

SELF-CONSISTENT SIMULATION OF CHARGED PARTICLES MOTION IN WAKE FIELDS BASED ON BOUNDARY INTEGRAL EQUATION

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Abstract

This paper presents a wake field numerical simulation by using a time domain boundary integral equation. The use of the boundary integral equation gives us great advantages in the wake field calculation because this numerical scheme can include the charged particle fields in the form of the Lienard-Wiechert fields and enables us to treat various curved trajectories of particles. The author has been developing this kind of wake field simulation code, and short range simulations were presented. [1][2] The field simulations however often encountered numerical instabilities in long range calculations. This paper presents a stable scheme of the time domain boundary integral equation method for axi-symmetric systems and a basic formulation of 3D code.

1 INTRODUCTION

The use of a time domain boundary integral equation gives us great advantages in the wake field calculation because this numerical scheme can include the charged particle fields in the form of the Lienard-Wiechert fields, and this enables us to treat various curved trajectories of particles in the simulation. The flexibility in setting the charged particle trajectory means possibility of self-consistent numerical simulation of high energy charged particles motion in the wake fields. Accordingly it will contribute to large amount of useful applications such as a bunch compressor, SASE FEL etc. Beyond this background, the author has been developing the time domain boundary integral equation

method (TBIE), and presented short range numerical simulations. [1][2] The field simulations however often encountered numerical instabilities in long range calculations, which are caused by various kinds of numerical noises. In this paper, it is reported that a quite stable scheme of TBIE is constructed making strict evaluation of the conservation law of surface charge and the causality relation, and its numerical example related to a disk load structure is shown. Moreover, three dimensional formulation of TBIE is given for a general model shape. To roughly estimate required storage memory of the 3D code, one can find that the straight expansion of the 2D scheme to 3D one requires us quite much memory and CPU time, and any efforts of memory reduction are essential. This memory reduction efforts are also shown here.

2 TIME DOMAIN BOUNDARY INTEGRAL EQUATION METHOD [1][2]

The master equations of wake fields phenomena are the inhomogeneous wave equations whose inhomogeneous terms are moving point charged particles,

$$\partial^\nu \partial_\nu A^k(x^i) = \frac{1}{\epsilon_0 c^2} J^k(x^i) \quad (1)$$

where A^k is the four electromagnetic potentials, J^k is the four electric current vector, c is the light velocity and ϵ_0 is dielectric constant in vacuum. Applying the Green theorem in 4D time-space,

$$\int_{\Omega} (\Psi \partial^\nu \partial_\nu \Phi - \Phi \partial^\nu \partial_\nu \Psi) d\Omega = \int_S (\Psi \partial_\nu \Phi - \Phi \partial_\nu \Psi) dS^\nu \quad (2)$$

(Ω , S are any functions, Ω, S are 4D volume and its boundary surface) under the perfect conductor boundary conditions,

$$\mathbf{E} \times \mathbf{n} = 0, \quad \mathbf{B} \cdot \mathbf{n} = 0 \quad (3)$$

(\mathbf{n} is a unit normal vector on the boundary) the following Boundary Integral Equation (BIE) can be derived,

$$\mathbf{E}(t, \mathbf{x}) = \mathbf{E}_{LW}(t, \mathbf{x}) - \frac{1}{4\pi} \int \frac{dS'}{|\mathbf{x} - \mathbf{x}'|} \mathbf{B}_t(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}, \mathbf{x}') - \frac{1}{4\pi} \int dS' \left[\frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} + \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} \frac{\partial}{c \partial t} \right] E_n(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}, \mathbf{x}') \quad (4-1)$$

$$\mathbf{B}(t, \mathbf{x}) = \mathbf{B}_{LW}(t, \mathbf{x}) - \frac{1}{4\pi} \int dS' \left[\frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} + \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} \frac{\partial}{c \partial t} \right] \times \mathbf{B}_t(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}, \mathbf{x}') \quad (4-2)$$

where \mathbf{E}_{LW} and \mathbf{B}_{LW} are the Lienard-Wiechert fields. Combining the BIE with the continuity relation of the surface charge,

$$\frac{\partial \sigma}{\partial t} + \text{div } \mathbf{K} = 0 \quad (5)$$

(σ is the surface charge density, \mathbf{K} is the surface current density) the final form of BIE are obtained as follows,

$$\dot{\mathbf{E}}(t, \mathbf{x}) = \dot{\mathbf{E}}_{LW}(t, \mathbf{x}) - \frac{1}{4\pi} \int dS' \left[\frac{\ddot{\mathbf{B}}_t}{|\mathbf{x}-\mathbf{x}'|} + \frac{c^2(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} \text{div}\mathbf{B}_t + \frac{c(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2} \text{div}\dot{\mathbf{B}}_t \right] \quad (6)$$

