

WAKE FIELDS DUE TO PERIODIC ROUGHNESS IN A CIRCULAR PIPE

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Abstract

The wake fields generated by a relativistic particle traveling in a long beam pipe of circular cross section with rough surface have been studied by means of standard theory based on hybrid modes in a periodically corrugated wave guide. Slow waves whose amplitude and frequency depend on the corrugation depth can be excited by the beam; the features of the resulting longitudinal wake fields are investigated.

1 INTRODUCTION

The effect of surface roughness is a subject arisen in the design of machines with extremely short bunches of the order of tens of microns. In this case, in fact, the surface roughness may be a source of wake fields which might significantly increase the beam emittance and the energy spread. Recently, a corrugation of the LHC (Large Hadron Collider) beam pipe has been proposed in order to reduce the reflectivity of the walls, and therefore to decrease the heat load on the dipole beam screen due to photoelectrons accelerated by the proton beam [1]. In LCLS (Linac Coherent Light Source) the surface roughness is due to residual defects in workmanship, and may be responsible of the longitudinal emittance growth due to wakefields.

In this paper we review the problem of the wake fields produced by an ultra-relativistic charge traveling inside a beam tube with a periodic corrugation making use of a standard theory based on the hybrid modes propagating in the waveguide [2, 3]. The paper is structured as follows: in Section 2 we describe the method used, in Section 3 we present the results: first the dispersion relation for the fields and the frequency where the synchronous wave can be excited; then, the amplitude of the field excited by the charge, the wake function and the coupling impedance.

2 THE METHOD

Let us consider a periodically corrugated waveguide with circular cross-section, with inner radius a and outer radius b . We model the wall roughness as a series of periodic (with period L) obstacles of height h ($h = b - a$) and thickness t (see figure 1). The charge travels along the z -axis; we assume $t \ll L$, $L \ll \lambda$ and the ohmic losses in the material negligible.

The periodicity of the geometry along the z -axis allows the use of Floquet's theorem which implies a field solution independent of the period L (obtained from a single cell). The steps are the following: at first we solve the homogeneous problem, finding the modes propagating in the

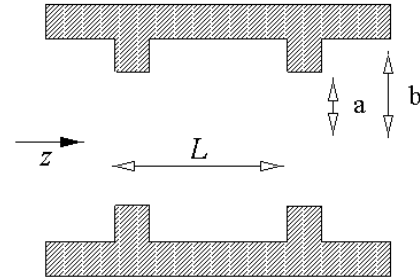


Figure 1: Schematic view of the waveguide and notation adopted.

waveguide and their features (the dispersion equation, the cut-off frequency and the frequency where the synchronous wave is excited). The dispersion relation is found by applying the continuity conditions for the field components over the boundary between the slot (the space inside the corrugation) and the internal region of the waveguide. The field inside the waveguide is considered as generated by the magnetic and the electric Hertz potentials along the z -axis. Then we apply the reciprocity principle, including the charge as an impulsive source, finding the coefficients used to express the electric field along the z -axis.

3 RESULTS

3.1 The Homogeneous Problem

The electromagnetic fields inside the corrugation are considered to be those due to propagating radial modes; higher-order evanescent modes are considered negligible, this assumption is justified under the hypothesis that the wavelength is much greater than the distance between two corrugations ($\lambda \gg L$). The components of the electromagnetic field are:

$$E_r^S = 0 \quad (1)$$

$$E_\phi^S = 0 \quad (2)$$

$$E_z^S = \sum_n [C_n J_n(k_0 r) + D_n Y_n(k_0 r)] \cos(n\phi) \quad (3)$$

$$H_r^S = \sum_n \frac{n}{j\omega\mu r} [C_n J_n(k_0 r) + D_n Y_n(k_0 r)] \sin(n\phi) \quad (4)$$

