AN IMPROVED METHOD FOR THE DIPOLE AND QUADRUPOLE MAGNET DESIGN

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Abstract

The dipole and quadrupole magnets are used frequently in the different types of accelerators. The solution of Laplace’s equation was introduced to calculate the pole shape based on the assumption that the pole surface is the equal scalar potential surface if the permeability is high enough [1,2]. During the calculation of the pole shape, one usually uses the error function to describe the distribution of the magnetic field or the gradient. However, we suffered some convergence problem in the solving process of the equal potential equation with the high order derivatives of the error function. In this paper, a new function is opted to describe the distribution of the field for the dipole or the gradient for the quadrupole. The solutions of the 3D Laplace’s equation determined by the distribution from this new function are investigated widely. A dipole magnet with the resulting shape of the pole is designed by the numerical computation to confirm the performance of this new function and is fabricated based on such a design for the experimental study. The pole shape of a quadrupole magnet with the gradient distribution by the new function is also described in this paper.

1 INTRODUCTION

The dipole and quadrupole magnets are playing an essential role in many accelerators and their transport lines. If the various species of ions are expected to be accelerated, stored and transported, e.g. the synchrotron for proton, light-heavy ions, polarized ions and electrons, the beam lines for RIB facilities, etc, the dipoles and quadrupoles used for these machines are provided with large dynamical field range. It is important to obtain the wide good field region through the whole dynamical field range. The field quality of the dipoles and quadrupoles is chiefly based on the pole shape. In RCNP, an effort to improve the field quality is made and some magnets are developed [3]. In this paper, an improved method for the dipole and quadrupole magnet design will be introduced.

2 C-MAGNET DEVELOPMENT

2.1 The solution of Laplace’s equation

The solution of Laplace’s equation was introduced to calculate the pole shape. In the current free region, the flux density $\vec{B}$ can be expressed with the scalar potential φ, which is governed by the Laplace’s equation. From the Laplace’s equation, the equation used to define the pole surface can be derived if the permeability of the magnet is high enough. During the derivation for the equation of the pole surface, one uses the error function to represent the field distribution and know the expression for the equation of the pole surface written as follows.

$$\frac{d}{2}=\sum_{\mu=0}^{\infty}\frac{(-1)^{\mu}}{(2\mu+1)!}y^{2\mu+1}\left\{\sum_{i=0}^{\mu}C_i^{\mu}\left[\frac{\partial}{\partial x}^{2(\mu-i)}g(x)\right]\cdot\left[\frac{\partial}{\partial s}^{2i}G(s)\right]\right\} \tag{1}$$

where d is the height of gap, C_i^{μ}, $g(x)$ and $G(s)$ are given by:

$$C_i^{\mu}=\frac{\mu!}{i!(\mu-i)!}$$

$$g(x)=\left[efr\left(\frac{x+x0}{\sigma_x}\right)-efr\left(\frac{x-x0}{\sigma_x}\right)\right]\Big/efr\left(\frac{x0}{\sigma_x}\right)$$

$$G(s)=\left[efr\left(\frac{s+s0}{\sigma_s}\right)-efr\left(\frac{s-s0}{\sigma_s}\right)\right]\Big/efr\left(\frac{s0}{\sigma_s}\right)$$

$g(x)$ and $G(s)$ are the field distribution on the median plane along the x and s direction respectively. The coordinates system used in this paper is defined as: s, the distance along the reference orbit, y, perpendicular to the pole surface, x, on the medium plane and perpendicular to s.

