TERAHERTZ WAKEFIELDS AND THEIR EFFECT ON THE SUPERCONDUCTING CAVITIES IN TESLA

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Abstract

Operation with very short bunches for the TESLA X-ray FEL generates wakefields in a frequency regime above the threshold for Cooper pair break-up (750 GHz) in superconducting Niobium. The impact of these wakefields on the cavity quality factor is investigated and an estimate of the additional heat load at the 2 K level in the TESLA linac during FEL-opration is given.

1 INTRODUCTION

The operation of a Free Electron Laser (FEL) in the vacuum ultraviolett or X-ray regime requires the acceleration of electron bunches with an rms-length of 25 to 50 µm and thus we consider a pulse length of 0.083 to 0.167 ps and a peak current of 5 kA. In TESLA it is foreseen to accelerate 11315 bunches within a time of 800 µs and with a spacing of 70 ns from bunch to bunch. The wake fields generated by these subpicosecond bunches extend into the threshold for Cooper pair breakup (about 750 GHz at 2 K) in superconducting Niobium. For this reason there has been a considerable concern whether the acceleration of such short bunches would be possible using the TESLA 9-cell superconducting cavities. A first approach to the problem ([1], [2], [3]) in which the wake field generation was based on a diffraction model led to the result that the superconducting cavities could indeed be operated with 25 µm bunches without suffering a breakdown of superconductivity (quench), however at the price of a reduced quality factor and an increased heat transfer to the superfluid helium bath. In the meantime the more conventional method of computing wake fields in the time domain by numerical methods has been extended into the very short bunch regime and has been applied for single cavities as well as a sequence of multicell cavities. As will be shown below, both methods lead to comparable results in the case of a single multicell cavity but the numerical approach predicts a much larger reduction of wake field losses in a long string of cavities.

2 THE DIFFRACTION MODEL

Following Bane and Sands [4] we consider a highly relativistic bunch travelling along the axis of a perfectly conducting beam pipe of radius a and passing through a pill-box cavity of gap length g. The electric field accompanying the bunch can be taken as radial. Its value at the pipe wall at $r=a$ is given in terms of the bunch charge q by

$$E(a) = \frac{q}{2\pi\varepsilon_0 a} \tag{1}$$

while the magnetic field is azimuthal and of magnitude $B(a)=E(a)/c$. For radiation in the THz regime the wavelength is much shorter than the pipe radius, hence close to the wall the electromagnetic field of the bunch can be replaced by a plane wave. This wave is diffracted at the entrance aperture of the cavity which we approximate as a straight edge [5]. The outward diffracted wave will than hit the cavity wall close to the exit aperture. For a monochromatic wave of intensity $I_0=I_0(\omega)$, the power scattered into the shadow region becomes

$$P(\omega) = I_0(\omega)\pi a\sqrt{\frac{cg}{\pi\omega}}. \tag{2}$$

With the Fourier transform $\tilde{\rho}(\omega)$ of a Gaussian shaped longitudinal charge distribution travelling in z direction with the velocity of light the wave intensity is

$$I_0(\omega) = \frac{1}{\mu_0 c}\left|\frac{\tilde{\rho}(\omega)}{2\pi\varepsilon_0 a}\right|^2 \tag{3}$$

and the energy per frequency intervall diffracted into the shadow region becomes

$$\frac{d}{d\omega}U(\omega) = \frac{q^2\sqrt{cg}}{4\pi^{5/2}\varepsilon_0 ac}\frac{exp(-\omega^2/\omega_c^2)}{\sqrt{\omega}}. \tag{4}$$

with the “cut-off” frequency $\omega_c=1/\sigma_t$. Integrating over all frequencies provides the energy deposited in the pill-box cavity by a single bunch

$$W_1 = \int_0^\infty U(\omega)d\omega = \frac{1.82q^2}{4\pi^{5/2}\varepsilon_0 ac}\sqrt{\frac{cg}{\sigma_t}} \tag{5}$$

According to Babinet’s Theorem the same amount of energy is diffracted into the illuminated region behind the diffracting aperture.

In multiple-cell cavities like the TESLA 9-cell cavity it turns out that the energy loss is weaker than $W_{9\text{-}cell} = 9W_1$. According to B. Palmer [8] the extent of the disturbance of the self field of the bunch inside the iris radius increases with the number of passed cells and reduces the intensity of the diffracted wave from cell to cell. For a TESLA 9-cell cavity one obtains $W_{9\text{-}cell} \approx 6W_1$ for the energy deposited in the 9-cell cavity by a single bunch.

In a periodic string of many cavities the energy loss will settle to a quasi equilibrium with the lowest possible energy loss. In this case the disturbance of the bunch self field has reached the bunch itself. In the quasi equilibrium state the energy loss of a single bunch in a 9-cell cavity becomes independent of bunch length and is further reduced by approximately a factor of 6 and we have $W_{9\text{-}cell} \approx W_1$. The energy loss in a 9-cell cavity is now nearly identical with the energy loss in a single pill-box cavity.

In FEL operation there will be N_b=11315 bunches per 800 μs long macropulse with a bunch charge of q=1,0 nC and σ_z=25 μm. The macropulse repetition rate is f_{rep}=5 Hz. Taking into an account the different radii of beam pipe and iris opening, the different cell length between end and middle cells and finally the round iris edge one obtains for the time averaged power scattered into the first 9-cell cavity [9] $P_{tot}=N_b \cdot W_1 \cdot f_{rep}$=2,8 W and in quasi equilibrium state only P_{tot}=0,47 W.