The unknowns in the above BIE are the tangential components of the magnetic field \mathbf{B}_t which is directly related to the surface current \mathbf{K} as follows,

$$\mathbf{B}_t = \frac{\mathbf{K}}{\epsilon_0 c^2} \quad (7)$$

Time domain property of the problem implies that the boundary values in different time are independent each other, but they are connected by the relation (6). Eq.(6) means that field values $\mathbf{E}(t, \mathbf{x})$ and $\mathbf{B}(t, \mathbf{x})$ are calculated by past values on the boundary surface $\mathbf{B}_t(t - |\mathbf{x}-\mathbf{x}'|/c, \mathbf{x}')$, and this gives us a time domain numerical simulation scheme. To discretize the unknown values $\mathbf{B}_t(t - |\mathbf{x}-\mathbf{x}'|/c, \mathbf{x}')$ on a direct product space $t \sim S$ (S is the boundary surface), the integral equations (6) are converted to the following matrix equation,

$$\sum_n [\mathbf{M}_n] [\mathbf{b}_n] = [\mathbf{e}_{LW}] \quad (8)$$

(where \mathbf{M}_n are boundary integral matrices, \mathbf{b}_n and \mathbf{e}_{LW} are discretized values of \mathbf{B}_t and \mathbf{E}_{LW} or \mathbf{B}_{LW}). Due to the retarded property of (6), Eq.(8) has a complicated structure shown in Fig.1.[3] All matrices in (8) are sparse but not symmetric. The number of matrices in (8) (or Fig.1) depends on individual problems (their boundary shape and size) and the time step size as well. To avoid numerical instabilities caused by numerical noises, causality lines should be exactly evaluated in the boundary integral (see Fig.2), and to do consistent evaluation of the conservation law of surface charge (5), an implicit scheme is essential. It is readily expected that all these conditions result in quite much memory and CPU time scheme.

3 MEMORY REDUCTION EFFORTS IN 3D FORMULATION

Even in standard size simulations, the number of mesh elements are of order 10,000, which means that the matrix size is a square of 10,000 (a hundred millions), and the number of the matrices in (8) is from fifty to a hundred. Therefore, even for single precision accuracy, required memory size will reaches to about 100 GB. To compress the sparse part of the matrices, the memory size can be reduced by 1/5, but this compression seems to be still not enough for uti-

lizing the numerical simulation of (8) on the presented computer performance. Accordingly additional memory reduction efforts are essential to do 3D wake field simulations based on the TBIE. It is found that a special class of 3D problems can be greatly reduced to small size simulations, for example, 3D wake field phenomena, which are caused by curved particle trajectories inside an "axi-symmetrical" accelerator tube. (see Fig.3) In that case, although the electromagnetic field profile is indeed 3D, the matrices in (8) can be condensed to 2.5 dimension size by using a group theory concept. (see Fig.4)

4 NUMERICAL EXAMPLE

Because of much memory requirement of 3D problem, as a first step, this paper shows an example of wake field simulation in an axi-symmetric system. The numerical model is the local part around a disk load structure of an accelerator tube. The thickness of the disk is 2 mm and the bunch length Δz is 3 mm. The bunch is assumed to have $\beta = 0.9997$ velocity (about 40 MeV). The cross section of the disk load structure are shown in Fig.5 and the surface current strength $|\mathbf{K}|$ behavior in time domain is shown in Fig.6. Due to the complicated reflection of electromagnetic fields, disturbance of \mathbf{K} continues for long time after the bunch pass through the structure.

5 SUMMARY

In this paper, a stable numerical simulation scheme of time domain boundary integral equation method is presented, and a typical numerical example of wake field phenomena around disk load structure is shown. Moreover, three dimensional formulation of TBIE is given for general model shape.

REFERENCES

- [1] H.Kawaguchi, M.Isoda and T.Honma, Boundary Elements XIX (Eds: M.Marchett et.al.) (1997), p.477.
- [2] H.Kawaguchi, M.Isoda and T.Honma, IEEE Trans. Magn. **33**[2] (1997), p. 1436.
- [3] H.Kawaguchi and M.Isoda, Procs of 6th European Particle Accelerator Conference (EPAC-98), p.1195.