We observe that the super high order derivatives of the error function should be calculated for solving the Eq. (1). And the derivatives of $g(x)$ and $G(s)$ are coupled together . Such a coupling of derivatives causes some convergence problems. One can truncate μ at a specific value, but the solution is not reasonable some time if μ is not high enough. To improve such a equal potential method for the pole shape representation, we introduce a new function to describe the field distribution in the no current source region. They are defined as:

$$g(x)=q(x)-f(x)$$
$$G(s)=Q(s)-F(s)$$

$$q(x)=\frac{1}{2(1-e^{-\frac{x0}{\sigma_x}})}\left[\frac{x+x0}{|x+x0|}-\frac{x-x0}{|x-x0|}\right]$$

$$f(x)=\frac{1}{2(1-e^{-\frac{x0}{\sigma_x}})}\left[\frac{x+x0}{|x+x0|}\cdot e^{-\frac{|x+x0|}{\sigma_x}}-\frac{x-x0}{|x-x0|}\cdot e^{-\frac{|x-x0|}{\sigma_x}}\right]$$

$$Q(s)=\frac{1}{2(1-e^{-\frac{s0}{\sigma_s}})}\left[\frac{s+s0}{|s+s0|}-\frac{s-s0}{|s-s0|}\right],$$

$$F(s)=\frac{1}{2(1-e^{-\frac{s0}{\sigma_s}})}\left[\frac{s+s0}{|s+s0|}\cdot e^{-\frac{|s+s0|}{\sigma_s}}-\frac{s-s0}{|s-s0|}\cdot e^{-\frac{|s-s0|}{\sigma_s}}\right]$$

Then, from 3D Laplace's equation, we get:

$$\frac{d}{2}=y\cdot g(x)\cdot G(s)+\sum_{\mu=1}^{\infty}\frac{(-1)^{\mu}}{(2\mu+1)!}y^{2\mu+1}\left\{\sum_{i=0}^{\mu}C_i^{\mu}\left[\frac{f(x)}{\sigma_x^{2(\mu-i)}}\right]\cdot\left[\frac{F(s)}{\sigma_s^{2i}}\right]\right\} \qquad (2)$$

It can be found that the derivatives are disappeared in the Eq. (2). So, we can get the solution with very high μ by the proper numerical method. We used this improved equal potential method to design a prototype C-magnet. A typical field distribution along the beam direction *s* defined by the new function is shown in Fig. 1. The pole shape will be fixed after the solving of Eq. (2). Fig.2 is extracted from the design stage, which gives the pole shape of the C-magnet with the different truncated μ. It shows us that the higher μ is necessary in the practical design. Because the field distribution along *x* direction is affected strongly by the coils and return yoke, one should not expect to use a single function to represent the field distribution in the whole space. We use two sets of the different parameters to define two distribution functions *g1(x)* and *g2(x)*, which represent the field inside and outside the pole respectively.

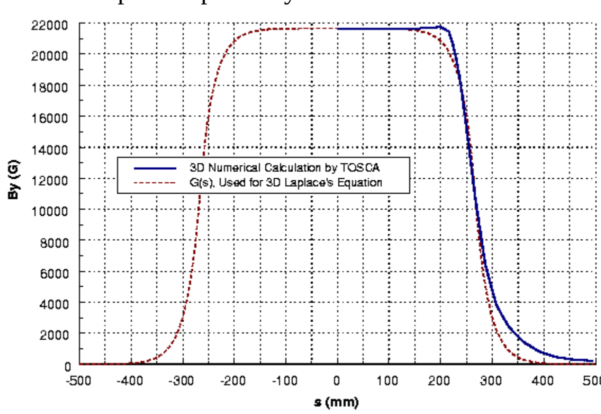

Fig. 1 Field distribution along reference orbit

2.2 Numerical Design

Based on the resulting pole shape defined by Eq. (2), a dipole magnet is modeled by TOSCA [4]. The modeled

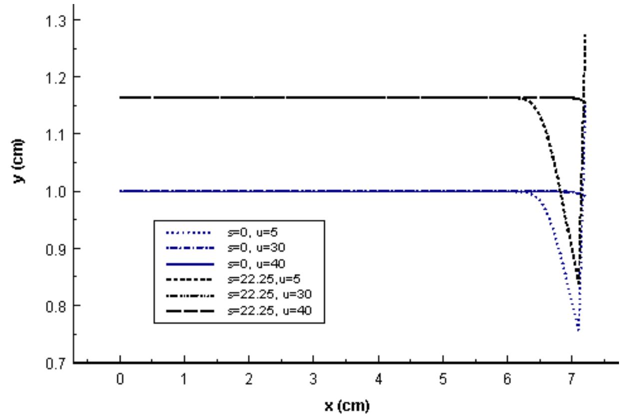

Fig. 2 Pole shape calculated by different truncated μ

magnet is shown in Fig. 3 and the field along *s* got from TOSCA is also plotted in the Fig. 1. The field around *s*=200 mm are different with that from the function *G(s)*. We think it is due to the saturation of the end piece. At *s*=350 mm, the higher field are because of the closer coils which also shown in Fig. 3. The good field regions for

