

ROLE OF LANDAU DAMPING IN THE QUANTUMLIKE THEORY OF CHARGED-PARTICLE BEAM DYNAMICS

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Abstract

The longitudinal dynamics of a charged-particle beam in a circular accelerating machine with an arbitrary coupling impedance is described with an integro-differential nonlinear Schrödinger equation. By using the Wigner transform, it is shown that the above longitudinal dynamics is governed, in phase space, by a kinetic-like equation. Remarkably, this quantumlike approach is capable of reproducing the phenomenon of Landau damping, extending in this way previous analysis carried out within the Thermal Wave Model.

1 INTRODUCTION

The standard theory of collective longitudinal relativistic high-energy bunch dynamics in accelerating machines is based on kinetic theory described by the Vlasov equation [1, 2]. This equation is in general coupled with a set of equations for the field produced by the bunch itself (wake-field self-interaction) which accounts for the interaction with the surroundings [3]. An important parameter to be specified for this self-interaction is the coupling impedance [3], which in general is a complex quantity. In this framework, a special role is played by the phenomenon of Landau damping [4] which provides for a stabilizing effect of the system.

Coherent instability of charged-particle beams in a circular accelerating machines has been recently described also in a quantumlike domain [5, 6], within the context of the Thermal Wave Model (TWM) [7]. In the above quantumlike approaches it is assumed that the longitudinal beam dynamics, in the case of negligible radiation emission effects, is governed by the following nonlinear Schrödinger equation (NLSE):

$$i \frac{\partial \Psi}{\partial s} + \frac{\eta \epsilon}{2} \frac{\partial^2 \Psi}{\partial x^2} + \hat{\mathcal{Z}} [|\Psi|^2 - |\Psi_0|^2] \Psi = 0 \quad , \quad (1)$$

where s and x are the timelike ($\equiv ct$) and the longitudinal configurational space coordinates, $\Psi(x, s)$ is a complex function, called beam wave function (BWF), whose squared modulus is the beam density, ϵ is the longitudinal beam emittance, η is the slip factor, and $\hat{\mathcal{Z}}$ is the following Hermitian linear-integral operator: $\hat{\mathcal{Z}} [f(x, s)] = \mathcal{X} f(x, s) + \mathcal{R} \int f(x, s) dx$. \mathcal{R} and \mathcal{X} are the resistance and the reactance of the system per unity length, respectively. In (1), Ψ_0 is the BWF corresponding to the equilibrium state of the system; consequently,

$|\Psi|^2 - |\Psi_0|^2$ represents the density perturbation of the beam. A standard analysis of the modulational instability has been carried out in configuration space to describe coherent instability, first in the case of $\mathcal{X} \neq 0$, $\mathcal{R} = 0$ (purely reactive impedance) [5], and more recently in the general case $\mathcal{X} \neq 0$, $\mathcal{R} \neq 0$ [6]. The first case corresponds to the standard NLSE of the cubic form, whilst the second one corresponds to a NLSE which includes a "memory term". The main results of the above modulational instability analysis of Eq. (1) were the following: (i). the coherent instability, was interpreted as a modulational instability; (ii). no Landau damping was predicted in configuration space.

In this paper we want to show that the quantumlike description provided by the above NLSE (1) is capable of predicting the phenomenon of Landau damping which appears in competition with the coherent instability. This is done in the phase-space framework associated with Eq. (1). This transition is naturally performed by using the Wigner transform of the BWF $\Psi(x, s)$. This allows us to write a sort of von Neumann equation for the Wigner function $\rho_w(x, p, s)$ [8], $p \equiv dx/ds$ being the conjugate momentum associated with x .

2 KINETIC-LIKE EQUATION

We transit from configuration space to phase space by the following Wigner-like transform $\rho_w(x, p, s)$ [8]:

$$\rho_w = \frac{1}{2\pi\eta\epsilon} \int_{-\infty}^{\infty} \Psi^* \left(x + \frac{y}{2}, s \right) \Psi \left(x - \frac{y}{2}, s \right) e^{ipy/\eta\epsilon} dy \quad , \quad (2)$$

provided that the normalization condition of ρ_w over all the phase space is assumed. Note that (2) implies that $|\Psi|^2 = \int_{-\infty}^{\infty} \rho_w(x, p, s) dp$. Furthermore, we observe that, if Ψ satisfies Eq. (1), then ρ_w satisfies the following von Neumann-like equation:

