HALO PARTICLE CONFINEMENT IN THE VLHC USING OPTICAL STOCHASTIC COOLING*

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Abstract

Beam halo particles following the extreme trajectories near the physical aperture limit radiate Smith-Purcell radiation when moving over a diffraction grating. This grating can be used as a pick-up and a kicker for optical stochastic cooling of the halo particles. In this application cooling would have the effect of slowing down the halo particle diffusion onto the aperture. Cooling efficiency would quickly diminish with the distance from the aperture and would only affect the halo particles. A preliminary analysis of this system is considered.

1 INTRODUCTION

This paper summarizes a study of a possible application of the Optical Stochastic Cooling (OSC) [1] to the Very Large Hadron Collider (VLHC) [2]. Here we consider a high field option of the VLHC. All ring and beam parameters related to our study are given in Table 1 (Ref. [3]).

Table 1: The VLHC parameter list

Beam energy, E_0	TeV	50
Revolution frequency, f_0	Hz	3156
Total number of protons, N		5×10^{14}
Normalized emittance, ϵ_n	m-rad	2.5×10^{-6}
Energy spread, σ_e		0.9×10^{-5}
SR damping time, τ_{SR}	hrs	1.3

The VLHC proton beam is too intense for the OSC to work with all beam particles and cool the emittance and energy spread of the entire beam. (As we will see later it would then require tens of kilowatts of optical power to get the damping time comparable to a synchrotron radiation damping time.) Instead, we consider another mission and that is to counterbalance the slow diffusion of protons towards the aperture at large amplitudes. In this regard Smith-Purcell radiation of particles moving over diffraction gratings seems to match perfectly the role of an 'optical' pick-up and kicker.

2 SMITH-PURCELL RADIATION

A relativistic particle moving over a diffraction grating (see, Fig. 1) radiate light with the wavelength [4]:

$$\lambda = \lambda_g \left(\frac{1}{2\gamma^2} + \frac{\theta^2}{2} \right), \quad (1)$$

where λ_g is the grating period, γ is the Lorentz factor, and θ is the observation angle which is presumed to be small.

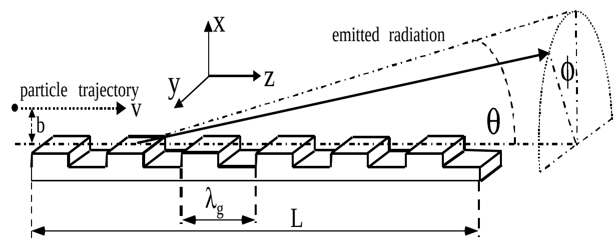

Figure 1: Diagram showing notations. Particle trajectory is in z direction, and x is normal to the grating plane.

Using Eq.46 from Ref. [5] we find the number of photons emitted spontaneously by the particle into a half-space above the grating [6]:

$$n_{ph} \simeq \pi\alpha R^2 \frac{(\gamma\theta)^2}{1+(\gamma\theta)^2} e^{-2kb\sqrt{\frac{1}{\gamma^2}+\frac{\theta^2}{2}}}, \quad (2)$$

where $\alpha \simeq 1/137$ is the fine structure constant, b is the distance from the grating to the particle, $k = 2\pi/\lambda$ is the wave number, and R^2 is grating efficiency [5]. The particle radiate a light pulse with the number of optical cycles equal to the number of grating periods $M = L/\lambda_g$. Correspondingly, the bandwidth of the signal is $\Delta\lambda/\lambda \simeq 1/M$. The photons are emitted into the angle $\theta \pm \Delta\theta$, where $\Delta\theta = \theta/4M$, and the diffraction-limited size of the radiation source is $d = (k\theta)^{-1}$.