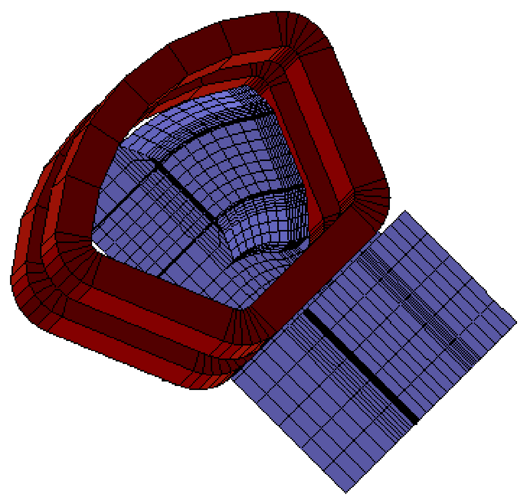

Fig. 3 3D FEM calculation for the C-magnet

various field intensities are estimated by POISSON [5]. From Fig. 4, it can be found that the good field regions are improved, especially at the high field range. The good field regions are extended to about ±4.8 cm when the uniformity kept better than $\pm 5.0\times 10^{-4}$. The numerical results show us that the pole shape, including the end pieces defined by Eq. (2) provides a good field distribution in the gap. The overview parameters of the final design is listed as following:

- Gap = 2.4 cm
- Width of pole = 17.5 cm
- Angle of bend = 90 degrees
- Radius of curvature = 34 cm
- Entrance pole rotation angle = 20 degrees
- Exit pole rotation angle = 20 degrees
- Field index n=0

The fabrication of the C-magnet has been finished. The magnet is installed in RCNP and is ready for measurement. The experimental study will be started soon.

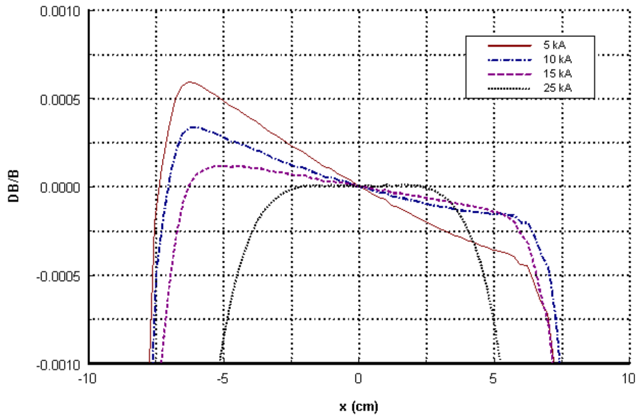

Fig. 4: The good field region for various field intensity

3 POLE SHAPE OF QUADRUPOLE

By using the method of separation of variables, the solution of 3D Laplace's equation in cylindrical coordinates will define an equal potential equation for the pole surface. For the pure quadrupole, the scalar potential will be:

$$\phi = A\cdot\sin(2\theta)\cdot\sum_{\mu=1}^{\infty}\frac{(-1)^{\mu}}{4^{\mu-1}(\mu+1)!(\mu-1)!}G^{(2\mu-2)}(s)\cdot r^{2\mu} \tag{3}$$

where A is a constant, $G(s)$ is the distribution function for the field gradient along the beam direction. We notice that the derivatives of $G(s)$ are not exactly zero at the point ($z=0$, $\theta=45°$, $r=R$) where R is the bore radius of the quadrupole. That means we cannot have a brief expression for the equal potential equation like that shown in the reference [1]. From Eq. (3), substituting for $G(s)$ by the function in section 2.1, we then have:

$$\phi = A\cdot\sin(2\theta)\cdot\left\{-\frac{1}{2}r^2\cdot Q(s)+F(s)\cdot(2\sigma_S)^2\cdot J_2\left(\frac{r}{\sigma_s}\right)\right\}$$

where J_2 is the 2nd order Bessel's function of the first kind. When $z=0$, $\theta=45°$, $r=R$, the potential is:

$$\phi_0 = A\cdot\left\{-\frac{1}{2}\frac{R^2}{1-e^{-s0/\sigma_s}}+\frac{e^{-s0/\sigma_s}}{1-e^{-s0/\sigma_s}}(2\sigma_S)^2\cdot J_2\left(\frac{R}{\sigma_S}\right)\right\}$$

So, we get the equal potential equation for the pole shape of quadrupole as:

$$-\frac{1}{2}\frac{R^2}{1-e^{-s0/\sigma_s}}+\frac{e^{-s0/\sigma_s}}{1-e^{-s0/\sigma_s}}(2\sigma_S)^2\cdot J_2\left(\frac{R}{\sigma_S}\right)$$
$$=\sin(2\theta)\cdot\left\{-\frac{r^2}{2}\cdot Q(s)+F(s)\cdot(2\sigma_S)^2\cdot J_2\left(\frac{r}{\sigma_s}\right)\right\} \tag{4}$$

In principle, the pole shape defined by this equation should eliminate not only the integrated, but also the local multipoles. Since we have not taken the effect from the coils into account in the above calculation, the multipoles will still exist in the practical quadrupole. At the center of the magnet, the pole shape got from this method is same as hyperbola. As the increasing of *s* toward to the both end of the magnet, the pole shape becomes a little bit flat as shown in Fig. 5. This gives us the answer that why one needs to shim the hyperbola in the common experimental way.

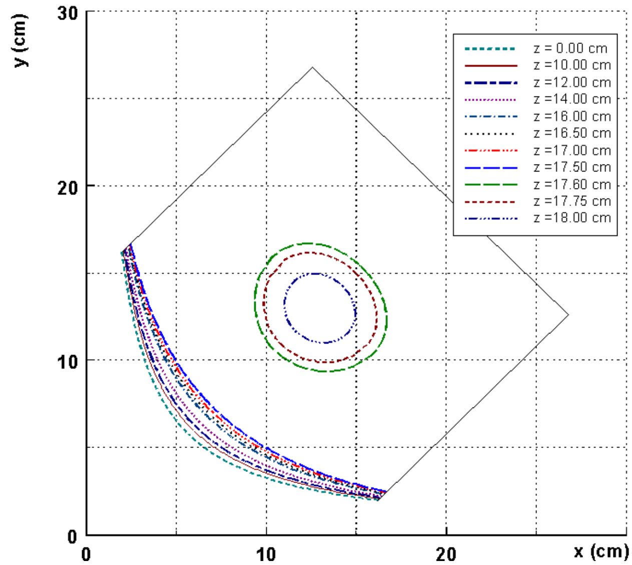

Fig. 5 The pole shape of quadrupole got from Eq. (4)

4 SUMMARY

An improved method for the dipole and quadrupole magnet design is introduced. Based on it, a C-magnet is designed with the assistance of the numerical method because we ignore the effect of coils and we cannot understand the saturation in the iron by the improved method. Now, the fabricated C-magnet with the pole shape, including the end pieces are optimized by this method is ready for measurement. After the measurement of the C-magnet, the performance of this improved method for the quadrupole will be investigated in more detail.

REFERENCES

[1] B.Langenbeck, IEEE Trans. On Mag 24(1988) 1369.
[2] S. Ninomiya, K. Sato, etc., Proc. Of 12th Jap. Symp. on Accelerator Science & Technology, 1999
[3] T.Zhang, K.Sato, S. Ninomiya, etc., RCNP Annual Report, 2000 (to be published).
[4] OPERA is a finite element code developed by Vector Field Ltd., VF-09-96-D4(1996)
[5] Poisson is a two-dimensional code at the Los Alamos National Laboratory, LA-UR-87-126(1987)