MEASUREMENTS OF COHERENT TUNE SHIFTS AND HEAD-TAIL GROWTH RATES AT THE CERN SPS

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Abstract

As part of an ongoing effort to monitor SPS impedance changes with regard to the upgrading of this machine as injector into the LHC, the coherent tune shifts and head-tail growth rates in the SPS were measured for single proton bunches at 26 GeV. From these measurements the real and imaginary components of the transverse broadband impedance can be estimated.

1 INTRODUCTION

Several measurements and calculations similar to those described in this paper have been performed in the past. Most of them are quite old and, moreover, there has been a significant spread in the vertical and horizontal broadband impedance parameters obtained (covering about a factor of 3, from 12 to 48 MΩ/m in Z_v/Q). Our aim is to measure an observable with an uncertainty below 20%. This would allow us to monitor the improvements planned to reduce the impedance of the SPS as injector into the LHC.

The measurements were all done using single and relatively short bunches ($\sigma_z \approx 16\,\text{cm}$ or $\sigma \approx 0.55\,\text{ns}$) injected at 26 GeV in the SPS-MD cycle. The fixed beam energy of 26 GeV was imposed by beam availability. In the future, we plan to confirm these measurements at a higher energy, to exclude any bias from space-charge effects [1, 2] and to measure over a larger range of chromaticity.

The measurements were performed close to "standard tunes" ($Q_x = 26.62, Q_y = 26.58$). Chromaticity was carefully measured and corrected in order to be slightly positive. The octupole components in the machine were compensated. The damper was off. The tune measurements were done using 1 mm (nominal) kicks. With these settings and for small intensities ($\sim 10^{10}$ protons), one obtains rather clean sinusoidal oscillations with little damping, observable online over $2^{12} = 4096$ turns using the SPS tune application.

Ideally, the bunch dimensions and in particular the bunch length should not vary from one intensity to another. The best compromise was achieved by adjusting the beam in the PS for the highest intensity first (close to 10^{11} protons per bunch), and then reducing it by vertical scraping in the PS. In this way, the bunch length and horizontal beam size remained nearly constant (whereas the vertical size changes by a factor of 4 within this range of bunch population). In the future we plan to use the new scraping facilities in the SPS.

On the SPS side, the 200 MHz rf was adjusted to obtain good capture and matching. In order to be independent of injection optimisation and to have shorter bunches with a larger effect on the coherent tune shift, the rf was ramped adiabatically to 3 MV nominal (corresponding to about 2.5 MV measured) just before the time of the measurements.

An approximately constant voltage of $V_{\text{rf}} = 0.8\,\text{MV}$ was used on the first MD on the 23/08/1999. A shorter, better controlled bunch length was obtained in the subsequent MD's using the voltage ramp described above. The bunch length was systematically recorded. A good knowledge of the bunch length σ is needed to extract the parameters of the broad band impedance model. Since the bunch length is not constant we use the average $\langle\sigma\rangle$ of all individual length measurements in our calculations. The r.m.s. spread in the measured bunch length is taken to be the error in the determination of σ and will lead to an error in the impedance estimate. The measured values are summarised in Table 1. Note that the measurement on the 13/08/1999 was done without ramp of the rf-voltage, *i.e.* with longer bunches.

Table 1: Bunch length measurements

date	σ [ns]
13/08/99	0.77 ± 0.14
23/08/99	0.47 ± 0.05
17/09/99	0.53 ± 0.02
10/11/99	0.58 ± 0.03

The bunch mode spectrum for these σ extends up to $f = 1/(2\pi \cdot \sigma) \approx 300\,\text{MHz}$.

Both the tune and the growth rate were measured using the frequency analysis method [3], which is a refined Fourier analysis with a Hanning filter that can be applied on experimental or tracking data. For more information on these studies see [4].

