A METHOD FOR SIMULTANEOUS OPTIMISATION OF ORBIT AND DISPERSION IN STORAGE RINGS

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Abstract

An algorithm for the simultaneous optimisation of orbit and dispersion in a storage ring is presented. Based on orbit and dispersion measurements the algorithm determines the optimal corrector settings in order to simultaneously minimize the r.m.s. orbit, the r.m.s. dispersion and the r.m.s. strength of the dipole correctors. A number of different options for error handling of beam position monitors, weighting, and correction have been introduced to ensure the stability of the algorithm in the environment of a large accelerator. Experimental results are presented for the LEP collider demonstrating the efficiency of the method. The use of this correction algorithm for LEP in 1999 allowed achieving about a factor of two smaller vertical emittances than in previous years.

1 INTRODUCTION

Storage rings are either used for particle physics studies with colliding beams or synchrotron radiation applications. In both cases the performance of the storage ring depends crucially on the vertical emittance ε_y. Neglecting eventual beam-beam effects we can write

$$\varepsilon_y \simeq \varepsilon_{y0} + \kappa\varepsilon_x + C \cdot \left(E \cdot D_y^{rms}\right)^2 \tag{1}$$

with ε_{y0} being the emittance contribution from quantum excitation with finite emission angles, κ being the emittance coupling, ε_x the horizontal dispersion, C a constant, E the beam energy, and D_y^{rms} the r.m.s. vertical dispersion. For an ideal planar storage ring we have $D_y^{rms} = 0$ and $\kappa = 0$. The ideal vertical emittance is given by ε_{y0} and approaches zero for all practical considerations.

Unavoidable imperfections and eventual coupling fields from solenoids result in non-zero emittance coupling and vertical dispersion. Numerous correction algorithms in storage rings aim at minimising the vertical emittance. We present an algorithm implemented for emittance optimisation in the Large Electron Positron collider LEP [1, 2]. There are 500 beam position monitors installed in LEP measure the beam position in both planes. 261 horizontal and 312 vertical orbit correctors are available for orbit steering.

The effect from a given dispersion on the emittance scales the square of the beam energy. The coupling effects from the experimental solenoids scale with $1/E$ so that the total coupling contribution $K\varepsilon_x$ due to the solenoids scales with beam energy. The vertical emittance in LEP with high beam energies ($E = 90 - 104$ GeV) is therefore mainly given by the dispersion term [3]. To improve LEP performance work on dispersion correction was started in 1998. We used a deterministic orbit and dispersion correction scheme ("dispersion-free steering"), originally proposed and developed for the SLAC linac [4, 5, 6]. It was later implemented for the PEP-2 storage ring [7].

2 FORMALISM

The formalism is shortly reviewed. More details are given in [8]. The beam position shall be measured with a set of N beam position monitors (BPM) which are distributed over the ring. The orbit is corrected with a set of M dipole magnets (correctors). The beam position at the BPMs is represented by the vector $\vec{u}$ of dimension N, the corrector strengths (kicks) by the vector $\vec{\theta}$ of dimension M. The response matrix $\mathbf{A}$ (dimension $N \times M$) describes the relation between corrector kicks and beam position changes at the monitors. An element A_{ij} of the response matrix corresponds to the orbit shift at the i^{th} monitor due to a unit kick from the j^{th} corrector.

The measured dispersion at the BPMs is represented by the vector $\vec{D}_u$ (dimension N). $\mathbf{B}$ is the $N \times M$ dispersion response matrix, its elements B_{ij} giving the dispersion change at the i^{th} monitor due to a unit kick from the j^{th} corrector.

In order to correct orbit and dispersion simultaneously a set of corrector kicks $\vec{\theta}$ must be found that solves the following system of linear equations:

$$\begin{pmatrix} (1-\alpha)\vec{u} \\ \alpha\vec{D}_u \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \end{pmatrix} \vec{\theta} = 0 \tag{2}$$

The weight factor α is used to shift from a pure orbit (α = 0) to a pure dispersion correction (α = 1). In general the optimum closed orbit and dispersion r.m.s. are not of the same magnitude and α must be adjusted for a given machine. α can in principle be evaluated from the BPM accuracy and resolution.

In general the number of BPMs (N) and the number of correctors (M) are not identical and Equation 2 is either over ($N > M$) or under constrained ($N < M$). In the former and most frequent case Equation 2 can not be solved exactly. Instead an approximate solution is found by least square minimisation of the quadratic residual

$$S = (1-\alpha)^2\|\vec{u} + \mathbf{A}\vec{\theta}\|^2 + \alpha^2\|\vec{D}_u + \mathbf{B}\vec{\theta}\|^2 \,. \tag{3}$$

Singular response matrices are a well known problem of orbit corrections. The singularities are related to redundant correctors, i.e. areas of the machine where the sampling of the orbit is insufficient. Such situations yield numerically unstable solutions where large kicks are associated

