CONTROLLABLE CHAOTIC MOTION OF THIRD-INTEGER RESONANT EXTRACTION WITH RF ELECTRIC FIELDS

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Abstract

An RFQ(Radio-Frequency Quadrupole) electric field device was installed at one of the two HIMAC synchrotrons in order to study a process of third-integer resonant extraction with rf electric fields. In mono-frequency constant-amplitude operation of the RFQ device, we encountered a new and unique result. The beam spill shows an exponential decay with a long time-constant despite a constant sextupole magnetic field for separatrix formation. From measurements of extraction efficiency and time constant as function of frequency and amplitude of the rf field, it is known that particles circulating with tunes at the low side of a tune distribution are extracted. This extraction process turned out to be a controllable chaotic motion occuring in a non-linear oscillating system with a time-periodic perturbation. Theoretical predictions of chaos give a close agreement with the experimental results .

1 INTRODUCTION

A slow kick-extraction with a transverse rf electric field has been developed at the HIMAC synchrotrons at NIRS. This unique method abbreviated by RF-KO-SE utilizes frequency and amplitude modulation, FM&AM, [1] for realizing a high extraction efficiency due to the fact that tunes of the circulating particles are distributed in a wide range due to a non-linear oscillating system.

Although the RF-KO-SE method with FM&AM is being used extensively in a daily operation for production of beams synchronized to patient respiration for cancer therapy, it was not known what really happens during this extraction. The purpose of the present study is to clarify this extraction process .

In Chapter 2, experimental results of the extraction efficiency and the time constant are given together with theoretical predictions of chaos in order to show the close agreement. In Chapter 3, the theory of chaos is outlined on the basis of circle mapping and the Fokker-Planck equation. In Chapter 4, some remaining problems are discussed which need further studies.

2 COMPARISON OF EXPERIMENTAL RESULTS WITH THEORETICAL PREDICTIONS OF CHAOS

When an RFQ field is applied to particles circulating in a synchrotron without a sextupole magnetic field for the separatrix formation, the equation of motion is reduced to the Mathieu's equation. The strict resonance occurs independently of the amplitude of the RFQ field and can be written as

$$f_{RFQ} = |2\nu \pm m| \times f_{rev} (m = 0,1,2,\cdots) \qquad (1)$$

where f_{RFQ} is the mono-frequency of the RFQ field, f_{rev} the revolution frequency of the beam, ν the betatron tune, and m an appropriate integer. When necessary for presentation of experimental data, the mono-frequency is reduced to a tune, ν_{RFQ}, based on Eq.(1).

We started an experimental study of the third-integer resonant extraction using the RFQ field with a mono-frequency constant-amplitude operation of the RFQ and encountered a new and unique result as shown in Fig.1. The beam spill shows a very low ripple with a long time-constant of 150 msec in an exponential decay despite the fact that all working parameters of the RFQ field and the magnetic sextupole field were kept constant during the extraction. The extraction efficiency, which is defined as ratio of the total amount of extracted particles to a total amount of circulating particles before extraction, was 16% at best. While this is rather large, it is significantly less than 100%.
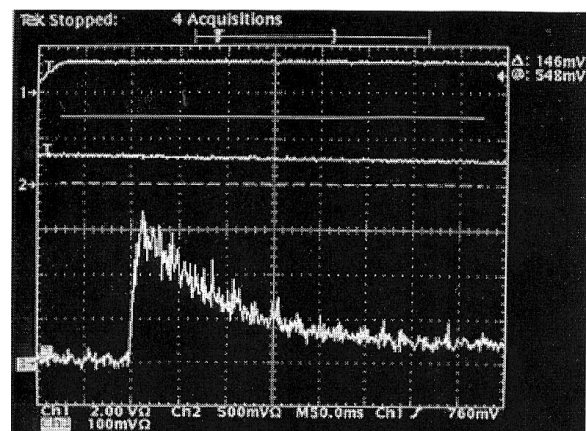

Fig.1 Typical time structure of the extracted beam. The lower curve shows a beam spill with very low ripple and a long time-constant in the exponential decay .

2.1 Extraction Efficiency

Dependence of the extraction efficiency on the mono-frequency at constant amplitude of a rf power of 1 kW was observed as shown in Fig.2. A tune distribution of the circulating particles was observed by a side-band spectrum of the synchrotron oscillation is also shown in Fig.2. It should be noted that the RFQ field operating at a mono-frequency in the vicinity of the central tune of the circulating particles can no longer extract particles despite the fact that the mono-frequency satisfies the resonant condition for some particles.

