NUMERICAL SIMULATIONS OF SPACE CHARGE COMPENSATION EFFECTS IN ION BEAMS

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Abstract

Two axisymmetric, transverse models of space-charge compensation are proposed in order to compute the self-consistent potential of a proton beam and of the plasma generated by the beam when it evolves in a residual gas and ionises its molecules. The first model consists in a stationary hydrodynamic description. The limitation of this model is shown in an experimental case where electrons are not thermalized. A kinetic model which computes the build up of space-charge compensation towards a stationary state is presented. Its numerical resolution is based on an explicit PIC method. Tests of this model on the same experimental case are shown and compared to measurements.

1 INTRODUCTION

Space charge compensation occurs in the low energy part of ions accelerators, where some residual gas remains at a density which can be much higher than the beam density. Electrons and residual gas ions are produced from collisions between the beam ions and the residual gas molecules. When the beam pulse is continuous, the production rate of electrons and residual gas ions is constant and writes :

$$\frac{\partial n_\alpha}{\partial t} = \sigma_i n_g n_b v_b = \frac{n_b}{\tau_i} \tag{1}$$

where $\alpha = e$ or $\alpha = i$, n_e and n_i denoting respectively electrons and residual gas ions density, n_g is the residual gas density, n_b is the beam ions density, v_b is the velocity of beam ions, σ_i is the cross section of the ionisation process, and τ_i is a characteristic time defined by this relation.

At the beginning of the process of particle production, the self-consistant electric potential of the medium is mainly due to the beam space charge, therefore the residual gas ions are expelled radially out of the beam while the electrons oscillate inside. If we assume that this dynamic goes on as long as ionization lasts, there will be as much electrons as beam ions within the beam after τ_i, which then represents the characteristic time of the phenomenon.

For long-pulsed, positive ion beams, time-resolved measurements have shown in different situations that space-charge compensation evolves towards a stationary state where the beam is partially compensated [1], [2].

To be able to describe the transport of a continuous compensated beam, one need first to have a precise knowledge of the transverse self-consistent electric field, or equivalently, of the electric potential, that we denote $\phi(r)$ (r is the radial coordinate), once a stationary state is reached. To compute this potential, we present in this paper two different models : one is based on a hydrodynamic description and the other uses kinetic equations. These are transverse models, where we assume that the beam is an infinite cylinder of radius a confined in a tube of radius R.

The case of SILHI

SILHI is the ECR High Intensity Light Ion Source studied in CEA-Saclay, France. It delivers a 95 keV, 100 mA proton beam which is transported in a Low Energy Beam Transport line (LEBT), where the residual gas, H_2, is at a pressure of around 5×10^{-5} hPa. The models presented in this paper are used in the situation of SILHI's beam.

Within the frame of a collaboration between SEA at CEA-Saclay, SP2A at CEA-Bruyères and the Institut für Angewandte Physik in Frankfurt, time-resolved measurements of space-charge compensation, related in [1], have been made at the end of the LEBT thanks to a residual gas ion analyser. The potential ϕ decreases with respect to r inside the beam, this enables the residual gas ions to escape. The ions produced on the axis are then collected with a maximum kinetic energy E_{max}, while the ions coming from the edge of the beam have a minimum energy E_{min}. From the measurements, one can get the potential drop in the beam and the total potential drop :

$$e\delta\phi(a) := e(\phi(0) - \phi(a)) = E_{max} - E_{min} \; , \tag{2}$$

$$e\delta\phi(R) := e(\phi(0) - \phi(R)) = E_{max} \; . \tag{3}$$

In a situation of a 92 keV, 61 mA beam, with radius $a = 10$ mm, whith $R = 135$ mm, and for a gas pressure $P = 3.8 \times 10^{-5}$ hPa, the measurements gave $\delta\phi(a) \approx 16$ V, and $\delta\phi(R) \approx 48$ V [1]. In the sequel, we take advantage of these values to fit and to test the models.

2 HYDRODYNAMIC MODEL

We describe the residual gas ions as a cold transverse beam, i.e. we neglect their velocity dispersion. This dispersion is very small compared to the average velocity of ions when the potential is flat inside the beam. The density n_i and the velocity u_i of ions are then solution of the system of conservation laws:

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial r n_i u_i}{\partial r} = \dfrac{n_b}{\tau_i} \; , \\ \dfrac{1}{r}\dfrac{\partial r n_i u_i^2}{\partial r} = -\dfrac{e}{m_i} n_i \dfrac{\partial \phi}{\partial r} \; , \end{cases} \tag{4}$$

with the symmetry condition $u_i(0) = 0$.

We assume that the electrons are at thermodynamic equilibrium in the potential well, their density is then given by the Maxwell-Boltzmann distribution :

$$n_e = n_{e0} \exp\left(\frac{e\phi}{kT_e}\right) , \quad (5)$$

for a given electron density on axis n_{e0}, and a given temperature kT_e.