3 OPTICAL STOCHASTIC COOLING

Recall that OSC obeys the same principles as the well established microwave stochastic cooling [7] but operates at optical frequencies and explores the superior bandwidth of optical amplifiers, $\sim 10^{14}$ Hz [8]. In the original proposal [1] the undulators played the role of 'optical' pick-ups and kickers. However they are impractical in proton rings exceeding few TeVs. For example, in the case of the VLHC the undulator for optical radiation at a fundamental harmonic would have a period $\sim$ 1 km. In this study we consider a diffraction grating for the role of an 'optical' pick-up and a kicker at multi-TeV proton energies.

Fig. 2 shows a schematic of a cooling section in the ring. It consists of two gratings, an optical amplifier and a bypass. Moving near the pick-up grating, the particle radiates an electromagnetic (EM) wave. The wave goes to the optical amplifier and then proceeds to the kicker grating. The particle traverses the bypass and meets this wave at the beginning of the kicker grating. A subsequent interaction

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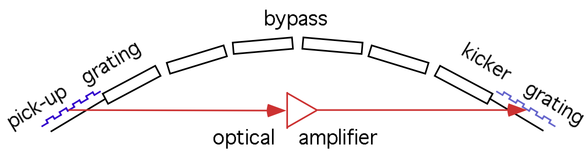

Figure 2: A schematic of the OSC system.

between the particle and the amplified wave as they pass together through the kicker grating results in the change of the particle energy [1]:

$$\Delta E_i \simeq 2\eta g n_{ph} \hbar\omega \cos\left[\omega\left(\tau_i - \tau\right)\right], \tag{3}$$

where η is the efficiency of the interaction, $\hbar$ is the Plank's constant, $\omega = kc$, c is the speed of light, g is the amplitude gain of the amplifier, $n_{ph}\hbar\omega$ is the energy of a particle spontaneous emission in the pick-up and kicker gratings (n_{ph} is defined by Eq.2), τ_i is the time-of-flight between the gratings for an arbitrary particle identified by the index i, τ is the time required for light to pass between the gratings including the time delay in the amplifier.

The time τ_i depends on the particle energy and transverse coordinates, and time-off-flight parameters of the bypass lattice. This dependence is used to arrange a correcting kick to the particle each time it passes the cooling section. It works as follows. The time τ is adjusted to yield: $\omega\tau = \omega\tau_0 + \pi/2$, where τ_0 is the time-of-flight between gratings of the equilibrium particle. Then $\cos\left[\omega\left(\tau_i - \tau\right)\right]$ in Eq. 3 equals to $\sin\left[\omega\left(\tau_i - \tau_0\right)\right]$. The time-off-flight parameters of the bypass lattice are chosen such that $\omega\left(\tau_i - \tau_0\right) \leq 1$ for all halo particles and that particles with the coordinate and energy off-sets $x_i, \delta E_i$ (x stands for all transverse coordinate and angles) are delayed or advanced relative to the equilibrium particle depending on the sign of the off-set. It leads to a condition that ΔE is proportional to a linear combination of x_i and δE_i. In this way we produce a correcting kick in energy, which is also transformed into a correcting kick in the coordinate using a non zero dispersion function at the location of gratings (similar to Palmer's method of stochastic cooling [7]), so the energy damping is equal to the coordinate damping.

However in planar rings like the VLHC, the dispersion function is in the horizontal plane and, therefore, the coordinate cooling would take place in the horizontal plane. To do it in the vertical plane one needs to introduce a coupling into the ring, so that cooling can be shared between the two planes. The global coupling does not seem attractive since it constrains the choice of the betatron tunes. More practical is to introduce the local coupling before the pick-up grating and cancel it after the kicker grating. It must be done in the dispersion free region by the skew-quadrupoles rotating the normal mode of betatron oscillation by 45°. In this way we preserve the dispersion in the horizontal plane and allow sharing of the energy kick between horizontal and vertical mode of betatron oscillations.

