LINEAR EFFECTS OF CROSSING ANGLE AND DISPERSION AT THE INTERACTION POINT

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Abstract

The presence of large dispersion and crossing angle at the interaction point is studied within the linear approximation of the beam-beam force. The betatron and synchrotron tune shifts are computed.

1 INTRODUCTION

Recently the tau-charm factories are planned seriously where the monochromatization are considered [1]. The idea of the monochromatization is that a rather large dispersion D exists at the interaction point (IP), having the opposite signs for e^+ and e^- beams, to make the spread of the collision energy much smaller than the nominal one, $\sqrt{2}\sigma_\epsilon^0$.

The dispersion at the IP is known as a source of the synchro-betatron coupling. In standard textbooks such as Ref. [2], it appears that the definition of the dispersion is based on the assumption that the energy of an electron is constant, and the synchrotron degree of freedom is not affected at all. This approach appears to be intuitively valid when the absolute value of the synchrotron tune ν_z is very small, but we showed [3] that even this is not true. The usual definition of the dispersion does not work under such a situation.

In this paper we discuss the symplectic effects of the dispersion and crossing angle at the IP paying attention to the mutual interaction between the betatron and the synchrotron degrees of freedom within the linear and weak-strong approximation of the beam-beam force.

2 ONE TURN MATRIX

We assume that there is only one interaction point and consider the vertical and longitudinal motions only. The physical variables for the betatron and synchrotron motions are $\mathbf{x} = (y, p_y, z, \varepsilon)$ where y is the vertical coordinate, $p_y = m\gamma(dx/ds)/p_0$ the vertical momentum normalized by the momentum p_0 of the reference particle (a constant), $z = s - ct(s)$, $\varepsilon = (E - E_0)/E_0$, E_0 being the energy of the reference particle, and γ the relativistic factor of the nominal particle energy.

The strong beam is regarded as a special "focusing quadrupole magnet".

The one turn matrix from the IP ($s = 0_+$) to IP($s = 0_-$), excluding the beam-beam kick, can be put in the following form[4].

$$M_{arc} = M(0_-, 0_+) = H_0 B_0 \hat{M}_{arc}\, B_0^{-1} H_0^{-1}, \quad (1)$$

where

$$\hat{M}_{arc} = \begin{pmatrix} r(\mu_y^0) & 0 \\ 0 & r(\mu_z^0) \end{pmatrix}, \quad (2)$$

$$r(\mu_{y,z}^0) = \begin{pmatrix} \cos\mu_{y,z}^0 & \sin\mu_{y,z}^0 \\ -\sin\mu_{y,z}^0 & \cos\mu_{y,z}^0 \end{pmatrix}, \quad (3)$$

$$B_0 = \begin{pmatrix} b_y^0 & 0 \\ 0 & b_z^0 \end{pmatrix}, b_{y,z}^0 = \begin{pmatrix} \sqrt{\beta_{y,z}^0} & 0 \\ 0 & 1/\sqrt{\beta_{y,z}^0} \end{pmatrix}, \quad (4)$$

$$H_0 = \begin{pmatrix} I & h_0 \\ \tilde{h}_0 & I \end{pmatrix}, \quad h_0 = \begin{pmatrix} 0 & D_0 \\ 0 & 0 \end{pmatrix}, \quad (5)$$

and $\tilde{h}_0$ is the symplectic conjugate of h_0, $\tilde{h}_0 = j h_0^t j$. Here j is the 2×2 symplectic metric ($j_{11} = j_{22} = 0, j_{12} = -j_{21} = 1$) and $\mu^0 = 2\pi\nu^0$ with ν^0 being the nominal tunes and β^0 is the nominal betatron function at the IP ($\beta_z^0 \equiv \sigma_z^0/\sigma_\varepsilon^0$, where σ_z^0 and σ_ε^0 are nominal bunch length and energy spread, respectively). Note that H_0, B_0, and $\hat{M}_{arc}$ are symplectic $H_0^t J H_0 = J$, etc. where $J = diag(j, j)$ is the 4×4 symplectic metric. The nominal synchrotron tune ν_z^0 is negative for conventional electron machines with positive momentum compaction factor. We will however consider both signs for ν_z^0 because the option of the negative momentum compaction factor[6] is being considered, which makes the ν_z^0 positive. We have assumed that the IP is a symmetric point with respect to betatron and synchrotron motions. We have also implicitly assumed that the dispersion does not exist in cavities. The matrix H_0 decouples the betatron and the synchrotron motions in a symplectic way. One can regard Eq.(1) as the definition of the dispersion, D_0.