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$$H_\phi^S = \frac{k_0}{j\omega\mu} \sum_n [C_n J'_n(k_0 r) + D_n Y'_n(k_0 r)] \cos(n\phi) \quad (5)$$

$$H_z^S = 0 \quad (6)$$

where $e^{j(\omega t - \beta'_n z)}$ is assumed and

$$\beta'_n = \sqrt{k_0^2 - k_t^2} \quad (7)$$

β'_n is the hybrid-mode propagation constant, k_0 is the free-space propagation constant and k_t is the transverse propagation constant. The field inside the waveguide is considered as generated by the Hertz potentials along the z-axis:

$$\Pi_{ez} = \sum_n A_n J_n(k_t r) \cos(n\phi) e^{j(\omega t - \beta'_n z)} \quad (8)$$

$$\Pi_{mz} = \sum_n B_n J_n(k_t r) \sin(n\phi) e^{j(\omega t - \beta'_n z)} \quad (9)$$

which are related to the field by the relation:

$$\mathbf{E} = -j\omega\mu\nabla \times \Pi_{\mathbf{m}} + (k^2 + \nabla\nabla\cdot)\Pi_{\mathbf{e}} \quad (10)$$

$$\mathbf{H} = j\omega\epsilon\nabla \times \Pi_{\mathbf{e}} + (k^2 + \nabla\nabla\cdot)\Pi_{\mathbf{m}} \quad (11)$$

The potential Π_{ez} generates TM_z modes and the potential Π_{mz} TE_z modes. The superposition of both modes gives the hybrid modes.

Applying the boundary conditions at $r = b$ we find:

$$E_z^S(b) = 0 \implies D_n = -\frac{C_n J_n(k_0 b)}{Y_n(k_0 b)} \quad (12)$$

and imposing it at $r = a$:

$$E_z^S = E_z \quad H_\phi^S = H_\phi \quad \text{for} \quad r = a \quad (13)$$

we find the dispersion relation for the hybrid mode.

The modes of interest are the TM_{0m} : the TE_z and the TM_{nm} (with $n \neq 0$) modes do not give any contribution in the application of the reciprocity principle.

The dispersion relation for $n = 0$ becomes:

$$\frac{J'_0(k_0 a)Y_0(k_0 b) - J_0(k_0 a)Y'_0(k_0 b)}{J_0(k_0 a)Y_0(k_0 b) - J_0(k_0 b)Y_0(k_0 a)} = \frac{k_0 J'_0(k_t a)}{k_t J_0(k_t a)} \quad (14)$$

In the hypothesis of small corrugations ($a \rightarrow b$) the cut-off frequency is found to be:

$$f_{co} = \frac{c}{2\pi} \left(\frac{\xi_{01}}{a+h} \right) \quad (15)$$

where ξ_{01} is the first zero of the Bessel function of first kind and order zero; in the same hypothesis we found the frequency where the synchronous wave is excited (crossing frequency):

$$\bar{f}_{cr} = \frac{c}{2\pi} \frac{\xi_{01}}{\sqrt{ah}} \quad (16)$$

In figure 2 is reported the Brillouin diagram when $h/a = 0.1$.

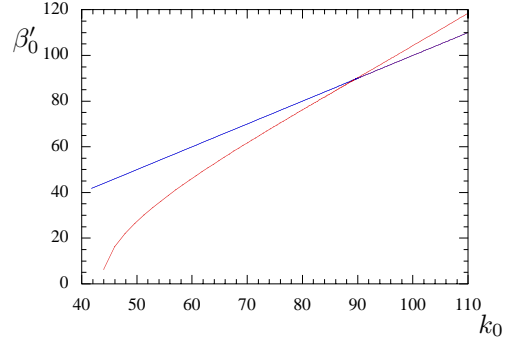


Figure 2: Brillouin diagram for a circular cross-section waveguide of radius a with periodic corrugations of depth h and $h/a = 0.1$.

The red line is the dispersion curve for the TM_{01} mode in the corrugated wave-guide and the blue one is the straight line $\beta'_0 = k_0$. For $\beta'_0 > k_0$ the wave is slow and can be synchronous.

3.2 Including the Sources

Once derived the modes of structures, having solved the homogeneous problem, the field generated by a point charge can be found by means of the Lorentz reciprocity principle [4].

The current density of a point charge traveling on-axis, \mathbf{J} , used in the reciprocity principle, is modeled as an impulsive source

$$\mathbf{J}(r, \phi, z; \omega) = q \frac{\delta(r)}{r} \delta(\phi) e^{-j \frac{z}{v_p} \omega} \mathbf{z}_0 \quad (17)$$

where \mathbf{z}_0 is the unit vector along z-axis and q is the charge.