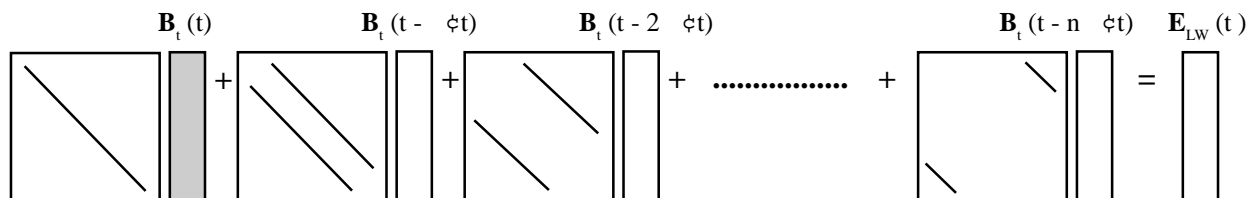


Fig. 1 Structure of discretized BIE

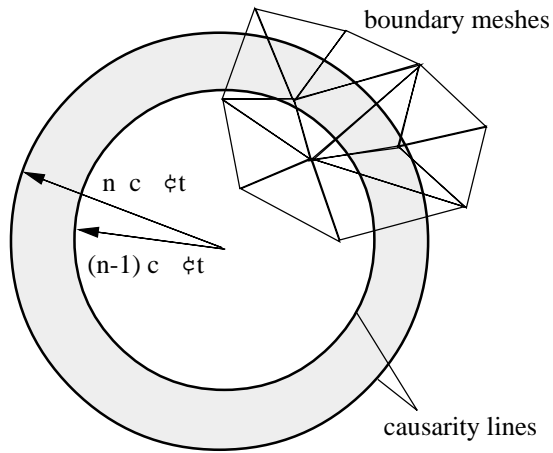


Fig. 2 Boundary meshes and causality lines

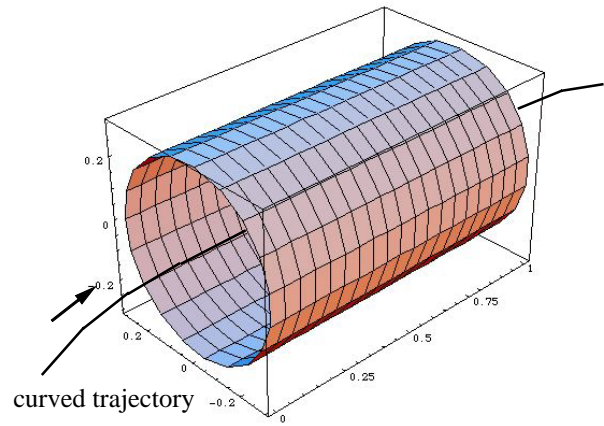


Fig. 3 Curved trajectory inside axis-symmetrical tube

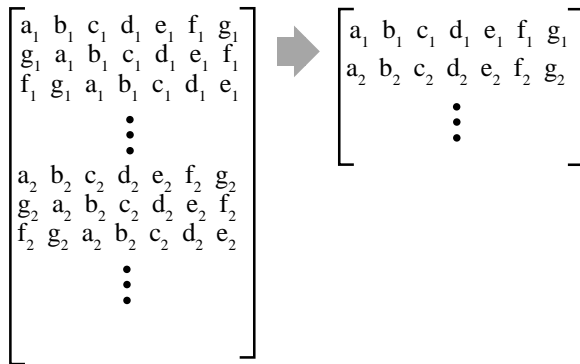


Fig. 4 Compression based on group theory concept

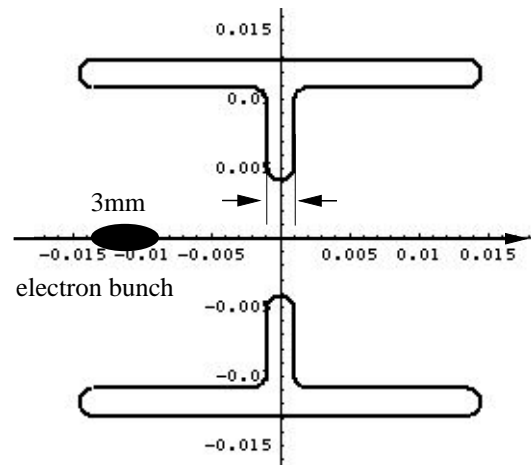


Fig. 5 Cross section of disk load structure

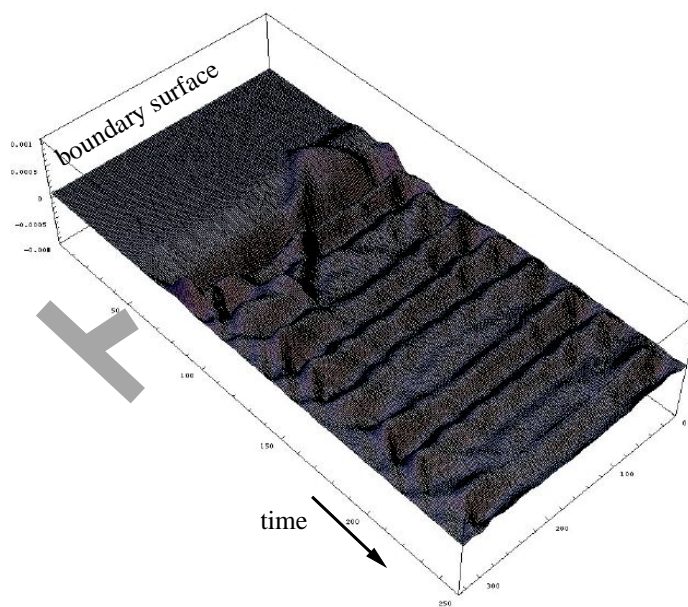


Fig. 6 Surface current strength behavior $|K|$ in time domain