$$\frac{\partial \rho_w}{\partial s} + p \frac{\partial \rho_w}{\partial x} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{\eta\epsilon}{2} \right)^{2n} \hat{\mathcal{L}}_n \rho_w = 0 \quad , \quad (3)$$

where:

$$\hat{\mathcal{L}}_n \equiv \eta\epsilon \frac{\partial^{2n+1}}{\partial x^{2n+1}} \left\{ \int_{-\infty}^{\infty} \hat{\mathcal{Z}} [\rho_w - \rho_0] dp \right\} \frac{\partial^{2n+1}}{\partial p^{2n+1}} \quad , \quad (4)$$

with ρ_0 defined from the following relationship: $|\Psi_0|^2 \equiv \int_{-\infty}^{\infty} \rho_0 dp$. In this scheme, the usual perturbative approach allows us to derive the linear dispersion relation and to carry out a stability analysis of the system.

3 LINEAR APPROXIMATION

Let us start from the equilibrium state: $\rho_w = \rho_0(p)$, and perturb the system according to:

$$\rho_w(x, p, s) = \rho_0(p) + \rho_1(x, p, s) \quad , \quad (5)$$

where $\rho_1(x, p, s)$ is a first-order quantity. Consequently, (3) and (4) can be linearized. Thus, by introducing the Fourier transform of $\rho_1(x, p, s)$, i.e.:

$$\rho_1(x, p, s) = \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} d\omega \tilde{\rho}_1(k, p, \omega) e^{ikx - i\omega s} \quad , \quad (6)$$

we easily get the following dispersion relation:

$$1 = iZ\eta\epsilon \int_{-\infty}^{\infty} \frac{\rho_0(p + \eta\epsilon k/2) - \rho_0(p - \eta\epsilon k/2)}{\eta\epsilon k} \frac{dp}{kp - \omega} \quad , \quad (7)$$

where we have introduced the coupling impedance as:

$$Z = \mathcal{R} + i k \mathcal{X} \equiv Z_R + i Z_I \quad , \quad (8)$$

where Z_R and Z_I denote real and imaginary parts of Z , respectively (note that k plays the role of harmonic number).

4 MONOCHROMATIC COASTING BEAM

In the limiting case of a monochromatic coasting beam, i.e.

$$\rho_0(p) = n_0 \delta(p) \quad , \quad (9)$$

(n_0 being constant), the wavepacket reduces to a monochromatic wavetrain. Thus, (7) becomes:

$$1 = -i n_0 \frac{Z}{k} \left[\frac{1}{\eta\epsilon k^2/2 + \omega} + \frac{1}{\eta\epsilon k^2/2 - \omega} \right] \quad , \quad (10)$$

from which we can see that ω can be complex, namely we can write $\omega = \omega_R + i\omega_I$. Then, by using (8), and separating (10) in its real and imaginary parts, we obtain:

$$Z_I = -\frac{\eta\epsilon n_0 k}{4\omega_I^2} Z_R^2 + \frac{\omega_I^2}{\eta\epsilon n_0 k} + \frac{\eta\epsilon^2 k^3}{4n_0} \quad . \quad (11)$$

Eq. (11), for given value of ω and k , determines a connection between Z_I , Z_R , and the growth rate ω_I . In the (Z_R, Z_I) -plane we have a family of symmetric parabolas around Z_I -axis, whose concavity orientation depends on the sign of $\eta\epsilon$, parameterized with respect to ω_I , with the following features.

For $\omega_I \rightarrow 0$, the parabolas collapse into a vertical straight line on the Z_I -axis given by the condition: $-\infty < Z_I \leq \eta\epsilon^2 k^3 / 4n_0$, for $\eta > 0$, and $\eta\epsilon^2 k^3 / 4n_0 \leq Z_I < \infty$, for $\eta < 0$.

The above straight line represents the stability region of the system which is enclosed by all the parabolas. This

means that all the points in the (Z_R, Z_I) -plane for which $Z_R \neq 0$, represent unstable states of the system.

