EFFECT OF NON-LINEARITIES ON BEAM DYNAMICS IN THE SNS ACCUMULATOR RING*

A.V. Fedotov, D.T. Abell, Y.Y. Lee, N. Malitsky, Y. Papaphilippou, and J. Wei
BNL, Upton, NY 11973, USA

Abstract

Uncontrolled beam losses lead to excessive radio-activation in high-intensity proton rings. Designing a successful high-intensity ring therefore requires a good understanding of the sources of such beam losses. This paper presents, for the Spallation Neutron Source (SNS) accumulator ring, dynamic aperture studies in which we explore the effect of non-linearities arising from kinematic terms and various magnet imperfections. We also discuss the impact on the design of the SNS accumulator ring.

1 INTRODUCTION

Special characteristics of the SNS ring are large beam emittances (up to $240\ \pi$ mm mrad), and large beam pipe apertures. Not surprisingly, this brings a variety of non-linear effects which are a direct consequence of large particle amplitudes. Such non-linearities can shift particles in undesired directions, dramatically decreasing the dynamic aperture. The study and understanding of these effects thus becomes very important. In this paper, we concentrate on some of such "special" (large amplitude) non-linear effects. We also present different mechanisms which contribute to amplitude dependent tune spreads, and report on dynamic aperture studies. These studies refer to the the hybrid lattice (FODO arcs and doublet straights) of the 220 m circumference SNS ring, and working point (Q_x, Q_y)=$(6.3, 5.8)$ [1].

2 KINEMATIC NON-LINEARITY

The kinematic non-linearity arises from high order terms proportional to the transverse momenta p_x, p_y in the expansion of the standard square-root relativistic Hamiltonian. The first correction to the tune shift comes from octupole-like terms: $p_x^4, p_x^2 p_y^2$ and p_y^4. A transformation to action variables results in the terms: $\gamma_x^2 J_x^2, \gamma_x \gamma_y J_x J_y, \gamma_y^2 J_y^2$. Thus, kinematic tune shift correction is governed by $\gamma_x^2 J_x, \gamma_x \gamma_y J_{x,y}$ and $\gamma_y^2 J_y$ pieces. We can now see what makes the SNS so "special" compared to typical machines where kinematic non-linearity is safely neglected. The first special characteristic of the SNS is that $\gamma_{x,y}$ functions are relatively large [1]. The second characteristic is that the beam emittances (actions) are also large, thus both effects produce noticeable kinematic tune shifts. Such kinematic tune shifts were observed in our numerical simulations.

Simulations for the SNS lattice were done with both Unified Accelerator Library (UAL) [2] and MARYLIE [4] codes, to ensure benchmarking of the UAL for the SNS application [3] and to be confident in our non-linear dynamics studies. MARYLIE is a thick element code with an approximate treatment of the equations of motion (we used MARYLIE's version 3.0 which expands the Hamiltonian through third order). UAL's tracking algorithm is TEAPOT [5], which uses thin elements but treats the non-linear equation of motion exactly. After detailed benchmarking of UAL with MARYLIE, we obtained perfect agreement between the codes. In Fig. 1 we show the tune shifts for the SNS ring produced solely by the kinematic corrections (no non-linear elements in the lattice or magnet errors are present). This plot is generated by launching particles in five different directions with the transverse amplitudes going up to $480\ \pi$ mm mrad. Data obtained with MARYLIE is presented by color circles, while UAL's data is given by white dots inside the color circles. These tune shifts also agree very well with the analytic predictions based on the perturbation theory.

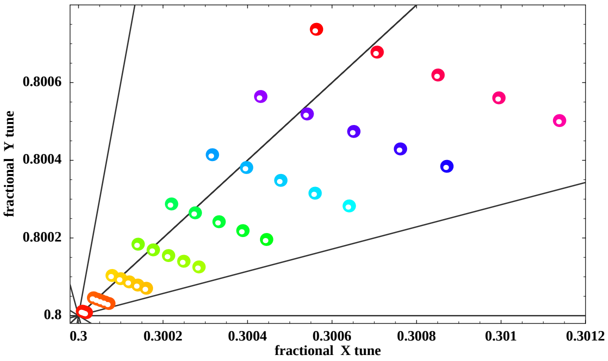

Figure 1: Kinematic non-linearity $X-Y$ foot-print.