2 HORIZONTAL AND VERTICAL TUNE SHIFT VS. BUNCH POPULATION

The vertical and horizontal tunes were obtained by kicking the beam and post-processing the time sequence (1024 to 4096 turns) of the beam position. Using the frequency analysis technique, the precision of the measurement was increased with respect to the standard Fast Fourier Transform available on the SPS acquisition system. We measured the tunes after the adiabatic ramp for bunch populations between 10^{10} and 5×10^{10}.

In Fig. 1 we show a typical measurement of the tune as a function of bunch population, for the horizontal and vertical plane. As expected from past measurements in the SPS,

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with increasing current the vertical tune decreases and the horizontal tune increases. The slope of these plots is related to the imaginary part of the impedance. The difference in sign and magnitude between the two planes is due to the flat dimensions of the chamber. The horizontal mean radius of the SPS chamber is about 7 cm and the vertical mean radius is about 2.4 cm.

The data was fitted to a straight line $f(x) = a \cdot x + b$. To obtain realistic errors for the slope, the uncertainties in each tune point (e_x, e_y) were scaled to obtain $\chi^2 = 1$ for the fit.

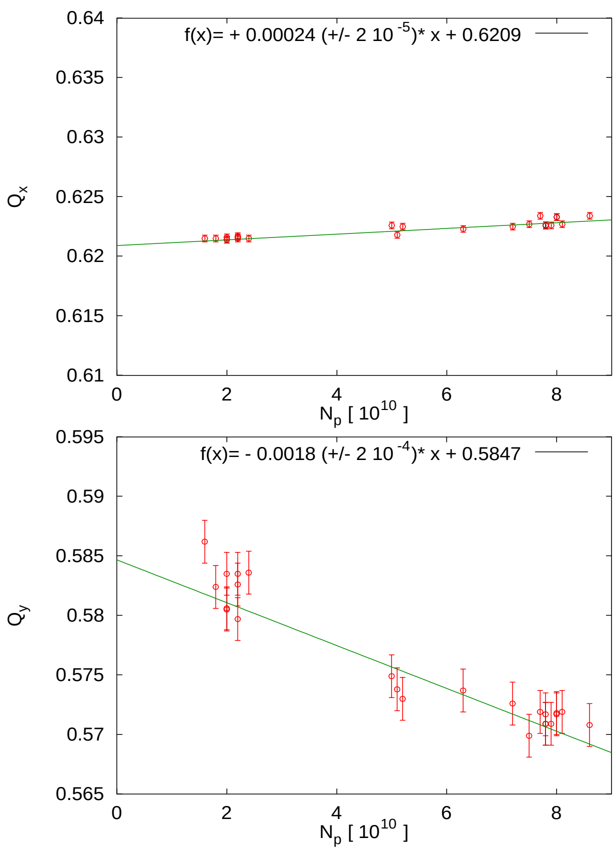

Figure 1: 13/08/1999 (V_{rf}=0.8 MV). Horizontal (top) and vertical (bottom) tune as a function of the bunch population and fit with errors. Tune error bars $e_y = 1.8 \times 10^{-3}$ and $e_x = 3 \times 10^{-4}$.

2.1 *Summary of tune shift measurements*

In Table 2 we summarise the slopes found and the errors.

Table 2: Coherent tune shift measurements

date	$\Delta Q_x/\Delta N_p[10^{10}]$	$\Delta Q_y/\Delta N_p[10^{10}]$
13/08/99	$(+24 \pm 2) \times 10^{-5}$	$(-18 \pm 2) \times 10^{-4}$
23/08/99	$(+58 \pm 6) \times 10^{-5}$	$(-29 \pm 1) \times 10^{-4}$
17/09/99	$(+21 \pm 4) \times 10^{-5}$	$(-36 \pm 2) \times 10^{-4}$
10/11/99	$(+23 \pm 2) \times 10^{-5}$	$(-29 \pm 1) \times 10^{-4}$