Finally, the potential is solution of Poisson equation :

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = \frac{e}{\varepsilon_0}\left(n_b + n_i - n_e\right) . \quad (6)$$

with the condition $\frac{\partial \phi}{\partial r}(0) = 0$, which comes from the fact that we look for bounded particle densities, and with $\phi(R) = 0$, which means that the tube is equipotential.

In the theoretical case where the beam is a slab evolving between two infinite grounded plates, one can derive a similar model where the different quantities depends on a cartesian transverse coordinate x. A numerical method to compute the solution of this model has been proposed in [3]. This method can also be used to solve the axisymmetric model (4) – (6).

2.1 *An example of the potential of a space-charge compensated beam*

The model derived hereabove approaches another model used to describe space-charge compensation, for instance in [4], where the dispersion of the residual gas ions velocity is not neglected. The simplification that we have made here, by treating ions as a cold transverse beam, allows easier computations, since no iteration method is required. Nevertheless, the assumption made restricts the field of application of this model, and before using it, we had to check that it could give reasonable results in cases tested with the model used in [4].

In this latter article, the authors present several diagnostics performed on a 10 keV, 143 μA H_e^+ beam, evolving in Helium at a pressure of 6.9×10^{-5} hPa; the beam radius is $a = 5$ mm and the radius of the tube is $R = 50$ mm. The two parameters n_{e0} and kT_e have been estimated in such a way that the numerical solution fits to the results of the diagnostics. In this case, it was found that $n_{e0} = 1.3 \times n_b(0)$ and $kT_e = 0.1\ eV$ give a good fitting. Here, we have solved numerically the hydrodynamic model (4) – (6) with these parameters. The result is presented on Figure 1 and approximates correctly the potential computed in [4]

2.2 *Application to SILHI*

We present here an attempt to use the hydrodynamic model in order to compute the potential in the experimental situation described in Section 1. We used the results of measurements (2) and (3) as two constraints to determine the parameters n_{e0} and kT_e. Two plots of the set of parameters allowing to fit each constraint are presented on Figure 2. The

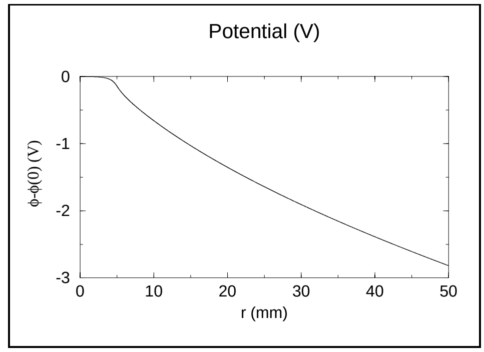

Figure 1: Potential solution of the hydrodynamic model in the case studied in [4].

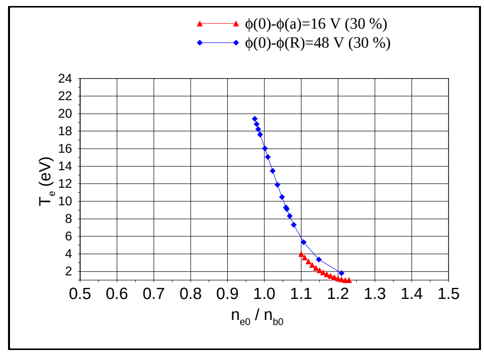

Figure 2: Set of parameters (n_{e0}, kT_e) for which the potential checks (2) or (3).

domain of investigation was : $0.5 n_b(0) \leq n_{e0} \leq 1.5 n_b(0)$, $1\ eV \leq kT_e \leq 24\ eV$. The beam profile was considered as uniform. Although the error margins choosen are large (the required tolerance is $30\ \%$ for each constraint) no couple of parameters could fulfill the two constraints simultaneously.

In fact the assumption of thermodynamical equilibrium is questionable in the case of SILHI. Let us precise this point. Firstly, using the ionization cross section value, $\sigma_i = 2.4 \times 10^{-20}\ m^2$, one can get $\tau_i \simeq 10\ \mu s$. Secondly, when they are produced by ionisation, electrons have an average initial kinetic energy of $10\ eV$, and most of them are trapped in the potential well whose depth is about $50\ V$. Hence, the electron temperature amounts roughly to $10\ eV$ and could not be lower than $1\ eV$, despite the lost of highly energetic electrons. The time of thermalization for such electrons is longer than $100\ \mu s$ [3], and thus could not be compared to the time scale of compensation τ_i.

