infty} E_n^{\,z} \cos(h_n z)\cos(\omega t),$$
$$E_r = \sum_{n=0}^{\infty} E_n^{\,r} \sin(h_n z)\cos(\omega t), \qquad (1)$$

where $E_n^{\,z} = E_n I_0(h_n r)$, $E_n^{\,r} = E_n I_1(h_n r)$, $h_n = h_0 + 2\pi n / D$, n=0, 1, ..., $\mu = h_0 D$ is a phase advance per period of structure D. Trajectories of particles in the field (1) may be expressed by the summation of slowly varying $\boldsymbol{r}_{sl}$ and rapidly oscillating $\tilde{\boldsymbol{r}}$ types of motion. By means of averaging method the motion equation for slow component can be obtained in the form

$$\frac{d^2 \boldsymbol{R}}{d\tau^2} = -\frac{\partial}{\partial \boldsymbol{R}} U_{eff}, \qquad (2)$$

where $U_{eff} = U_0 + U_1 + U_2 + U_3$ is the effective potential function:

$$U_0 = \frac{\beta_c}{2} e_s^{\,z} \sin\psi,$$
$$U_1 = \frac{1}{16}\sum_{n\neq s} \frac{\boldsymbol{e}_n^{\,2}}{\Delta^{-\,2}_{s,n}} + \frac{1}{16}\sum_{n} \frac{\boldsymbol{e}_n^{\,2}}{\Delta^{+\,2}_{s,n}},$$
$$U_2 = \frac{1}{16}\sum_{\substack{n\neq s\\ n+p=2s}} \frac{e_n^{\,z} e_p^{\,z} - e_n^{\,r} e_p^{\,r}}{\Delta^{-\,2}_{s,n}} \cos(2\psi), \qquad (3)$$
$$U_3 = \frac{1}{8}\sum_{n\neq s} \frac{\boldsymbol{e}_n \boldsymbol{e}_p}{\Delta^{-\,2}_{s,n}} \cos(2\psi),\ h_n - h_p = 2h_s.$$

Here $e_n = e\lambda E_n / 2\pi mc^2$, $\boldsymbol{R} = 2\pi \boldsymbol{r}_{sl} / \lambda$, $\tau = \omega t$, $\Delta_{s,n}^{\pm} = (h_n \pm h_s)/h_s$, n=s for synchronous wave. $\psi = \int dZ(1/\beta_c - 1/\beta)$; β_c, Z_c are synchronous particle velocity and coordinate, $\psi(Z_c) \equiv \psi_c = const$. The U_{eff} defines the system Hamiltonian

$$\frac{1}{2}\left(\frac{d\boldsymbol{R}}{d\tau}\right)^2 + U_{eff} = H \qquad (4)$$

and describes the 3-D dynamics of particle in the smooth approximation completely.

3 ANALYSIS OF THE EFFECTIVE POTENTIAL FUNCTION

The terms composing U_{eff} are determined by RF field spectrum. Item U_0 describes interaction of particle with only synchronous harmonic, which accelerates and defocuses the beam, i.e. extremum of the U_0 is saddle point. Term U_1 evaluates only transverse focusing and doesn't depend on phase and amplitude of the synchronous wave. These two items can circumscribe some kinds of APF (see below). If condition n+p=2s is satisfied, $U_2 \neq 0$. This term affects on both transverse and longitudinal motion. At last, in the case of $h_n - h_p = 2h_s$, $n \neq s$, summand U_3 also appears.