3 DODECAPOLE TUNE SHIFTS

Another "special" large amplitude contribution to the tune shifts is the fringe field effect. The bore of the SNS magnets is very large resulting in high aspect ratio (inner diameter over magnet length). Therefore, the contribution from magnet ends becomes significant. For the tracking approach two-dimensional (2-D) multipoles were extracted based on 3-D field calculations of the unshimmed magnet poles [1]. Detailed study of the fringe field effects showed that direct octupole-like transverse kick induced by the edge-effect of the quadrupoles is very significant and much bigger than the octupole contribution from the 2-D multipole expansion. Because of its importance, this effect

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is described in a separate paper [6]. Here we present the dodecapole tune shift contribution from the fringe field based on a 2-D multipole. In the absence of pole-tip shimming, the error of first allowed multipole in the SNS quadrupoles (dodecapole) is 120 units normalized to 10^{-4} of the main field at the reference radius [1]. In Fig. 2 we present the tune foot-print due to such large dodecapole component obtained with the UAL. Once again we launch particles in five different direction with the amplitudes up to 480 π mm mrad. Remarkably, these calculations were again in very good agreement with MARYLIE and the analytic estimates based on the first order perturbation theory [7]. For example, based on first order perturbation theory, the dodecapole tune shift due to a specific quadrupole for on-momentum particles (in the horizontal direction) is given by

$$\Delta Q_x = \frac{b_6 L \beta_x}{12\pi\rho}\left[\frac{15}{2}\beta_x^2 J_x^2 - 45\beta_x\beta_y J_x J_y + \frac{45}{2}\beta_y^2 J_y^2\right] \quad (1)$$

where b_6 is the dodecapole error in the quadrupole, β is the average beta function, and L is the length of the magnet. (Better agreement is achieved when values of local beta functions are taken instead.) Figure 3 shows how the tune spread is reduced with a dodecapole component of 12 units. This clearly indicates the necessity of a detailed pole-tip compensation. Based on measurement data from the AGS Booster magnets, this dodecapole contribution is not expected to exceed a few units in 10^{-4}[1]. When magnet field errors are compensated to expected level, resulting tune spread is very small (see Fig. 4).

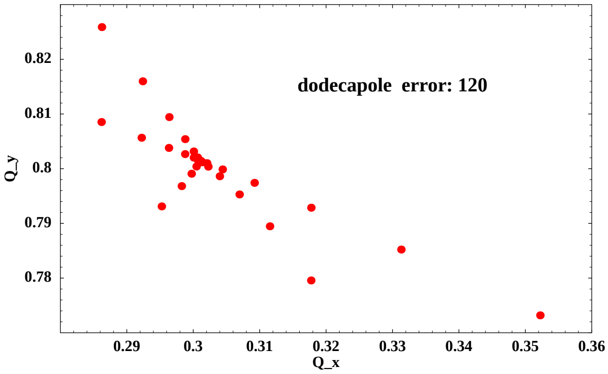

Figure 2: $X - Y$ foot-print for the dodecapole 2-D fringe field component of 120 units normalized to 10^{-4} of the main field at the reference radius.

4 EXPECTED TUNE SPREAD

In Table 1 we list the tune spread produced by various effects. The largest contribution clearly comes from the space-charge, and thus space-charge deserves serious study [8]. The next important contribution is introduced by the natural chromaticity. It requires compensation with multipole families of sextupoles [9]. The dodecapole fringe field tune shift is important but can be compensated by magnet end shimming. The next very important effect is the octupole-like 3-D fringe field tune shift (given at 480 π mm mrad). This effect and its compensation is described in [6]. Kinematic non-linearity tune shift is noticeable but still negligible as compared with the other non-linear effects in the SNS.

