NUMERICAL CALCULATIONS OF THE ELECTRON COOLING DRAG FORCE IN A MAGNETIC FIELD

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Abstract

The longitudinal drag force that the electrons in an electron cooler exert on a circulating ion beam has been measured at many electron-cooling installations and also at CRYRING. Although different theoretical models have been used for calculation of this drag force, the discrepancy between theory and experiment have sometimes been quite big due to the theoretical difficulty in treating the interaction between charged particles in a magnetic field. We here present the beginning of an attempt to numerically calculate the energy loss suffered by ions in binary collisions with electrons in the presence of a finite magnetic field. Results for the longitudinal drag force are presented for relative velocities between ions and electrons and magnetic fields that are relevant for electron cooling at CRYRING and similar storage rings.

1 THE PROBLEM

The magnitude of the drag force that an ion experiences when it moves through the electron beam of an electron cooler has been calculated, with or without the inclusion of a magnetic field, using a number of different approaches such as binary collisions [1], dielectric theory [2] or molecular-dynamics simulations [3]. To include the magnetic field in a realistic and precise way is, however, a difficult task. Generally speaking, the two-body problem with a magnetic field is much more complicated than that without a field since the motion can no longer be separated into that of the center of mass and a relative motion. The field causes, for instance, the system to show chaotic behaviour for certain parameter ranges.

In our case, the electron is more or less strongly bound to a field line and it can approach the ion several times during successive cyclotron orbits. For large impact parameters this is conventionally regarded as the ions colliding with an electron "disk", whose charge is smeared out over the cyclotron orbits and which has a very small transverse motion, leading to an increased drag force. Indeed, a drag force larger than what has been calculated from theories that do not take the magnetic field into account has been observed at several cooler installations.

Another feature of magnetized collisions, in contrast to classical Rutherford scattering, is that negative and positive ions have different drag forces. This effect that also has been observed experimentally [4].

Here we present the beginning of an attempt to get accurate numerical values for the electron-cooling drag force in the presence of a finite magnetic field using a binary-collision approach. The aim is to calculate the energy loss for heavy ions passing through an electron gas that has the anisotropic temperature and the magnetization that are characteristic of an electron cooler. Outside the scope of this investigation, however, is the inclusion of plasma effects or electron–electron interactions.

2 THE METHOD

The method we use to calculate the energy loss of an ion colliding with an electron is simply to numerically integrate the classical equations of motion of the two particles. We define the magnetic field to be in the positive z direction, and the terms 'longitudinal' and 'transverse' refer to the direction of the field.

The initial condition is that the electron is performing gyro motion on a circle around the origin in the plane $z = 0$ with velocity $v_{\mathrm{e},0}$. The ion starts far below that plane and moves toward it with a velocity $v_{\mathrm{i},0}$ directed along the z axis. It scatters against the electron and continues until it no longer is influenced by the force of the electron, whereupon the energy loss suffered by the ion is computed. The energy loss thus gives the longitudinal drag force for an ion that has an initial velocity that is purely longitudinal. This energy loss or drag force is then integrated over all impact parameters of the ion and over all phases of the electron gyro motion (or, equivalently, over an impact-parameter plane assuming that the electron always starts in the same (x, y) coordinate). The drag force is finally normalized to an electron density of 1×10^{14} m^{-3}.

The force acting between the ion and the electron is a screened Coulomb force, and the Debye length is used as the screening length. Since the Debye length

$$\lambda_{\mathrm{D}} = \left(\frac{\epsilon_0 kT}{n_{\mathrm{e}} q^2}\right)^{1/2}$$

depends on the electron density n_{e} and the electron temperature T, there are two additional choices to be made here. We have used an electron density of 1×10^{13} m^{-3}, which is typical for CRYRING, and as the electron temperature we have taken $kT_{\mathrm{e}} = m_{\mathrm{e}} v_{\mathrm{e},0}^2/2$. However, for a few different combinations of $v_{\mathrm{e},0}$ and $v_{\mathrm{i},0}$ other values of λ_{D} were also used in order to test how sensitive the results are to the choice of λ_{D}, see below.

The calculations were performed for a magnetic field of 0.1 T, but the results can easily be transformed to other fields. It is readily seen that the equations of motion are