It is also shown in Fig.2 that a theoretical prediction of chaos is in close agreement with the experimental result.
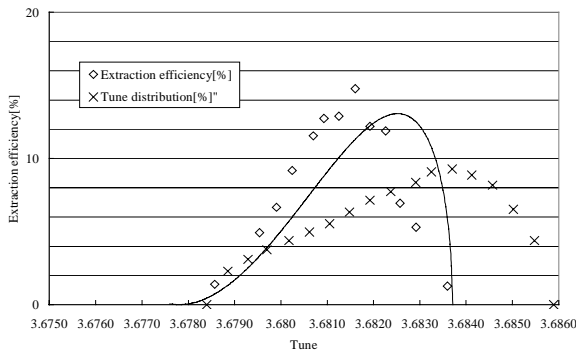

Fig.2 Comparison of dependence of the extraction efficiency on the mono-frequency of the RFQ field between the experimental result and the theoretical prediction. Cross marks show a tune distribution in an arbitrary unit.

The dependence of the extraction efficiency on the rf power is shown in Fig.3. The observed extraction efficiency tends to saturate with increasing rf power and well agrees with the theoretical prediction using a relation of $P^{1/4}$,where P is the rf power. This dependence is different from a relation of $P^{1/2}$ which may be obtained from a perturbation in a linear oscillating system.
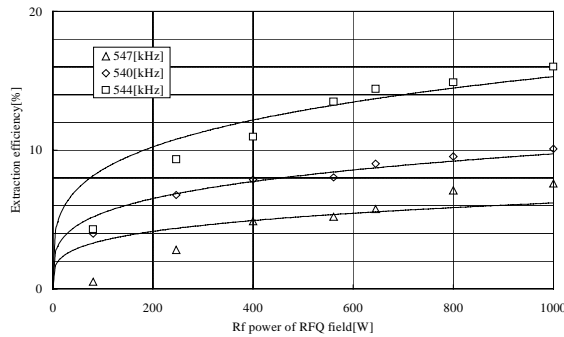

Fig.3 Comparison of dependence of the extraction efficiency on the rf power of the RFQ field between the experimental results and theoretical predictions.

2.2 Time Constant

We also measured time constants as function of the mono-frequency and the rf power. The observed time-constants tend to increase sharply towards the central tune and to decrease with increasing rf power. Time constants as long as 800 msec could be observed but the extraction efficiency was quite low.

Theoretical predictions agree well with experimental results especially for the rf power dependence using a relation of $P^{-5/8}$. This unusual relation arises apparently from a periodic perturbation of a non-linear oscillating system .

3 OUTLINE OF THEORY OF CHAOS

In contrast to the usual approach of shrinking the

separatrix for extraction, it can be seen from Fig.4 that in case of chaos in a third-integer resonant extraction with electric rf fields a separatrix disappears as a stochastic layer. Particles moving in the layer are extracted slowly through an extraction separatrix due to diffusion. It should be noted that the stochastic layer occurs always in the vicinity of the separatrix and an extraction separatrix is always associated with the separatrix.

The present formulation of chaos follows the usual description of a non-linear oscillating system with a periodic perturbation, although an extraction associated with chaos has not been discussed previously.
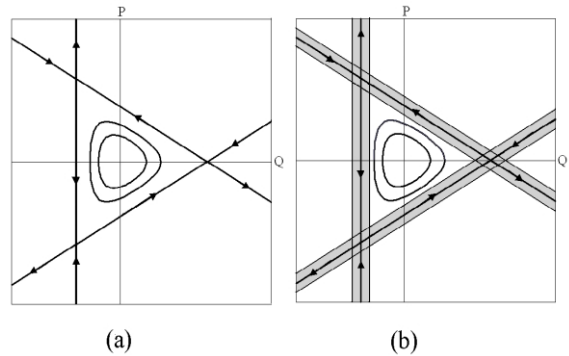

Fig.4 (a)Separatrix of the third-integer resonance without periodic perturbation. (b) Disappearance of separatrix as a stochastic layer due to a periodic perturbation.

3.1 Extraction Efficiency

It can be seen from Fig.4 that the extraction efficiency is the ratio of the total number of particles in the stochastic layer to the total number of particles in an area surrounded by the original separatrix. In order to evaluate the width of the layer, the action-angle variables (J, θ) are used and the Hamiltonian is assumed to consist of two parts:

$$H(J, \theta, \phi) = H_0(J) + \varepsilon V(J, \theta, \phi). \qquad (2)$$

The first term describes an unperturbed non-linear oscillating system and the second a periodic perturbation. The phase variable ϕ is equivalent to a time variable and denotes the well-known phase variable, which is defined to describe a betatron oscillation.