In the case of the VLHC we deal with a situation where the maximum possible amplifier power sets the upper limit for the damping rate. In this case we can consider only a correction effect of the particle on itself via the feedback loop of the pick-up, amplifier and the kicker and ignore the heating effect due to the particle interaction with the EM wave emitted by neighboring particles. Then the energy damping time can be estimated as the number of passes through the cooling system that is needed for an accumulation of the correction equal to the beam energy spread, i.e.

$$\frac{1}{n_{OSC}^2} \simeq \frac{\Delta E^2}{\sigma_e^2 E_0^2} = 4\pi\eta^2 R^2 \frac{\alpha P \hbar\omega}{N f_0 \sigma_e^2 E_0^2} e^{-2kb\sqrt{\frac{1}{\gamma^2}+\frac{\theta^2}{2}}}, \tag{4}$$

where n_{OSC} is the damping time expressed in the number of passes through the cooling system and $P = Ng^2 n_{ph}\hbar\omega f_0$ is the average power of the optical amplifier. Eq. 4 is written for $\gamma\theta > 1$.

4 COOLING OF HALO PARTICLES

In the following analysis we presume that after the accelerator reaches 50 TeV the gratings can be put at $10\sigma_\perp$ distance from the orbit[2], where $\sigma_\perp$ is the beam transverse size. To supress the Smith-Purcell radiation of the beam core particles and still be sensitive to the radiation of the particles from the tails of the distribution we need the exponential factor in Eq. 2 to drop somewhat moderately for $b \leq 2\sigma_\perp$ and to drop dramaticaly for $b \simeq 10\sigma_\perp$. In this study we assume five orders of magnitude in this attenuation that seems to be sufficient to bring the output power of optical amplifiers to reasonably achievable levels. Because of this large factor special care has to be taken to ensure that the amplifiers are protected from extra loads by any stray photons originated from the radiation of the beam core particles somewhere upstream of the gratings. This can be done by limiting the acceptance of the amplifiers with synchrotron radiation masks and by tapering the magnetic field of the bending magnets near the gratings.

Using Eq. 2 and assuming a difference of seven orders of the magnitude between the intensity of the radiation of particles positioned right on the orbit and at $10\sigma_\perp$ we get:

$$2k\sigma_\perp\sqrt{\frac{1}{\gamma^2}+\frac{\theta^2}{2}} = 0.5\ln\ 10. \tag{5}$$

Then considering the physical aperture at the gratings of ± 2 mm, i.e. $\sigma_\perp \simeq 0.2$ mm, we calculate $\theta \simeq 0.5$ mrad. Finally, using this θ in Eq. 1 with $\lambda = 800$ nm (chosen in the middle of the amplifier bandwidth) we find $\lambda_g \simeq 6$ m. Thus, with $M = 10$ the total length of the grating is gigantic, 60 m.

[1] Eq. 3 can be obtained by considering in the far field region a total energy of the field of the amplified wave $\mathcal{E} \propto g\sqrt{n_{ph}\hbar\omega}\sin(\omega t + \omega\tau)$ and the field of spontaneous emission of the particle $\mathcal{E}_{se} \propto \sqrt{n_{ph}\hbar\omega}\sin(\omega t + \omega\tau_i)$ [9].

[2] It is possible to combine the grating with the aperture mask.

The diffraction limited size of the source of Smith-Purcell radiation in the above defined conditions is rather small, $\simeq 0.25$ mm. Thus the radiation emitted by halo particles spreading along the vertical line near the left and right gratings in Fig. 3 can be viewed as being emitted by several independent sources and can therefore be amplified by the same number of optical amplifiers. Currently, we consider ten optical amplifiers (five per each side) collecting the light emitted near the horizontal plane, i.e. near the plane with the dispersion function.

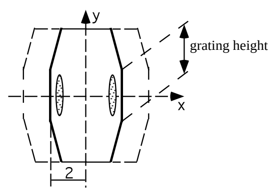

Figure 3: The cross-section of the vacuum chamber. Dashed lines show grating position at injection.