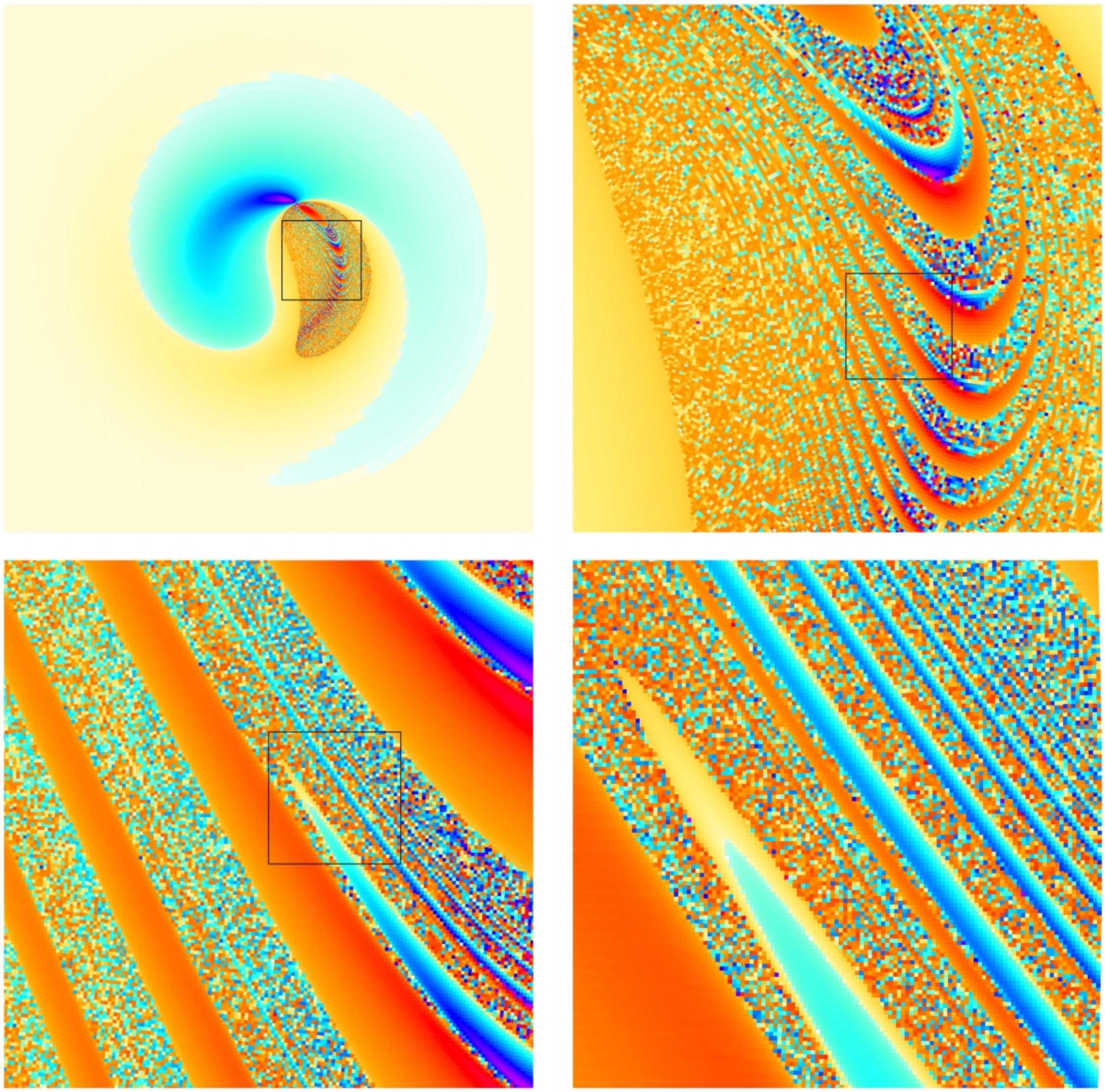

Figure 1: Drag force as a function of the impact parameter of the ion, colour coded such that blue indicates a positive force and red a negative force, more intense colour representing stronger force. The size of the upper left picture is $20 \times 20\mu$m and the black squares are blown up in the subsequent pictures.

invariant under a scaling of variables such that $r \to kr$, $v \to k^{-1/2}v$, $F \to k^{-2}F$, $B \to k^{-3/2}B$, etc., r representing distances, v velocities, F forces and B magnetic fields and k being an arbitrary scale factor. As long as the scaled $v_{\rm e,0}$ and $v_{\rm i,0}$ are within the range that we have computed, we can thus obtain the drag force for any value of the magnetic field.

The ions used are deuterons, i.e., particles with charge 1 and mass 2. The reason is to eventually allow a comparison with the cooling forces measured at CRYRING, although the results should be essentially independent of the ion mass.

It should also be said that a difference between these simulations and collisions in a real electron cooler is end effects in the cooler. We assume that the ions come from $z = -\infty$ and move to $z = +\infty$ which in reality is not the case because of the finite time of interaction available as the ions pass through the cooler.

3 RESULTS

In figure 1 is shown, as an example, the drag force calculated for initial conditions $E_{\rm e,0} = 0.00625$ eV and $v_{\rm i,0} = 7000$ m/s, where $E_{\rm e,0} = m_{\rm e}v_{\rm e,0}/2$. (Since the transverse electron motion often is characterized by the energy or temperature, we specify the electron's energy rather than velocity, whereas we label the ion by its velocity since drag forces usually are plotted as a function of velocity).