For small k , the last term in (11) can be neglected, obtaining:

$$Z_I = -\frac{\eta\epsilon n_0 k}{4\omega_I^2} Z_R^2 + \frac{\omega_I^2}{\eta\epsilon n_0 k} \quad . \quad (12)$$

This relation shows clearly that the stability region (for $Z_R = 0$) corresponds to the interval of Z_I satisfying the condition:

$$Z_I \eta < 0 \quad , \quad (13)$$

and, consequently, instability is obtained for

$$Z_I \eta > 0 \quad , \quad (14)$$

which coincides with the well known coherent instability condition derived in the standard theory.

We can conclude that, as in the standard description, in the case of monochromatic coasting beam, the quantum-like description predicts that coherent instability exists and Landau damping does not exist.

5 COASTING BEAM WITH ARBITRARY $\rho_0(P)$

If the coasting beam is not monochromatic, an analysis of dispersion relation (7) can be carried out by considering the limit of small k , but keeping $\eta\epsilon$ finite and ω arbitrary. This corresponds to assuming that $|\eta|\epsilon k \ll 1$.

First of all, we observe that, since $|\eta|\epsilon k \ll 1$, we have:

$$\frac{\rho_0(p + \eta\epsilon k/2) - \rho_0(p - \eta\epsilon k/2)}{\eta\epsilon k} \approx d\rho_0/dp \equiv \rho'_0 \quad . \quad (15)$$

Consequently, Eq. (7) becomes:

$$1 = i\eta\epsilon Z(k, \omega) \int_{-\infty}^{\infty} \frac{\rho'_0}{kp - \omega} dp \quad . \quad (16)$$

Eq. (16) is identical to the linear dispersion relation that in the standard theory, and the limit of small wavenumbers considered above allows us to predict a sort of weak Landau damping, as described in the standard theory, as well. In fact, the dispersion relation (16) can be cast as:

$$V_R + i V_I = - \left[i \int_{PV} \frac{\rho'_0}{p - \beta_{ph}} dp + \pi \rho'_0(\beta_{ph}) \right]^{-1} \quad , \quad (17)$$

where

$$V_R + i V_I = \eta\epsilon \left(\frac{Z_R}{k} + i \frac{Z_I}{k} \right) \quad , \quad (18)$$

and $\int_{P_V}(\dots)dp$ accounts for the principal value. This equation determines a relationship between V_R , V_I , and β_{ph} . In principle, β_{ph} is a complex quantity. Thus, we put:

$$\beta_{ph} \equiv \gamma_R + i \gamma_I . \quad (19)$$

Consequently, curves in the V_R - V_I plane for different growth rates γ_I can be plotted. It is clear from (17) and (18) that they agree with the ones given in the standard theory [1]. We would like to stress that these plots would represent a sort of universal stability chart predicted by the present quantumlike theory.

6 CONCLUSIONS AND PERSPECTIVES

In this paper a quantumlike approach in phase space has been used to describe the nonlinear collective longitudinal dynamics of a coasting beam in a circular accelerating machine. Within the framework of TWM, this dynamics is governed by a nonlinear integro-differential Schrödinger-like equation. By means of the Wigner transform, a kinetic-like equation, similar to the one based on the Vlasov equation, has been obtained. With a perturbative approach we have recovered coherent instability as well as predicted the phenomenon of the Landau damping, in a way fully similar to the one usually predicted by the conventional theory of accelerator physics [1, 2]. Physically, this result is due to the stabilizing role of Landau damping which competes with the coherent instability. This result improves the ones obtained in previous works, in which coherent instability was recovered but Landau damping was not predicted [5, 6].

In conclusion, the very well known charts of coherent instability, including the stabilizing effect of Landau damping [1] are successfully recovered by the present quantum-like approach.

In perspective, on the basis of the present analysis, a possible development of the results given in this paper should be done to prove that Landau damping is predictable also in configuration space. In fact, the configuration space description provided by Eq. (1) is fully equivalent to the one provided by von Neumann equation (3), according to the well known properties of the Wigner transform (2). Consequently, all the information contained in the phase-space distribution ρ_w are also contained in Ψ . We must conclude that, in the quantum-like domain, the phenomenon of Landau damping must be predicted not only in phase space but also in configuration space. On the basis of this statement, it seems clear that the difficulty met in predicting it in configuration space lies in the method used to analyze the modulational instability. It seems that the right way to predict the phenomenon of Landau damping also in configuration space may be solving a suitable inhomogeneous eigenvalue problem to carry out the instability property of a small perturbation of a non-homogeneous background. Such an investigation is now in progress and considered by the authors to be discussed in a future work.

7 REFERENCES

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