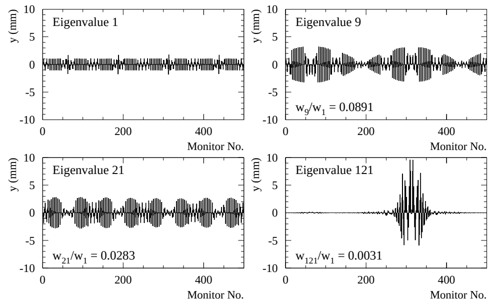

Figure 1: Example of normalised eigenvectors ($w_1\vec{v}^{(i)}/w_i$) for the vertical orbit of LEP (α = 0.2, β = 0). The main harmonics are the tune Q for eigenvector 1, $Q \pm 1$ for eigenvector 9 and $Q \pm 4$ for eigenvector 21. The eigenvectors associated to small eigenvalues (bottom right) often correspond to long "bumps".

to minor changes in the orbit. A standard cure consists in disabling a subset of correctors and removing the corresponding lines from the linear system of Equation 2. Regularisation can also be obtained by extending Equation 2 to constrain the size of the kicks:

$$\begin{pmatrix} (1-\alpha)\vec{u} \\ \alpha\vec{D}_u \\ \vec{0} \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \\ \beta\mathbf{I} \end{pmatrix} \vec{\theta} = \vec{d} + \mathbf{T}\vec{\theta} = 0 \ . \quad (4)$$

Here, $\vec{0}$ is a null vector of dimension M, $\mathbf{I}$ a unit matrix of dimension $M \times M$ and β is a kick weight. The quadratic residual is now:

$$S = (1-\alpha)^2\|\vec{u}+\mathbf{A}\vec{\theta}\|^2 + \alpha^2\|\vec{D}_u+\mathbf{B}\vec{\theta}\|^2 + \beta^2\|\vec{\theta}\|^2 \ . \quad (5)$$

Large kicks are suppressed since they receive a penalty which can be adjusted with β.

The SVD algorithm [9, 10] is a powerful tool to handle singular systems and to solve them in the least square sense. It finds eigenvectors $\vec{\vartheta}^{(i)}$ that are linear combinations of all dipole correctors. The M vectors $\vec{v}^{(i)}$ are orthogonal but not normalised

$$(\vec{v}^{(i)} \cdot \vec{v}^{(j)}) = \vec{v}^{(i)\mathbf{t}}\ \vec{v}^{(j)} = \vec{\vartheta}^{(i)\mathbf{t}}\ \mathbf{T}^{\mathbf{t}}\mathbf{T}\vec{\vartheta}^{(j)} = w_i^2\delta_{ij} \ . \quad (6)$$

The eigenvalues (or weights) w_i are a quantitative measure of the orbit and dispersion response to a given $\vec{\vartheta}^{(i)}$. Small eigenvalues correspond to singular solutions where combinations of correctors lead to essentially no response on the measured orbit or dispersion.

Examples of eigenvalue spectra and eigenvectors for LEP are shown in Figures 1 and 2. The eigenvectors associated to the largest eigenvalues correspond to orbit and dispersion changes that contain strong harmonics close to the tune. They are combinations of a large number of small corrector kicks, but their effect on the orbit and dispersion is large because the kicks add up resonantly, as can be seen in Figure 2. It can be shown that the harmonics of the

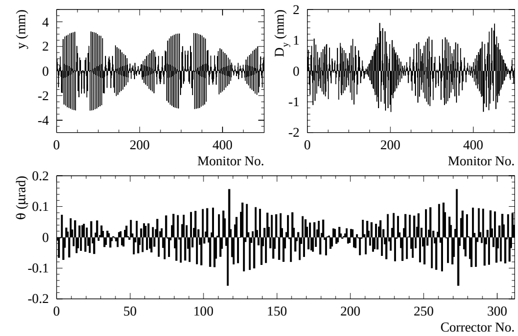

Figure 2: The orbit (top left) and dispersion (top right) components of $\vec{v}^{(9)}$ are shown together with the corrector setting corresponding to $\vec{\vartheta}^{(9)}$ (bottom).

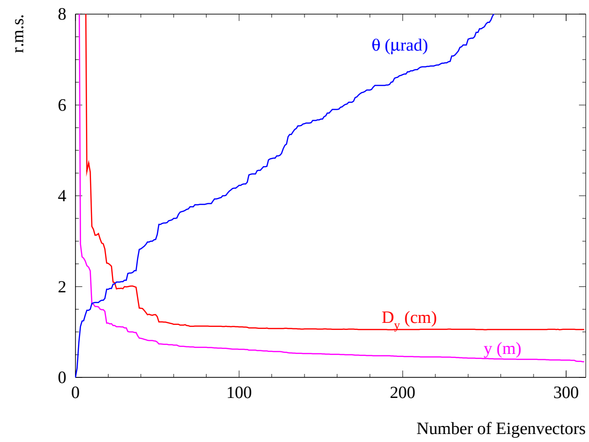

Figure 3: Predicted r.m.s. of the vertical orbit (y), dispersion (D_y) and corrector kicks (θ) as a function of the number of used eigenvectors (for α = 0.2 and β = 0).

eigenvectors with the largest eigenvalues always reflect the machine (super) symmetries [11]. Small eigenvalues often correspond to orbit and dispersion bumps.