In the hypothesis of small corrugation depth ($h \rightarrow 0$) it is immediate to find the expression of the electric field along the z-axis

$$E_z(r, \phi, z; \omega) = -\frac{q}{\bar{\omega}_{cr} \epsilon \beta'_0 a^2 F(k_t a)} k_t J_0(k_t r) \times \left[\delta\left(\frac{k_0}{\beta} - \beta'_0\right) + \delta\left(\frac{k_0}{\beta} + \beta'_0\right) \right] e^{-j\beta'_0 z} \quad (18)$$

where $\bar{\omega}_{cr}$ is the crossing frequency (expressed by 2π times the equation 16), ϵ is the dielectric constant in free-space, $F(k_t a) = J_1^2(k_t a) - J_0(k_t a)J_2(k_t a)$ and $J_0(k_t a)$, $J_1(k_t a)$

and $J_2(k_t a)$ are the Bessel functions of first kind and order 0, 1 and 2, respectively. For relativistic particles ($\beta \rightarrow 1$) the electric field is

$$E_z(z; \tau) = -\frac{8qZ_0ch}{(\xi_{01})^2\pi a^3} \cos(\bar{\omega}_{cr}\tau) e^{-j\beta'_0 z} \quad (19)$$

where c is the light velocity in free-space and Z_0 is the free-space characteristic impedance.

3.3 Longitudinal Coupling Impedance and Wake Function

Following the standard definition of the longitudinal coupling impedance per unit length [5]:

$$\frac{\partial Z(\omega)}{\partial z} = -\frac{1}{q} E_z(x=0, y=0, z, \omega) e^{j\omega z/c}, \quad (20)$$

from equation 18 we get

$$\frac{\partial Z_z(\omega)}{\partial z} = \frac{4Z_0ch}{(\xi_{01})^2\pi a^3} [\delta(\omega - \bar{\omega}_{cr}) + \delta(\omega + \bar{\omega}_{cr})] \quad (21)$$

Again from the definition [5], it is easy to get the longitudinal wake function per unit length

$$\frac{\partial w(\tau)}{\partial z} = -\frac{E_z(z; \tau)}{q} e^{j\omega z/c}. \quad (22)$$

From equation 19:

$$\frac{\partial w_z(z; \tau)}{\partial z} = \frac{8Z_0c}{(\xi_{01})^2\pi a^3} h \cos(\bar{\omega}_{cr}\tau) \quad (23)$$

3.4 The LCLS Case

As an example we report the application of the theory developed above to the case of LCLS [6]. We consider the case of a rectangular bunch, of temporal dimension $2T$, where T is given by $T = \frac{\sqrt{3}}{c}\sigma_l$, being σ_l ($= 15 \mu m$) the longitudinal dimension of the bunch.

We find that the crossing frequency is given by

$$\bar{f}_{cr} = 2.29 \cdot 10^{11} \frac{1}{\sqrt{h}} \quad [Hz] \quad (24)$$

and the amplitude of the wake function per unit length is given by

$$w'_0 = 3.2017 \cdot 10^{18} h \quad [V/Cm] \quad (25)$$

Let us focus our attention on the energy spread, that is given by [7]

$$\frac{\Delta E^{rms}}{E_0} = \frac{w'_0 D Q}{2E_0} \left[\frac{1}{2(\bar{\omega}_{cr} T)^2} \left(1 - \frac{\sin(4\bar{\omega}_{cr} T)}{4\bar{\omega}_{cr} T} \right) - \left(\frac{\sin(\bar{\omega}_{cr} T)}{\bar{\omega}_{cr} T} \right)^4 \right]^{\frac{1}{2}} \quad (26)$$

where $E_0 = 14.35 GeV$ is the total energy of the electron beam, $D = 112 m$ is the total length of the path followed by the beam and $Q = 1 nC$ is the bunch charge. In figure 3 is reported the energy spread vs. h [m].

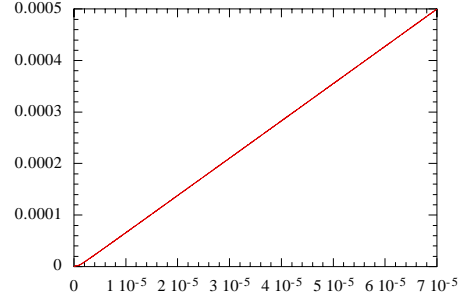


Figure 3: Energy spread for the circular cross-section waveguide vs. h .

The design parameters require the energy spread to be less than $5 \cdot 10^{-4}$, which is verified for value of h of the order of tens of microns.

4 CONCLUSIONS

We have derived the longitudinal wake due to a periodic corrugation in a circular beam pipe. The amplitude of the wake function is proportional to h and the crossing frequency to $1/\sqrt{h}$.

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