Note that the suffix 0 refers to all the unperturbed quantities evaluated without the presence of the beam-beam interaction. Let us turn on the beam-beam interaction at the IP. For the headon collision, the linearized beam-beam force is represented by the matrix

$$M_{bb} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -4\pi\xi_0/\beta_y^0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (6)$$

and $\xi_0 = N r_e \beta_y^0 / 2\pi\gamma\sigma_y^0(\sigma_y^0 + \sigma_x^0)$. is the vertical (nominal) beam-beam parameter, N the number of particles in the strong beam, r_e the classical electron radius, σ_x^0 (σ_y^0) the nominal horizontal (vertical) beam size. Moreover it is

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Table 1: Standard parameters used. They give $\chi = .2$

β_y^0	0.03m	β_z^0	26.3m
ϵ_y^0	$4 \cdot 10^{-9}$m	ϵ_z^0	$3.8 \cdot 10^{-6}$m
σ_ε^0	$3.8 \cdot 10^{-4}$	σ_z^0	0.01m
T_y	1000	T_z	500

$\sigma_y^0 = \left[\beta_y^0\epsilon_y^0 + D_0^2\epsilon_z^0/\beta_z^0\right]^{1/2}$, where ϵ_y^0 and $\epsilon_z^0 \equiv (\sigma_z^0\sigma_\epsilon^0)^{1/2}$ are the vertical and longitudinal emittances, and all quantities are evaluated at the IP.

The introduction of the crossing angle is rather straightforward: following [5] we introduce the transverse Lorentz boost to make the collision headon. The mapping at the IP is as follows:

$$\mathbf{x}(0_-) \xrightarrow{\mathcal{L}} \mathbf{x}^*(0^*) \xrightarrow{B-B} \mathbf{x}^{*'}(0^*) \xrightarrow{\mathcal{L}^{-1}} \mathbf{x}(0_+). \tag{7}$$

At IP a boost map is applied to the (physical) particle coordinates $\mathbf{x}(s = 0)$ to perform a Lorentz trasformation ($\mathcal{L}$) which makes the collision headon. Then, M_{bb} is applied in the boosted frame. The coordinates are then trasformed back to the original frame using the inverse boost map ($\mathcal{L}^{-1}$). Therefore, the complete one turn matrix eveluated in the laboratory frame is

$$M = \mathcal{L}^{-1}M_{bb}^{1/2}\mathcal{L}M_{arc}\mathcal{L}^{-1}M_{bb}^{1/2}\mathcal{L}, \tag{8}$$

$$\mathcal{L} = \begin{pmatrix} 1 & 0 & \tan\phi & 0 \\ 0 & 1/\cos\phi & 0 & 0 \\ 0 & 0 & 1/\cos\phi & 0 \\ 0 & -\tan\phi & 0 & 1 \end{pmatrix}. \tag{9}$$

Here ϕ is the half crossing angle (vertical crossing is assumed). Note that $\mathcal{L}$ is not symplectic but the combination $\mathcal{L}^{-1}M_{bb}\mathcal{L}$ is so and thus the whole one turn matrix M is symplectic, too. Also note that the Lorentz transformation itself is nonlinear[5] and $\mathcal{L}$ is its linearized form.

2.1 *Linear Instabilities*

The real (perturbed) tunes can be obtained from the eigenvalues of M. Since M is symplectic, we can calculate the eigenvalues in a straightforward manner. The expression is, however, too long. Instead, we first discuss approximation. To lowest order in ξ_0, we get the betatron and synchrotron tune shifts as follows:

$$\nu_y^0 \to \nu_y^0 + \xi_0\cos\phi\left(1 + \frac{D_0^2\tan^2\phi}{(\beta_y^0)^2}\right), \tag{10}$$

$$\nu_z^0 \to \nu_z^0 + \xi_0\cos\phi\frac{\beta_z^0}{\beta_y^0}\left(\tan^2\phi + \frac{D_0^2}{(\beta_y^0)^2}\right). \tag{11}$$