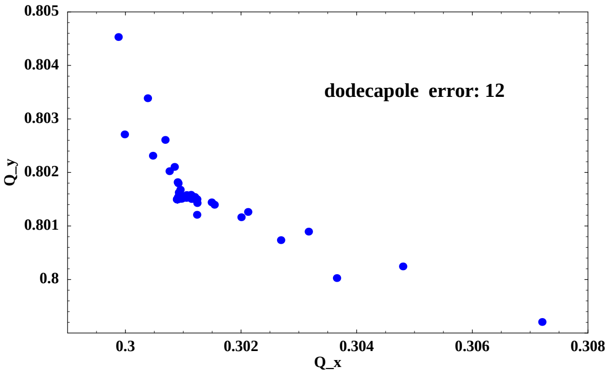

Figure 3: $X - Y$ foot-print for the dodecapole 2-D fringe field component of 12 units in 10^{-4}.

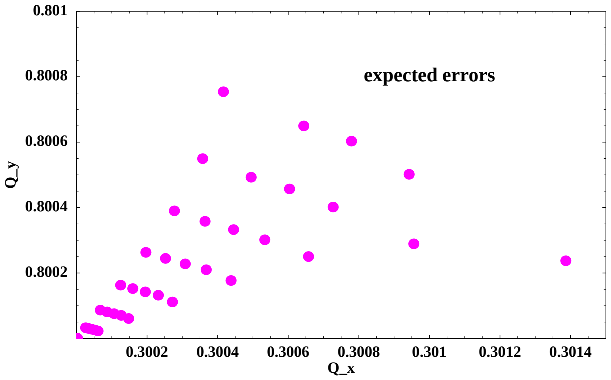

Figure 4: $X - Y$ foot-print for expected errors, assuming that integrated fringe field contribution is corrected.

Table 1: Tune spread due to various mechanisms

Mechanism	**Tune spread**
Space-charge (2 MW beam)	0.15 – 0.2
Chromaticity (1% $\Delta p/p$)	$\pm$0.08
Uncompensated dodec. component	0.06 (480π)
Uncompensated oct.-like fringe field	0.025 (480π)
Compensated errors	0.025 (480π)
Kinematic non-linearity	0.001 (480π)

5 DYNAMIC APERTURE

Figures 5, 6 and 7 show the impact of magnet field errors and the importance of field compensation and orbit correction. Six-dimensional tracking was performed using UAL. Particles are launched in five transverse directions, with increasing betatron amplitudes, for three momenta $\Delta p/p = 0, \pm 0.7\%$. The average dynamic aperture and the statistical errors are obtained from the results of 10 random seeds. The green curves show the combined effect resulting from the integrated 2-D fringe field component of the uncompensated magnet pole-tips and misalignments. The violet curves indicate the sole effect of 2-D integrated fringe field for uncompensated magnet pole-tips. The red curves represent the combined effect of pessimistic field errors (10^{-3} level) and misalignments. With field errors in both dipoles and quadrupoles at 10^{-4} level, magnet misalignments can still cause significant closed orbit offset and coupling, requiring careful corrections. Finally, the blue curves show the case of expected errors (10^{-4} level) [1] with corrected misalignments.