For the average output power of the amplifier we use 20 W (200 W in total), thus leaving some room open for a technological progress in the future. Further assuming $\eta = 0.5$, $R^2 = 1$ and taking $N = 5 \times 10^9$ we calculate from Eq. 4:

$$n_{OSC}(x) \simeq 3 \times 10^5 \, e^{0.57(10 - x/\sigma_\perp)}, \tag{6}$$

where x is the distance from the beam orbit. This is equivalent to $\sim$ 100 sec of damping time for halo particles at $10\sigma_\perp$. Now we can define the diffusion of the halo particles that can be counterbalanced by this damping:

$$D(x) = \frac{\Delta\sigma_\perp^2}{\sigma_\perp^2} \leq \left(\frac{x}{\sigma_\perp}\right)^2 \left(\frac{1}{n_{OCS}} + \frac{1}{n_{SR}}\right), \tag{7}$$

where $n_{SR} \simeq 1.5 \times 10^7$ is the synchrotron radiation damping expressed in the number of orbit turns. We call $D(x)$ critical diffusion when it is exactly equal to the right side of Eq. 7. The plot for critical diffusion at different amplitudes is shown in Figure 4.

To help evaluate the impact of the projected damping on the performance of the VLHC, we did similar calculations for the beam parameters of the Large Hadron Collider: $E_0 = 7$ TeV, $\sigma_e = 10^{-4}$, $f_0 = 11.25$ kHz, $N = 3 \times 10^9$, and compared them with the simulated diffusion from Ref. [10]. This is shown in Fig. 5.

The sharp rise of the simulated diffusion near $6\sigma_\perp$ is due to the long-range beam-beam effects, i.e. in a condition when protons appear close to the center of the other beam in parasitic crossings. It is then evident that the OSC can not stop diffusion ariving from long-range beam-beam effects if they would have similar stengths at the VLHC as at the LHC. But OSC still outperform the synchrotron radiation damping and could stop particles before they reach the amplitudes where the long range beam-beam effects become significant.

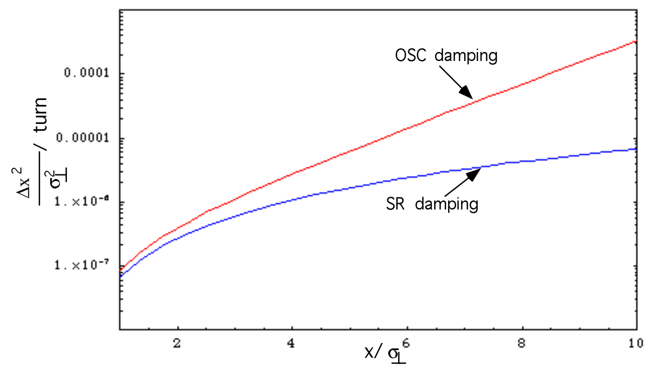

Figure 4: Critical diffusion versus the amplitude. Bottom curve shows only effect of synchrotron radiation (SR) damping. Top curve shows the combined effect of SR damping and OSC.

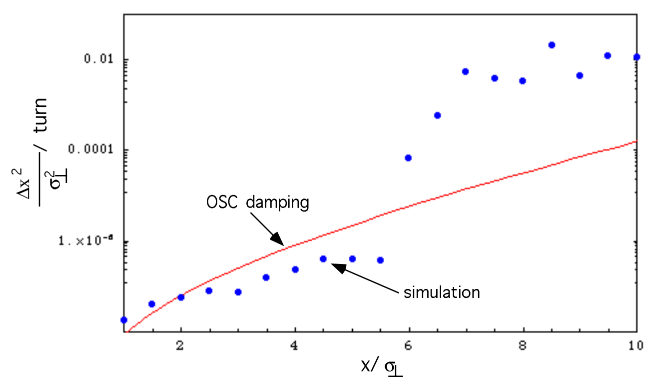

Figure 5: The LHC example. Solid line is the critical diffusion from this analysis. Dots are the simulated diffusion from [10].

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