This item influences on phase and radial dynamics, but in some cases it can be not equal to zero in two waves approach. The extremums of U_2 and U_3 are also saddle point. The necessary condition of simultaneous transverse and longitudinal focusing is existence of total minimum of U_{eff}. In this case the effective potential function is 3-D potential well in the beam frame. Expanding the U_{eff} near synchronous particle coordinate we can formulate these condition as

$$\omega_\psi(e_s, e_n, \dots)^2 > 0 \qquad \omega_R(e_s, e_n, \dots)^2 > 0 . \quad (5)$$

The important restriction on choice of space harmonics amplitudes can be obtained from the condition of non-overlapping for different waves resonances in the phase space. This restriction defines limits of averaging method applicability. Moreover, it's the necessary condition of longitudinal stability. Using the Hamiltonian (4) and analyzing the bunch form in the 4-dimensional phase space it is possible to find the relationship between the defined longitudinal acceptance and the limit value of transverse emittance, which provides the maximal transmission coefficient.

4 CLASSIFICATION OF ACCELERATING SYSTEMS

The space harmonics distribution of the field is a base of RF focusing systems classification. Let's present the dimensionless amplitudes of space harmonics on the cavity axis as $e_n = \alpha_n e_{max}$, $\alpha_n < 1$. The value e_{max} connected with the structure breakdown voltage is defined by cavity construction and further will be fixed. Therefore, the problem is to find the optimal set of weight coefficients $\{\alpha_n\}$ for different RF focusing versions.

4.1 Synchronous and one nonsynchronous harmonics.

Let's start from the system with synchronous and one nonsynchronous harmonics. Adding the saddle term U_0 and the focusing one U_1 it is possible to achieve the U_{eff} to have a shape of 3-dimensional well. Now it's suitable to define the set of weight coefficients as $\{\alpha_s = \alpha, \ \alpha_n = 1\}$. Than condition (5) takes the form:

$$\alpha \sin\psi_c < \frac{3}{8} e_{max} \left(\frac{1}{\Delta^{+\,2}_{s,n}} + \frac{1}{\Delta^{-\,2}_{s,n}} \right) \left(\frac{h_n}{h_s} \right)^2 + \frac{3}{32} \alpha e_{max} \quad (6)$$

At first let's consider the field with phase advance $\mu = 0$ and $s \geq 1$. System {s=1, n=0} is the ordinary Alvarez-type structure. Here the focusing effect is absent since $U_1 = const$. The case {s=2, n=0} is the same structure with doubled period. Systems {s=1, n=2}, {s=2, n=1} are distinguished by absence of the zero harmonic. Moreover, the structure {s=1, n=2} provides pretty effective transverse focusing but it's hard to realize this.

Now let's discuss the case of $\mu = \pi$ and $s \geq 0$. System {s=0, n=1}, {s=1, n=0} are the Wideroe-type structures which can be realized by alternating of drift tubes with different length and different internal diameter. In the case {s=0, n=1} $U_3 \neq 0$, but influence of this term is negligibly.

In the simplest case of APF mentioned in paper [2] the accelerating wave is faster than the focusing one (s<n) and harmonics amplitudes decrease with growth of their number ($e_s > e_n$) for the ordinary structure period. Thus the condition (6) can be satisfied only in the case of $\sin\psi_s << 1$. It leads to small longitudinal acceptance. But the condition (6) is satisfied also with a large phase capture ($\sin\psi_s$ ~1) if the amplitude of focusing harmonic e_n is greater than the amplitude of the acceleration one e_s ($\alpha << 1$). The acceleration gradient $dW_s / dz = 0.5 \alpha E_{max} \cos\psi_s$ is proportional to α. It restricts the parameter α from below. This version of the RF focusing can be created by special construction of structure period containing two and more acceleration gaps. The condition (6) also gives that systems with s>n are ineligible because of weak transverse focusing. The combination {s=1, n=2} also allows to achieve effective transverse focusing, but the structures with a large harmonic number are not effective. The value of field amplitude which corresponds to separatrixes overlapping decreases pretty fast versus growth of harmonic number. Creation of the structure with n>2 is difficult, because it's necessary to set many acceleration gaps per period. Choice of Hamiltonian initial value depends significantly on size of the channel aperture and on magnitude of the synchronous velocity. The depths of U_{eff} intersections by planes ($R = 0$) and ($\psi = \psi_c$) should be congruent for providing of effective phase capture and transverse focusing.