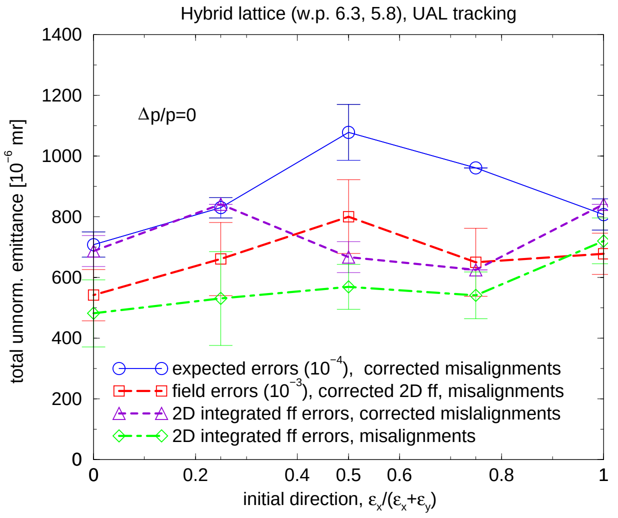

Figure 5: Dynamic aperture for on momemtum particles

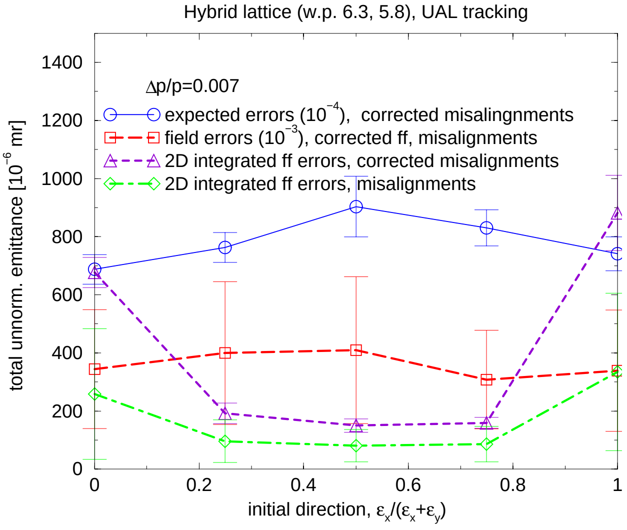

Figure 6: Dynamic aperture for particles with initial momentum $\Delta p/p = 7\%$.

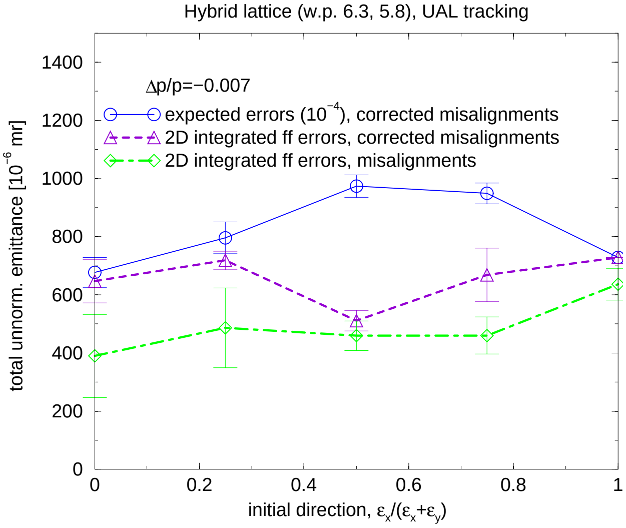

Figure 7: Dynamic aperture for particles with initial momentum $\Delta p/p = -7\%$.

6 CONCLUSIONS

In this paper we present a study of the amplitude-dependent tune shifts, including kinematic non-linearity which becomes noticeable for such machines as the SNS. Their effect on dynamic aperture for the SNS ring is also discussed.

7 ACKNOWLEDGMENT

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REFERENCES

[1] J. Wei at al., Low-loss design for the high-intensity ring of the SNS, submitted to PRST Accel. Beams; also BNL/SNS Tech Note 66 (1999).

[2] N. Malitsky and R. Talman, AIP 391 (1996).

[3] N. Malitsky et al., PAC99 (New York 1999), p. 2713.

[4] A. Dragt et al., MARYLIE 3.0 User's Manual, University of Maryland, Physics Department Report (1999).

[5] L. Schachinger and R. Talman, Particle Accel., Vol. 22 (1987), p. 35.

[6] D.T. Abell et al., Dynamic analysis of magnet field imperfections in the SNS accumulator ring, these proceedings.

[7] J. Wei, Workshop on LHC interaction region correction systems, BNL Report 52575 (1999), p. 87.

[8] A.V. Fedotov et al., Halo and space-charge issues in the SNS accumulator ring, these poceedings.

[9] N. Tsoupas et al., Chromatic correction and optical compensation in the SNS ring using sextupoles, these proceedings.