BEAM PROPERTIES IN THE SNS ACCUMULATOR RING DUE TO TRANSVERSE PHASE SPACE PAINTING*

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Abstract

Beam properties resulting from transverse phase space painting for the injection of the Spallation Neutron Source (SNS) accumulator ring are summarized. The focus is on the creation of closed orbit bumps that give suitable distributions at the spallation neutron target, at the same time minimizing the beam losses through tail/halo formation. Related issues such as foil traversal rate and foil temperature/lifetime are also studied. Finally, injection mismatch is investigated for large linac emittance to reduce the beam loss through scattering.

1 INTRODUCTION

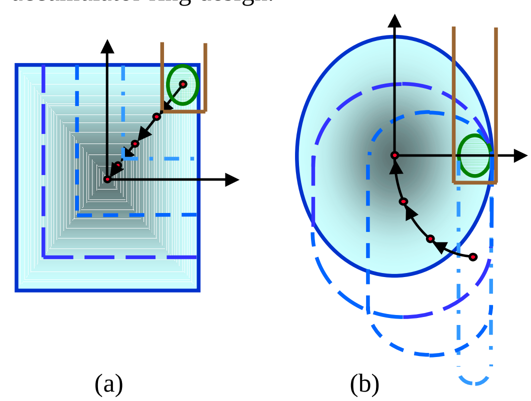
Injection painting is a multi-turn injection process with a controlled phase space offset (6-D) between the centroid of the injected beam and the closed orbit in the ring to achieve a different particle distribution from the injected beam. If the longitudinal beam manipulation is decoupled from transverse phase space painting, as is the case for the SNS accumulator ring injection [1], we can study the longitudinal painting separately [2]. In this paper we study beam properties due to transverse phase space painting [3], which will be implemented for the SNS accumulator ring injection. The injection bumps will be optimized in order to:

(a) Satisfy the target requirements (Table 1);
(b) Reduce beam losses due to space charge;
(c) Reduce foil-hits, therefore reduce beam losses at foil while maintaining an adequate foil lifetime.

Table 1 Beam requirements at the target.

Beam horizontal dimension	200 mm
Beam vertical dimension	70 mm
Time-averaged beam current density over beam footprint	< 0.143 A/m^2
Beam power within target and outside nominal spot	< 10 %
Peak time-averaged beam current density over 1 cm^2	≤ 0.25 A/m^2

The circulating beam properties are critically dependent on the choice of painting schemes and the optimization of injection orbit bumps. There are two basic painting schemes — correlated (Fig.1(a)) and anti-correlated painting (Fig. 1(b)) — incorporated in the SNS accumulator ring design.



(a) (b)

Fig. 1 Illustration of basic painting schemes in (*x-y*) space: (a) correlated painting; (b) anti-correlated painting. In both graphs, the small green ellipse represents the injected beam on the foil. The closed orbit (red dots) moves in the direction indicated by the black arrows. The beam shape and size from beginning to the end of injection are characterized progressively by the dot-dashed, short-dashed, long-dashed and solid lines.

2 ANALYTICAL EXPRESSION

Assume the linac beam with a Gaussian distribution (σ_{0x}, σ_{0y}) is continuously injected into the ring during interval [0, *t*]. At time *t*, the centroid of injected beam is at ($X_c(t)$, $X'_c(t)$, $Y_c(t)$, $Y'_c(t)$) and the closed orbit is at ($X_o(t)$, $X'_o(t)$, $Y_o(t)$, $Y'_o(t)$). If painting in (x, x') phase space is independent (y, y'), the (x, y) space distribution can be expressed as:

$$n(x,y,t) = \int_0^t n_x(x,t_1)\, n_y(y,t_1)\, dt_1 \qquad (1)$$

with [4]

$$n_x(x,t_1) = \frac{1}{\pi\sigma_{0x}^2}\int_{|x|}^{+\infty} e^{-\{[a_x - c_x(t_1)]^2/2 + |a_x c_x(t_1)|\}/\sigma_{0x}^2}\, \frac{a_x I_0\left(\frac{a_x c_x(t_1)}{\sigma_{0x}^2}\right)}{(a_x^2 - x^2)^{1/2}}\, da_x \qquad (2)$$

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where $I_0(z)$ is the modified Bessel function of order zero, and $c_x(t)$ is the effective 2-D phase space offset between the two centers $c_x(t) = \{\Delta X^2(t)+[\beta_x \Delta X'(t) + \alpha_x \Delta X(t)]^2\}^{1/2}$ with $\Delta X = X_C - X_O(t)$, $\Delta X' = X'_C - X'_O(t)$. In expression (1), $n_y(y, t_1)$ can be obtained from Eq. (2) by replacing x by y. In the derivation of Eq. (1), space charge and other non-linear effects are not included and the following assumptions have been made:

- The painting in (x, x') phase space is controlled independently from the (y, y') painting. Therefore, the 4-D problem is reduced to 2-D.
- The bumps change slowly compared to the betatron oscillation (for the SNS, $<|dx/dt|>|_{bump}$ = 40m/sec, and $<|dx/dt|>|_{beta} = 1\times10^6$m/sec).