The figure shows the force as a function of the impact parameter of the ion (radial coordinate) and the phase of the electron's gyro motion (azimuthal coordinate). The black squares in the first three panes indicate the part that is magnified in the following pane. The force is colour coded, and blue indicates a positive force (the ion is accelerated by the electron) while red indicates a negative force (the ion is decelerated). The reader is referred to the CD or web version of these proceedings rather than the book in order to view the colours. The region immediately above and to the right of the origin in the yin-yang-like shape of the upper left picture corresponds to a positive force, i.e., the electron being scattered backward. The rest of the picture, including the large impact parameters, shows negative force. An exception is a chaotic region, characteristic of the dynamics of this system, where the particles have several close encounters. In these encounters, the electrons again can scatter either forward or backward.

More specifically, the large bands in figure 1 correspond to the second encounter taking place one, two, etc., cyclotron periods after the first encounter. The finer bands, seen in the two lower panes, are caused by a third close encounter at different cyclotron periods after the second one, etc.

This pattern also hints at the computational difficulty in obtaining results that converge reasonably well with respect to both the spatial resolution in the impact parameter plane and the time resolution of the integration.

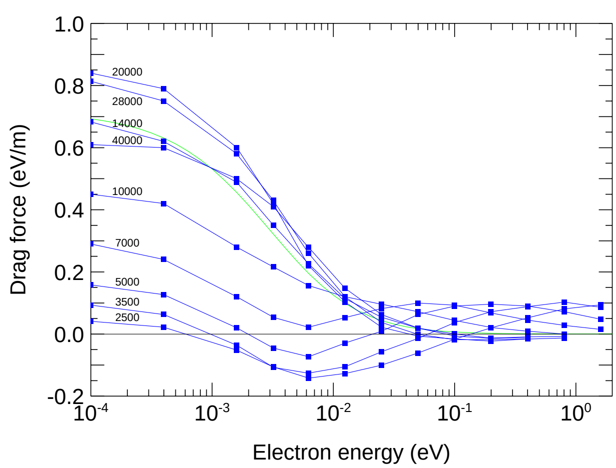

Figure 2: Longitudinal drag force as a function of electron energy. The curves represent different ion velocities between 2500 and 40000 m/s. The smooth curve is theory without magnetic field (see text).

The drag force has been computed for, so far, nine different ion velocities between 2500 and 40000 m/s, and electron energies range between 4×10^{-4} eV and 1.6 eV. A summary of the results are shown in figure 2. As already stated, the forces were calculated for a magnetic field of 0.1 T, but can easily be rescaled. Values at, e.g., the 0.03 T normally used at CRYRING are thus obtained by multiplying ion velocities by 0.67, electron energies by 0.45 and drag forces by 2.23 if they still are to be normalized to an electron density of 1×10^{14} m^{-3}.

As mentioned above, we have normally set the electron temperature kT in the expression for the Debye length equal to $E_{\mathrm{e},0}$. For a few points at the highest and the lowest electron energies, $E_{\mathrm{e},0} = 4 \times 10^{-4}$ and 1.6 eV, and also one intermediate point, we checked the sensitivity of the results to the choice of kT by instead using $kT = 0.1$ eV in the Debye length. The maximum deviation was less than 0.05 eV/m , showing, as one would expect, that moderate changes in the Debye length do not critically influence the drag force.

At high ion velocities, it is expected that the effect of the magnetic field on the drag force is small. As a comparison we have thus included in figure 2 the longitudinal force for an ion velocity $v_{\mathrm{i}} = 40000$ m/s calculated according to the standard expression

$$\mathbf{F}(\mathbf{v}_{\mathrm{i}}) = -4\pi \left(\frac{q^2}{4\pi\epsilon_0} \right)^2 \frac{n_{\mathrm{e}} L_{\mathrm{C}}}{m_{\mathrm{e}}} \frac{\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{e}}}{|\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{e}}|^3}.$$

Here, L_{C} is the Coulomb logarithm which we have set to the rather small value of 2.5 in order to make the curve fit to the simulations. We assume that in order to reach agreement using a more appropriate value of the Coulomb logarithm, a still higher ion velocity is required.

4 CONCLUSION

We have calculated numerically the longitudinal drag force for ions with purely longitudinal velocities interacting with electrons in the presence of a finite magnetic field. We only have results for limited ranges of ion and electron velocities due to the amount of computing time required. We plan to extend the ranges and to include ions with transverse velocity components. The latter is, however, even more computationally demanding, since it involves integration over impact parameters in one more dimension. Also, we should be able to make comparisons between positive and negative ions.

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REFERENCES

[1] Ya.S. Derbenev and A.N.Skrinsky, Part. Acc. **8** 234 1978

[2] A.H. Sørensen and E. Bonderup, Nucl. Instr. Meth **215** 27 (1983)

[3] G. Zwicknagel, C. Toepffer and P.-G. Reinhard, Phys. Rep. **309** 117 (1999)

[4] N.S. Dikansky, N.Kh. Kot, V.I. Kudelainen, V.A. Lebedev, V.V. Parkhomchuk, A.A. Seriy, A.N. Skrinsky, B.N. Sukhina and V.D. Shiltsev, Proc. EPAC-88, p. 529