In order to correct orbit and dispersion in LEP k eigenvectors with the largest eigenvalues (the k "most efficient" linear combinations of correctors) are used. k is optimized from experience. Figure 3 shows the prediction for orbit, dispersion and corrector strengths in the case of a vertical bare orbit correction at LEP. Good corrections for the dispersion and the orbit are obtained with $k \approx 80$ eigenvectors (out of 312). Since the r.m.s. strength of the correctors increases with the number of eigenvalues, the corrector kicks can be controlled by limiting the number of eigenvectors for the correction. The trade-off between orbit, dispersion and r.m.s. corrector strength is seen in Figure 3.

The algorithm is quite robust against "wrong" BPM readings. Corrections are based only on the largest eigenvalues and act principally on the main harmonics of orbit and dispersion. Local structures cannot be easily produced and particularly suspicious data ("apparent π-bumps") be-

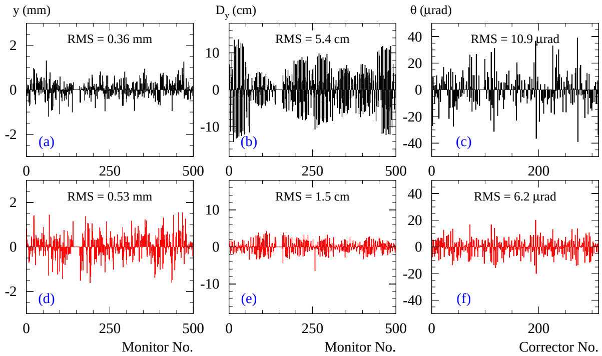

Figure 4: The measured vertical orbit (y), dispersion (D_y) and corrector kick strengths (θ) after a traditional bare correction of the LEP orbit using MICADO (i.e. α = 0) are shown on the three top figures (a), (b) and (c). The same quantities are shown after a correction with the DFS procedure on the bottom figures (d), (e) and (f). This experiment was performed with a single beam.

come more visible. The simultaneous correction of orbit and dispersion avoids spurious local bumps, since such bumps generate local orbit distortions but global dispersion waves. Iterations over the SVD solution allows disabling "bad" BPM readings for orbit and dispersion, such that they do not further constrain the least squares solution.

3 LEP RESULTS

Dispersion Free Steering was tested in 1998 and implemented in the LEP control system for the 1999 run. The response matrices are evaluated from the machine model with the MAD program [12]. The large energy loss per turn at high energy ($\approx$ 2%), which affects the response matrices of the two beams differently, is taken into account. Corrections can be evaluated for the individual beams, for both beams at the same time or for the average of the two beams (the most frequent case). The optimum value for α ranges from 0.1 to 0.3 and in general α is set to 0.2 (for an orbit r.m.s. expressed in mm and a dispersion r.m.s. expressed in cm). This value is in agreement with estimates based on the machine alignment and the accuracy of the dispersion measurement. Results are not very sensitive to the precise value of α. β is usually set to 0.1 to avoid problems with singular solutions localised in the low-beta insertions.

A "traditional" bare orbit correction using MICADO ($\alpha = 0$) is compared to a DFS correction with SVD in Figure 4. While the orbit r.m.s. is not affected significantly, the r.m.s. vertical dispersion is reduced from typically 5 to 1.0-1.5 cm which corresponds to the smallest achievable r.m.s. dispersion at LEP. For the available momentum range of $\Delta p/p \simeq 0.15\%$, a dispersion of 1 cm corresponds to a measured beam position shift of only 15 μm, at the limit of the LEP BPM resolution. The r.m.s. kick strength is reduced by almost a factor two.

The application of DFS showed a limitation in vertical dispersion due to vertical separation bumps in the odd IPs (the experiments are in the even IPs). This was improved with a local change of optics. The use of DFS then allowed reducing the vertical r.m.s. dispersion from 3-4 cm in 1998 to 1.5-2 cm in 1999. As a consequence the best emittance values during LEP luminosity running were reduced by almost a factor two [13]. Contrary to previous years where the search for good orbits was done empirically, in 1999 the baseline performance was established deterministically and much faster with DFS. From the reduction in dispersion and Equation 1 we would naively expect an improvement in vertical emittance by a factor of four. However, we observed a factor of two. The difference is explained by a larger beam-beam blow-up for smaller ε_y [13] and residual coupling between the two planes.

4 CONCLUSION

Dispersion Free Steering, a deterministic and simultaneous correction of the closed orbit and the dispersion, was implemented in LEP. The correction scheme is relying mainly on the SVD algorithm to solve the least square problem. The vertical dispersion in LEP was reduced to the expected minimum, only limited by residual dispersion generated from separation bumps and by the measurement noise. With DFS the empirical search for "Golden Orbits", yielding peak performance, was made deterministic and a significantly smaller residual dispersion was obtained. This resulted in a vertical emittance reduction of approximatively 50%.

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