3 GROWTH RATE VS. CHROMATICITY

We studied the head-tail mode for low currents ($N_p \approx 1.6 \times 10^{10}$ protons per bunch). Changing the strength of the sextupoles the chromaticity ($\xi = (\Delta Q/Q)/(\Delta p/p)$) was reduced by $\Delta\xi_y$ with respect to the settings used for the tune shift measurements. In Fig. 2 we show the vertical growth rate as a function of $\Delta\xi_y$. For the negative chromaticity measurements the bunch population was constant and equal to 1.6×10^{10}. The values at $\Delta\xi_y = 0$ were taken from the tune shift measurements which were performed with slightly positive chromaticity that led to damping of the centroid motion. These points were measured with a bunch population of $N_p = 10^{10}$ and $N_p = 2.2 \times 10^{10}$ and their values were rescaled by the intensity ratio to compare with the measurements at $N_p = 1.6 \times 10^{10}$.

The zero crossing of the linear fit suggests that our standard setting $\Delta\xi_y = 0$ corresponds to a chromaticity of $\xi_y = 0.011$.

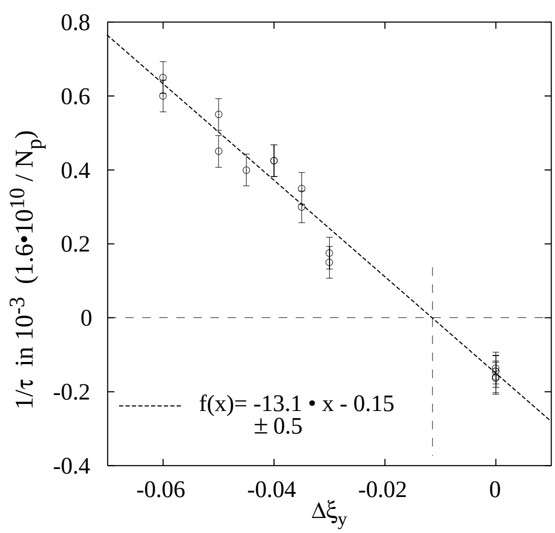

Figure 2: 10/11/1999. Growth rate of the vertical head-tail mode instability (in units of 10^{-3} turns^{-1}), as a function of the decrement of chromaticity. Error bars are $e = 0.043$ (in units of 10^{-3}).

4 BROAD-BAND IMPEDANCE FIT

The transverse impedance is modelled by an equivalent LRC resonator with resonance frequency $w_R = 1/\sqrt{CL}$, resistance R_s and quality factor $Q = R_s\sqrt{C/L}$, according to $Z_1^\perp = \frac{w_R}{w}\frac{Z_t}{1+iQ\left(\frac{w_R}{w}-\frac{w}{w_R}\right)}$ where $Z_t = c/w_R R_s$.

Let ξ be the chromaticity, η the slip factor ($\eta = 5.55 \times 10^{-4}$), w_0 the revolution frequency, $w_\beta = Q_\beta w_0$ the betatron frequency and $Q_\beta = 26.6$ the betatron tune. Defining $w_\xi = \frac{\xi w_\beta}{\eta}$ and $w_p = pw_0 + w_\beta$ with p an integer number we can evaluate the effective transverse impedance (see [5]) $(Z_1^\perp)_{eff} = \frac{\sum_{p=-\infty}^{\infty} Z_1^\perp(w_p)h_l(w_p-w_\xi)}{\sum_{p=-\infty}^{\infty} h_l(w_p-w_\xi)}$, where the impedance is convoluted with the bunch spectrum h_l for the $l = 0$ head-tail mode as defined for a Gaussian beam model $h(w_p) = e^{-w_p^2\sigma_z^2/c^2}$ with $\sigma_z = c\sigma$ the bunch length, c the speed of light and σ the r.m.s of the Gaussian distribution in units of time. The tune shift is given by

$$\Delta Q = \frac{\Omega - w_\beta}{w_0} \approx \frac{1}{w_0}\frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z}\Im(Z_1^\perp)_{eff} \quad (1)$$

with N_p the number of particles per bunch, e charge of the particle, $E = 26.017$ GeV the particle energy, and $T_0 = 2\pi/w_0 = 23.05$ μs the revolution period. Similarly the growth rate (in turns^{-1}) is given by

$$\frac{1}{\tau} \approx -T_0\frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z}\Re(Z_1^\perp)_{eff} \quad . \quad (2)$$

The real part of the effective impedance is different from zero only if the chromaticity is not zero. Above transition, this leads to a negative growth rate (damping) for positive chromaticity, and to a positive growth rate otherwise.