Equations of motion of the action-angle variables are approximated due to the existence of a periodic perturbation[2,3] as follows:

$$\begin{cases} \dfrac{d\theta}{d\phi} \sim \dfrac{\partial H_0}{\partial J} = \Omega(J) \\ \dfrac{dJ}{d\phi} \sim -\dfrac{\varepsilon}{\Omega(J)} \dfrac{dV}{d\phi} \end{cases} \qquad (3) \text{ In}$$

these equations, the angle variable itself is not affected by the perturbation directly so that a new phase variable φ which expresses a particle motion[2,3] is defined to evaluate a circle mapping :

$$\frac{d\varphi}{d\theta} = \Omega_0 - \Omega_Q \qquad (4)$$

where Ω_Q is the mono-frequency of the rf field and Ω_0 the revolution frequency of the central particle.

Here, a circle mapping is applied and a relation of the phase variable between the nth and $(n+1)^{th}$ revolution is defined as follows:

$$K = \left| \frac{\delta\varphi_{n+1}}{\delta\varphi_n} - 1 \right| \qquad (5)$$

where K is a coupling constant which is a measure of a non-linearity.

A critical condition of $K = 1$ corresponds to a boundary of the stochastic layer so that a width of the layer can be evaluated as a deviation of the Hamiltonian δH from the original separatrix H_{0C} and is obtained as :

$$\frac{\delta H}{H_{0C}} \propto \sqrt{\varepsilon(\Omega_0 - \Omega_Q)} \propto \sqrt{\varepsilon(\nu_0 - \nu_{RFQ})} \qquad (6)$$

where ν_0 is the central tune.

This expression gives the rf power dependence P $^{1/4}$ of the extraction efficiency as described before.

3.2 Time Constant

Particles moving in the stochastic layer diffuse outside through an extraction separatrix so that the particle density becomes thinner with time passing. Therefore a particle density distribution in the layer is considered to satisfy the Fokker-Planck equation:

$$\frac{\partial \rho(J,\phi)}{\partial \phi} = -\frac{1}{2}\frac{\partial}{\partial J}\left\{ D(J)\frac{\partial}{\partial J}\rho(J,\phi) \right\} \qquad (7)$$

where J is the action variable, $\rho(J,\phi)\,\delta J$ the probability of a particle between J and $J + \delta J$ at time ϕ, and $D(J)$ a diffusion constant.

It should be noted that the diffusion constant can be determined both in the absence and the presence of a periodic perturbation as follows[4]:

$$D(J) = \frac{J^2}{\tau_D} \qquad \text{without the perturbation}$$

$$\qquad (8)$$

$$D(J) = \left\langle \frac{(\delta J)^2}{\tau_S} \right\rangle \qquad \text{with the perturbation}$$

where τ_D is the time constant of the diffusion, δJ a change of the action variable due to a kick given by Eq.(3), and τ_S the time interval of the kick.

A change of the action variable is reduced to a deviation of the Hamiltonian by Eq.(3) and is expressed as a function of the mono-frequency and the constant amplitude of the RFQ field by Eq.(6). A theoretical prediction of the time constant is obtained as follows:

$$\tau_D \propto 1/\varepsilon^{\frac{5}{4}}/(\Omega_0 - \Omega_Q)^{\frac{5}{4}} \propto 1/\varepsilon^{\frac{5}{4}}/(\nu_0 - \nu_{RFQ})^{\frac{5}{4}} \qquad (9)$$

This expression gives the unusual rf power dependence P $^{-5/8}$ of the time constant as mentioned before.

4 DISCUSSION

It can be concluded that a chaotic motion occurs in the third-integer resonant extraction generated by electric rf fields with a close agreement between the experimental results and theoretical predictions. It is a controllable chaos because the extraction efficiency and the time constant can be chosen by appropriate combinations of the mono-frequency and the constant amplitude of the rf field.

The FM&AM is needed for a high extraction efficiency at HIMAC because all particles moving in a stable region surrounded by the stochastic layer are necessarily pushed out of the layer due to a diffusion which is caused by a stochastic FM&AM rf field. Good extraction will be realized in an appropriate combination of the mono-frequency and the FM&AM.

Although the present extraction intends to use the third-integer resonance $\nu=11/3$, it can be seen from Fig. 2 that the lowest tune in the tune distribution is higher than 11/3 so that the cause of a stop band should be studied. One of candidates is the non-linear force itself, that is, the magnetic sextupole field itself for the separatrix formation, which may cause a stochastic layer.

We carried out experiments not only using the RFQ field but also an RFD(Radio-Frequency Dipole) field but we could not find a difference of the extraction performance between them. This fact suggests that the chaotic motion is independent of the type of the periodic perturbation.

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