Besides the well known betatron tune shift, Eq.(10), the synchrotron tune shift Eq.(11) occurs due to D_0 and/or ϕ. Both tunes increase. Considering the motion of the eigenvalues on the unit circle in the complex plane, we can expect that the system becomes unstable when one of the following conditions applies: $\nu_y^0 \lesssim$ half integers (betatron instability); $\nu_z^0 \lesssim$ half integers (synchrotron instability); $\nu_z^0 + \nu_y^0 \lesssim$ integers (synchro-betatron instability). We can calculate the eigenvalues numerically and exactly: the linear motion is unstable when some of the eigenvalues of M is larger than unity in absolute value. In Fig. 1 we plot the instability regions in the parameters space in terms of the growthrate-1. (Hereafter a set of model parameters listed in Table 1 are used unless otherwise stated.) The three unstable regions stated above are clearly seen. The unstable regions become thick for larger values of ξ_0, D_0 and ϕ. As clear from the figure, a machine might be intrinsically more stable when $\nu_z^0 > 0$, because we can get rid of the synchrotron and synchro-betatron instabilities. We note that the region of the synchro-betatron instability (both upper and lower edges) moves in the $\nu_y^0 - \nu_z^0$ plane as ξ_0 changes, while the upper edges of the betatron and synchrotron instabilities are fixed. This "floating instability" seems to be typical to the sum resonance in the beam-beam interaction [7].

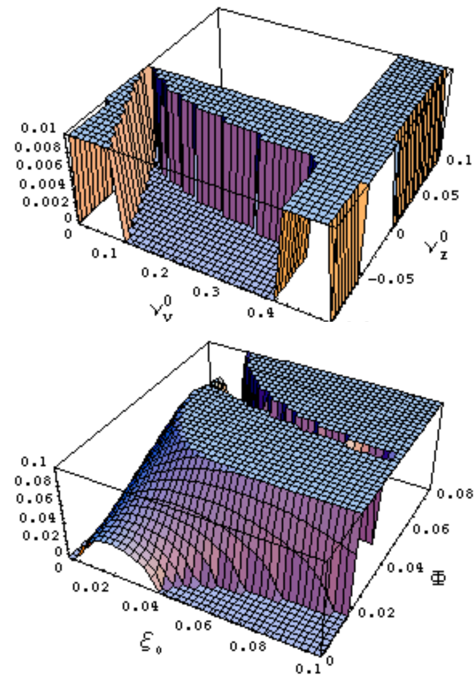

Figure 1: Crossing angle and dispersion at IP. The growthrate (-1) (top) in the (ν_y^0, ν_z^0) plane with $(\phi = 0.02, D_0 = 0.4\ m, \xi_0 = 0.05)$, (bottom) in the (ξ_0, ϕ) plane with $(D_0 = 0.4\ m, \nu_y^0 = 0.1, \nu_z^0 = -0.11)$. $(\xi_0 = 0.05, \nu_y^0 = 0.1, \nu_z^0 = -0.11)$.

Note that when $\nu_z^0 \lesssim 0$, the motion becomes unstable: $\nu_z^0 = 0$ is the singular point and the coasting beam approximation is quite dangerous in this case.

2.2 *Dispersion Only*

In simpler cases where only D_0 is not zero the eigenvalues can be expressed in rather simple manner. The perturbed tunes are given by rather short form [3]:

$$2\cos\mu_{\pm} = \cos\mu_y^0 + \cos\mu_z^0 - 2\pi\xi_0(\sin\mu_y^0 + \chi\sin\mu_z^0) \pm \sqrt{d}, \tag{12}$$

$$d = \left\{\cos\mu_y^0 - \cos\mu_z^0 - 2\pi\xi_0(\sin\mu_y^0 - \chi\sin\mu_z^0)\right\}^2 + +16\pi^2\xi_0^2\chi\sin\mu_y^0\sin\mu_z^0, \tag{13}$$

where the synchrotron tune shift factor is:

$$\chi = \frac{D_0^2}{\beta_y^0\beta_z^0} \sim \frac{D_0^2\sigma_\varepsilon^0}{\beta_y^0\sigma_z^0} \tag{14}$$