4.1 Tune shift

We fit the broad band resonator with a quality factor $Q = 1$ and a resonance frequency $w_R = 2\pi \times 1.3$ GHz [6].

The ratio $\Delta Q/\Delta N_p[10^{10}]$ is directly proportional to the impedance $Z_1^\perp$. For each plane we determine the impedance such that $\Delta Q/\Delta N_p[10^{10}]$ equals the slope found in our measurements. In Table 3 we summarise the impedances inferred from the tune shifts. The uncertainty reflects both the error of the fitted slope and the spread in the measured bunch length σ.

Table 3: Impedance results obtained by fitting coherent tune shifts with a broad-band model.

date	Z_v in MΩ/m	Z_h in MΩ/m
13/08/1999	25 ± 6	-3.3 ± 0.7
23/08/1999	24 ± 2	-4.8 ± 0.7
17/09/1999	33 ± 3	-2.0 ± 0.4
10/11/1999	30 ± 2	-2.4 ± 0.3
weighted average	28 ± 2	-2.6 ± 0.2

The averages and uncertainties from combining the four measurements are also given. The four numbers of Z_v are all compatible with the weighted average within 20%. The effect in the horizontal plane is much smaller, and clearly of opposite sign. The uncertainties given above, based only on the scatter in the data, are relevant for a comparison of data taken under similar conditions. To compare this impedance with other previous estimates one should also take into account the model dependence and the differences in beam parameters such as E, σ, ξ etc.

4.2 Growth rate

On the experiment of the 10/11/1999 (see Fig. 2), we found that the growth rate increases linearly with the decrement of chromaticity. This can be understood as follows. If the bunch is longer than the range of the wake field ($\sigma c > c/w_R = 3.6$ cm for $w_R = 2\pi \times 1.3$ GHz) then $(Z_1^\perp)_{eff} \approx Z_1^\perp(w_\xi)$. The growth rate $1/\tau$ which is proportional to $\Re(Z_1^\perp)_{eff}$ is then

$$\frac{1}{\tau} \approx -T_0\frac{N_p e c^2 Z_1}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z w_R}\frac{\xi w_\beta}{\eta} \quad (3)$$

which increases linearly with $-\xi$.

Using the complete formula for $(Z_1^\perp)_{eff}$, assuming $Q = 1$, the impedance that fits the measured dependence on the chromaticity is $Z_t = 8.3 \pm 0.6$ MΩ/m. This impedance is 3.7 times smaller than the impedance found by fitting the coherent tune shift.

We can fit both measurements simultaneously with $Z_t = 108$ MΩ/m by changing the quality factor to $Q = 3.6$. In this broad band model $Z_t/Q \approx 30$ MΩ/m.

5 CONCLUSIONS

Using single proton bunches at 26 GeV we have succeeded to measure the coherent tune shift, with a 20% reproducibility. This will allow us to experimentally document the impedance update of the SPS ring. From the coherent tune shift and head-tail mode instability studies we estimate the broad-band component of the vertical impedance to be of the order of $Z_v/Q \approx 30$ MΩ/m (for a quality factor $Q \approx 3.6$). The horizontal impedance is negative and small.

In the future, further measurements at different energies, bunch lengths and larger values of ξ will help understanding the role of space charge and probing the reliability of a broad-band impedance model. These measurements can be combined with the 1000 turns technique to localise in the ring the main sources of impedance [7].

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