These assumptions have been justified for the SNS injection. (See reference [5] for details).

From this analytical expression we deduce:

- If $c_x(t)=c_y(t)=0$, there is a singularity in the local density at the center of the beam. So, one should not have zero offset in 4-D phase space at any time.
- Correlated painting will give a rectangular shaped beam distribution with high density along the diagonal line, and low density on the x- and y-axes.
- If $\sigma_x << \Delta X(t)$, $a_x(t)$ and $\sigma_y << \Delta Y(t)$, $a_y(t)$, a careful choice of anti-correlated painting with offsets:

$$\Delta X(t) = A\left(\frac{t}{t_{inj}}\right)^{1/2}, \qquad \Delta X'(t) = -\Delta X(t)\frac{\alpha_x}{\beta_x} \qquad (3)$$

$$\Delta Y(t) = B\left(1-\frac{t}{t_{inj}}\right)^{1/2}, \quad \Delta Y'(t) = -\Delta Y(t)\frac{\alpha_y}{\beta_y} \qquad (4)$$

will give a 4-D KV-like transverse distribution at $t=t_{inj}$:

$$n(x,y,t_{inj}) = f_0(x,y,t_{inj}) + f_1(x,y,t_{inj}) \qquad (5)$$

where $f_0(x,y)$ is the zeroth order solution of Eq (1):

$$f_0(x,y,t_{inj}) = \begin{cases} \dfrac{t_{inj}}{\pi AB} & for \quad \dfrac{x^2}{A^2}+\dfrac{y^2}{B^2} \le 1 \\ 0 & for \quad \dfrac{x^2}{A^2}+\dfrac{y^2}{B^2} > 1 \end{cases}. \qquad (6)$$

It is an elliptically shaped uniform distribution, which can also be obtained by projecting a 4-D KV-distribution to (x, y) space. Defining

$$\eta = \sqrt{1-\frac{x^2}{A^2}}, \quad \xi = \frac{1}{\eta}\sqrt{1-\frac{x^2}{A^2}-\frac{y^2}{B^2}}, \qquad (7)$$

$$\zeta = \sqrt{1-\frac{y^2}{B^2}}, \quad \psi = \frac{1}{\zeta}\sqrt{1-\frac{x^2}{A^2}-\frac{y^2}{B^2}}, \qquad (8)$$

the first order term in Eq. (5) is given by [5]:

$$f_1(x,y,t_{inj}) = -\frac{\sqrt{2}\,t_{inj}}{\pi^{5/2}AB}\left\{\frac{\sigma_x}{A\zeta}\left[F(\frac{\pi}{2},\psi) - F(0,\psi) + \frac{x^2}{A^2\zeta^2\psi}\left(\Pi(\frac{\pi}{2},\psi^2,1) - \Pi(0,\psi^2,1)\right)\right]\right.$$
$$\left.\frac{\sigma_y}{B\eta}\left[F(\frac{\pi}{2},\xi) - F(0,\xi) + \frac{x^2}{B^2\eta^2\xi}\left(\Pi(\frac{\pi}{2},\xi^2,1) - \Pi(0,\xi^2,1)\right)\right]\right\} \qquad (9)$$

where F is the elliptic integral of the first kind and Π is the elliptic integral of the third kind. As long as $f_1(x,y,t_{inj}) << f_0(x,y,t_{inj})$, a KV-like distribution is maintained.

3 COMPUTER SIMULATIONS

The investigations are performed by tracking 2×10^4 macro-particles through the ring lattice, in the presence of space charge, with the simulation code SIMPSONS [6]. All the physical quantities used in the simulations (Table 2) are chosen according to the specifications in the design [1]. The results of simulations at working point (5.82, 4.80) from correlated and anti-correlated painting are shown in Figs. 2 and 3 respectively. In both figures, the simulation including space charge (red dots/lines) is compared to that without space charge (blue dots/lines) with identical physical and numerical parameters. The upper graphs show the distributions in (x, x') and (y, y') phase space. The lower two graphs show the distribution in (x, y) space and the distribution of 4-D phase space areas occupied by single particles.

Table 3 summarizes the beam properties due to correlated and anti-correlated painting. The distribution from correlated painting may satisfy the target requirements, but it could be difficult to preserve the beam shape. The distribution from anti-correlated painting is immune to the transverse coupling, but there are excessive halo/tails. In order to take advantage of both correlated and anti-correlated painting, an oscillating painting scheme was proposed at BNL [5]. However, it requires more aperture, and it is technically challenging for the power supplies. Reference [7, 8] give more detailed studies on the effects of space charge forces and magnet errors in the SNS accumulator ring.

Table 2 Design parameters used in the simulations.