The motion is stable if and only if $|\cos\mu_{\pm}| \le 1$ and $d \ge 0$. To lowest order in ξ_0, we get

$$\nu_y^0 \to \nu_y^0 + \xi_0, \quad \nu_z^0 \to \nu_z^0 + \xi_0\chi. \tag{15}$$

It may be useful to note that the synchrotron tune shift effect is remarkable for 1) large D_0, 2) large σ_ε^0, 3) small σ_z^0, 4) small β_y^0 and 5) $|\nu_z^0|$ small. The items 3), 4) and 5) are general tendency when we want to have large luminosity by making beam size small and avoiding synchro-betatron side bands[8]. The condition $\chi \ll 1$ is equivalent to $(D_0\sigma_\varepsilon^0)^2/\beta_y^0 \ll \epsilon_z^0$. On the other hand, for the monochromatization to be useful, it should be $\epsilon_y^0 \ll (D_0\sigma_\varepsilon^0)^2/\beta_y^0 \ll \epsilon_z^0$.

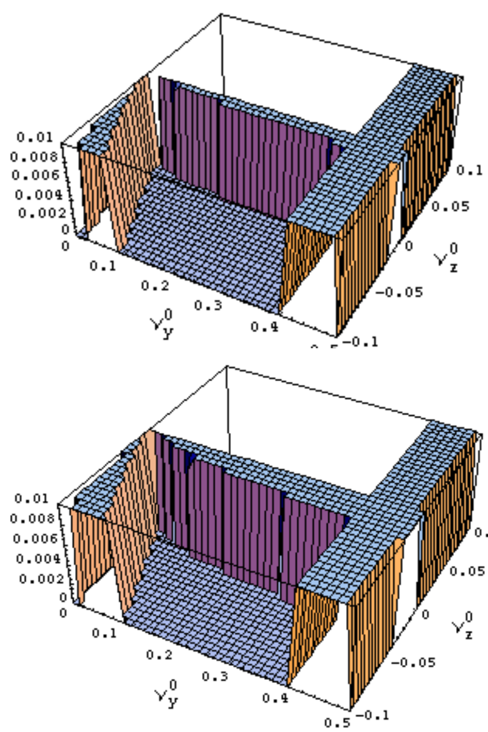

Figure 2: The growthrate (-1) in the (ν_y^0, ν_z^0) plane with $\xi_0 = 0.05$ in the presence of (top) dispersion only ($D_0 = 0.4\,m, \phi = 0$), (bottom) crossing angle only ($D_0 = 0, \phi = 0.02$).

2.3 *Crossing angle Only*

In the presence of crossing angle only at IP we get:

$$2\cos\tilde{\mu}_{\pm} = (\cos\mu_y^0 + \cos\mu_z^0) - 2\pi\xi_0\cos\phi(\sin\mu_y^0 + \frac{\beta_z^0}{\beta_y^0}\sin\mu_z^0\tan^2\phi) \pm \sqrt{\tilde{d}}, \tag{16}$$

$$\tilde{d} = [(\cos\mu_y^0 - \cos\mu_z^0) - 2\pi\xi_0\cos\phi(\sin\mu_y^0 + \frac{\beta_z^0}{\beta_y^0}\sin\mu_z^0\tan^2\phi)]^2 +16\pi\xi_0(\cos\mu_y^0 - \cos\mu_z^0)\frac{\beta_z^0}{\beta_y^0}\sin\mu_z^0\tan^2\phi\cos\phi. \tag{17}$$

To lowest order in ξ_0, we get

$$\nu_y^0 \to \nu_y^0 + \xi_0\cos\phi, \tag{18}$$

$$\nu_z^0 \to \nu_z^0 + \xi_0\frac{\beta_z^0}{\beta_y^0}\cos\phi\tan^2\phi. \tag{19}$$

3 CONCLUSIONS

Through the dispersion at IP, the synchrotron and betatron motions influence each other, giving several nontrivial strong effects, overlooked before, on the synchotron motion in addition to the well-known transverse effects for rather small values of ξ_0 [3].

In this paper we showed the linear instabilities due to the presence of both crossing angle and dispersion at IP. There does not seem to be any qualitative difference with respect to the presence of dispersion only studied in [3]. They act similarly with respect to the instability threshold, and just add to each other and no compensation mechanism exists.

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