3 KINETIC MODEL

The model used in this section was presented in [5]. Electrons and ions are described with their distribution function in phase space, $f_e(t, \mathbf{r}, \mathbf{v})$ and $f_i(t, \mathbf{r}, \mathbf{v})$, where $\mathbf{r}$ is the 2D transverse position in the tube and $\mathbf{v}$ is the 2D transverse velocity. The evolution of the distribution functions can be described with Vlasov equation, using a source term to modelize ionisation [3] :

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f_e + \frac{e}{m_e} \nabla \phi \cdot \nabla_{\mathbf{v}} f_e = \frac{n_b}{\tau_i} S_e(\mathbf{v}) \ , \quad (7)$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f_i - \frac{e}{m_i} \nabla \phi \cdot \nabla_{\mathbf{v}} f_i = \frac{n_b}{\tau_i} S_i(\mathbf{v}) \ , \quad (8)$$

$$-\Delta \phi = \frac{e}{\varepsilon_0} \left(n_b + \int f_i d\mathbf{v} - \int f_e \mathbf{v} \right) \ , \quad (9)$$

S_e and S_i are the probability densities of the initial velocities of electrons and ions (once they have just been produced). As a matter of fact, S_i is the velocity distribution of gas molecules.

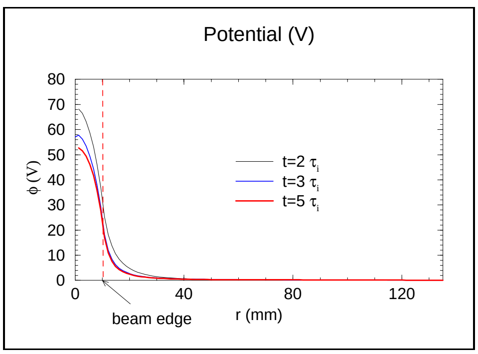

Figure 3: The steady state is nearly reached at $t = 3\ \tau_i$, time after which the evolution is neglectible.

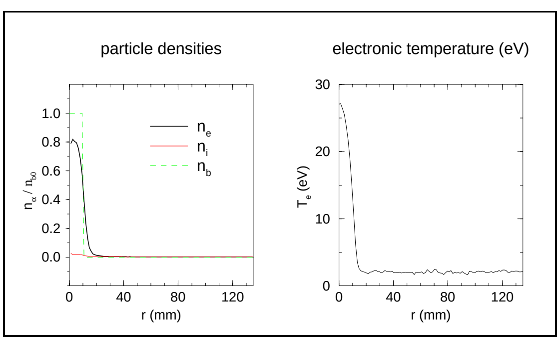

Figure 4: Electron density and temperature at equilibrium. This model shows a varying temperature profile.

It was proven (see [3]), in both plane symmetric and axisymmetric case, that this model does not admit any stationary solution when $S_e(\mathbf{v}) > 0$, for $|\mathbf{v}| < v_c$, where $v_c > 0$. Indeed, in this case, electrons emitted with a low kinetic energy accumulate inside the beam. In fact, some mecanisms which are not taken into account transfer energy to electrons allowing them to escape. These mecanisms have not been clearly identified. It is possible that they are due to the longitudinal electric field, created by the beam divergence : such a field accelerate electrons, enabling them to escape longitudinally [3].

In the present description, we modelize the effects of these mecanisms by assuming that the electrons are created with a minimum kinetic energy E_g. This energy does

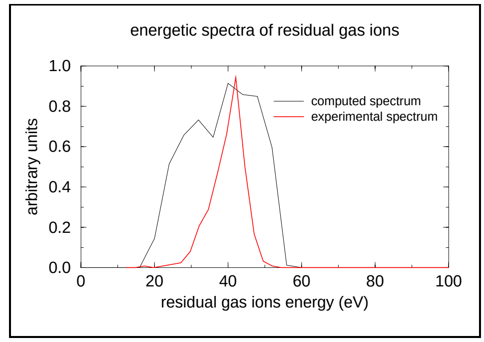

Figure 5: Comparison between a computed and an experimental spectrum at the wall. The spread in ions energy is reproduced by the model, but the shape of the experimental spectrum indicates that the potential profile is different.

not exist in reality, since the real distribution S_e is maximum at $\mathbf{v} = 0$. Nevertheless, this description allows to reproduce the evolution of space-charge compensation towards a stationary state. We have simulated this process, by solving this model with an explicit PIC method, using a Boris algorithm to integrate the particles motion in cylindrical geometry. The results of the simulation are presented on Figure 3, 4, 5, at $t = 5\ \tau_i$, when a stationary state has been reached. The energy E_g is a free parameter which has to be fixed as $E_g = e\delta\phi(R)$. Indeed, the system stops its evolution once all the emitted electrons have enough energy to overcome the potential barrier towards the tube wall.

4 CONCLUSION

A kinetic model of space-charge compensation based on Vlasov-Poisson system allows to compute the density and the temperature of electrons trapped in the potential well, which neutralise the beam space-charge. It requires the estimation of the total potential drop, which can be directly measured. It can replace a classical stationary model when the electrons cannot be described with the Maxwell-Boltzmann distribution.

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