4.2 Influence of the second nonsynchronous harmonic.

In practical realizing of optimal versions for accelerating cavity it's necessary to take into account contribution of higher space harmonics which amplitudes decrease versus growth of their number as a rule. The analysis of the U_{eff} allows to research the influence of these harmonics on beams dynamics. Let's add the second nonsynchronous harmonic to the system discussed above. Now the set of weight coefficients becomes to be

$\{\alpha_s = \alpha,\ \alpha_n = 1,\ \alpha_p \equiv \varepsilon < 1\}$. Two different cases are possible depending on type of structure. In the first $U_{2,3} = 0$. In the second case combinative effect of nonsynchronous waves leads to appearance of the crossed items $U_{2,3}$. Now the phase portrait of the system is changed significantly. The phase capture decrease and the acceleration gradient increase with growth of parameter ε, moreover the addition to acceleration gradient is proportional to ε. Choice of the structure period depends on magnitude of nonsynchronous harmonics. The second minimum of U_{eff} appears at some value of ε. It leads to formation of second bunch on the period.

4.3 Acceleration system without synchronous harmonic.

It can be seen from formula (3) that simultaneous phase and transverse stability may be achieved in the structure without synchronous wave ($\alpha_s = 0$). 3-D potential well can be created using influence of terms $U_{1,2,3}$. Here particle is accelerated and focused by field of the combinative wave (undulator way of acceleration, [8]). Such system has some nontrivial features. First, the beam is modulated on double frequency. Second, formulas (2), (3) contain the particle charge squared. It means that it's possible to accelerate both positive and negative ions with identical charge-to-mass ratio in one bunch. Therefore the space-charge compensation is available. It allows to increase the limit current.

5 COMPUTER SIMULATION

The detailed method of the field harmonic spectrum optimization and the RF focusing realizing was tested by numerical simulation of ion beam dynamics in polyharmonic field. The calculations were carried out both in polyharmonic and in averaged field. In the range of proton energy 0.1—2 MeV the results coincide up to some percents, what is a good confirmation of smooth approach. Influence of the Coulomb interaction on bunch motion was studied particularly by means of the "superparticles" method. Using the technique described above the acceleration system {μ=π, s=0, s=1} was calculated in detail. Numerical simulation shows that to achieve a large particles capture it's necessary to use bunching section, where phase of the synchronous particle decreases linearly and the field amplitude increases slowly, and accelerating section: $\psi_c = \pi/2 - z\pi/4L_{gr}$ if $L < L_{gr}$, $\psi_c = \pi/4$ if $L > L_{gr}$; $E(z) = E_{max}\sin^2(\pi z/2L_f)$ if $L < L_f$; $E(z) = E_{max}$ if $L > L_f$. The main characteristics of acceleration channel and injected beam are the next: generator wave length λ=2 m, field amplitude on the axis E_{max} =200 kV/cm, aperture radius a=0.6 cm, injection energy W=100 keV, maximal radius of the beam r_{max} =0.3 cm, α=0.1 $L_{gr}/L = 0.25$, $L_f/L = 0.25$. The simulation results for protons and deutons are coincided: exit energy is 1.3 MeV, acceleration gradient is 0.61 MeV/m, maximal exit current is 0.09 A.

Therefore, from the point of view of the transmission coefficient and the exit current the suggested system exceeds all known acceleration RF focusing structures and in some cases is close to Alvarez-type structure with quadruple focusing.

6 CONCLUSION

In the paper the new approach to RF focusing in the axisymmetric polyharmonic RF field of ion linac is suggested. The method of classification for all types of axisymmetric RF focusing including APF on the base of field harmonic spectrum analysis is offered. Choice and optimization of RF field conditions to obtain maximal transmission coefficient are formulated. Numerical simulation of beam dynamics in polyharmonic field is carried out. It shows a good agreement with the results obtained in smooth approximation. The concrete variants of parameters choice for RF focusing accelerator were reduced.

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