Beam Kinetic Energy	1 GeV
Beam Average Power	2.0 MW
Protons per Pulse	2.08×10^{14}
Pulse Repetition Rate	60 Hz
Ring Circumference	220.88 m
Proton Revolution Period	841 nsec
Injected Pulse length	546 nsec
Beam gap (injection)	295 nsec
Number of Turns Injected	1225
Ring Fill Time	1.03 msec
Horizontal tune ν_x	5.8 – 6.8
Vertical tune ν_y	4.8 – 5.8
Un-nor. Emittance (99%)	160 πmm-mr

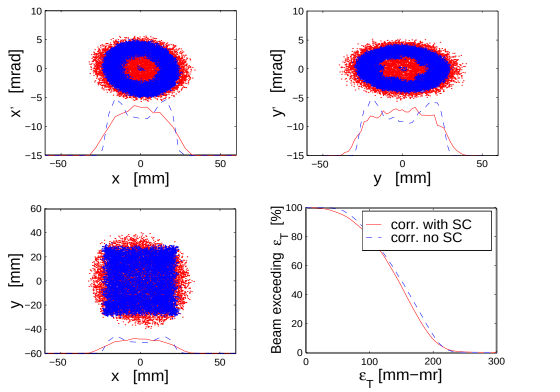


Fig. 2 Particle distribution and emittance distribution resulting from an example of correlated painting.

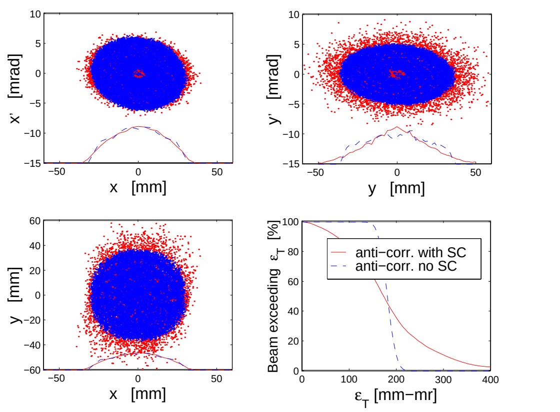


Fig. 3 Particle distribution and emittance distribution resulting from an example of anti-correlated painting.

Table 3 Beam properties due to correlated and anti-correlated painting in the SNS accumulator ring.

painting scenarios	correlated	anti-correlated
Beam shape without SC	Rectangular	Oval
Beam emittance evolution	Small to large	~ constant
Final emit. $\epsilon_x+\epsilon_y$(πmm-mr)	120+120	160
Est. foil-hitting rate	6.5	8.5
Est. max foil temp. (K)	1940	2070
Horizontal aperture (Δ_H)	1:1	1:1
Vertical aperture (Δ_V)	1:1	1:1.5
Susceptible to coupling	Yes	No
Capable for KV painting	No	Yes
Paint over halo	Yes	No
Horizontal halo/tail	Normal	Normal
Vertical halo/tail	Normal	Large
Satisfy target requirements	Likely	Not likely
Bump function candidates for optimization	Square root; exp(-t/0.3ms); Combination	Square root; exp(±t/0.6ms); Sinusoidal

4 INJECTION MISMATCH

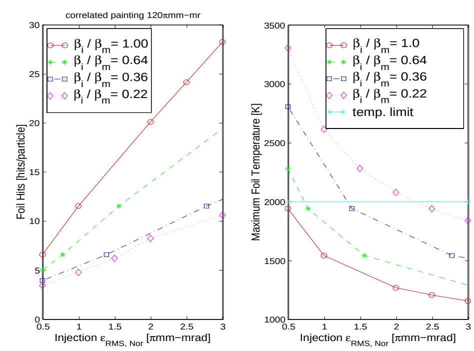
Twiss function mismatch can be used as a tool to reduce the foil-hitting rate, thereby reducing the beam loss due to scattering in the foil. A mismatched injection should satisfy two preferred conditions in order to stack the injected turns efficiently in phase space [9]:

$$\frac{\alpha_i}{\beta_i}=\frac{\alpha_m}{\beta_m}=-\frac{X'_C-X'_O}{X_C-X_O}\quad, \tag{12}$$

and

$$\frac{\beta_i}{\beta_m}\geq\left(\frac{\varepsilon_i}{\varepsilon_m}\right)^{1/3}\quad, \tag{13}$$

where α_i, β_i and α_m, β_m are the values at the injection point of the Twiss functions in the transfer line and the ring respectively. The foil hitting rate and maximum foil temperature for one matching condition and three different degrees of mismatch with correlated painting are shown in Fig. 4. Similar results with slightly higher values are found for anti-correlated painting [9]. A lower foil-hitting rate is accompanied by a higher foil temperature due to a smaller beam size. The mismatch for the SNS injection will be optimized to reduce the foil-hitting rate while maintaining an adequate foil lifetime.



(a) (b)

Fig. 4 Foil-hitting properties due to injection mismatch for correlated painting. (a) foil-hitting rate vs. injected beam emittance ε_{inj}; (b) maximum foil temperature vs. ε_{inj} .

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