For an rms bunch length σ_z=25 μm the cut off frequency is f_c=1,9 THz. By integration of Equation (4) one finds that 32 % of the intensity is above 750 GHz, the threshold frequency for Cooper pair breakup in niobium. Most of the diffracted radiation hits the next iris of the multicell cavity in a narrow ring shaped region close to the smallest diameter. The radiation pulse has a time duration less than a picosecond and hence the power density of the radiation impinging on the ring region at the next iris is so high that one could easily image an immediate breakdown of superconductivity in that region. Fortunately, this is not the case since most of the radiation is not absorbed but reflected by the superconductor. To compute the energy absorption in high frequency fields it is convenient to describe the superconductor by its frequency dependent surface resistance. The surface resistance of the high purity niobium used in

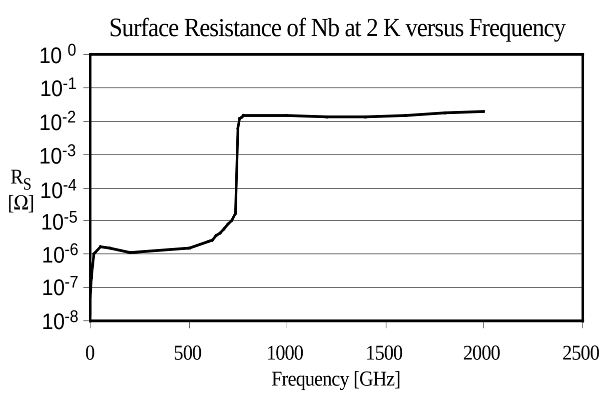

Figure 1: Surface resistance of Nb at 2 K as a function of frequency

TESLA cavities has been computed within the Eliashberg model [11]. For an operating temperature of 2 K the surface resistance is plotted in Figure 1 as a function of frequency. Starting from a value of about 20 nΩ at the fundamental mode frequency of 1.3 GHz the resistance rises to a few μΩ at 600 GHz and then exhibits a large step at 750 GHz to a value of 15 mΩ. This implies that the potentially dangerous THz radiation undergoes thousands of reflections before being absorbed by the niobium. The time of flight between to reflections is in the order of nanoseconds, hence the original picosecond radiation pulse is stretched in time by many orders of magnitude before absorption takes place. Also the spatial distribution is greatly altered by the reflections.

To study the process of multiple reflections in the 9-cell cavity we resort to the photon picture which appeared justified since the wavelength of THz-radiation is much smaller than the cavity dimensions. Due to elastic scattering at a boundary of elliptical shape it is evident that chaotic motion is present. In a Monte Carlo simulation of many thousand reflections almost every point on the cavity surface is hit by the photons, and there is a high chance that photons remain trapped in the 9-cell structure until they are absorbed [3]. The large number of reflections and their chaotic structure have the beneficial effect that the radiation power which eventually has to be absorbed by the niobium is distributed over the whole cavity surface and smared out in time by many orders of magnitude.

To get an estimation on the wake field load in a TESLA cavity we make the simplifying assumption that all wake field intensity above 750 GHz is absorbed in the cavity while all radiation below 750 GHz eventually leaves the cavity and is absorbed by a suitable material in the beam pipe sections [6]. The power absorbed in the 9-cell cavity is taken to be uniformly distributed over the whole inner surface of 0,8 m^2. Thus one finds for the first TESLA cavity $0.3 \cdot 2.$ W≈0.9 W of average and 200 W of instantaneous power are dissipated in the inner niobium surface of the cavity. In the quasi equilibrium state both values are reduced by a factor of 6 to 0.15 W average and 33 W instantaneous power. An important question is how much the temperature of the inner surface rises and how fast the temperature rise occurs. First we consider the stationary case with an incident power density of $\Phi \approx 200/0.8 = 250$ W/m^2. The wall thickness is 2.5 mm and the heat conductivity of our high-purity niobium at 2 K amounts to ≈7 W/m^2K [7], hence the temperature rise is about 0.19 K. Solving the time dependent heat equation one finds that the stationary state is achieved in about 100 μs, so stationary heat conduction applies for most of the 800 μs long macropulse and the inner surface assumes a temperature of 2.19 K for cooling with superfluid helium of 2 K. If the initial quality factor at 2 K is $Q_0 \approx 10^{10}$ then Q_0 drops to $\approx 9 \cdot 10^9$ at 2.19 K. The consequence is an increased surface heating by the fundamental 1.3 GHz mode. Taking that into an account we finally arrive at an even lower effectiv Q_0 of about $7 \cdot 10^9$. In the quasi equilibrium state the situation is much relaxed and the drop of Q_0 negligible.

3 NUMERICAL CALCULATION OF THE LOSS FACTOR

As we have allready shown in the previous section in the case of a periodic array of resonators, the induced wake fields cannot be simply calculated as the sum of single cavity contributions, because the field traveling with the bunch is modified due to the presence of the upstream discontinuities. To which extent the transition to the periodic regime actually applies in the TESLA linac can more accurately be answered by numerical calculations. For short bunches the interaction length of the inward deflected wave traveling with the bunch can become very large (up to tens of meters). Wakefield calculations in the time domain have been performed for up to 2 entire TESLA accelerator modules, each about 12 m long and containing eight 9-cell resonators, bellows and beam pipes [10]. The minimum possible bunch length for reasonable numerical effort is σ_z=50 µm. For this case, the results show still a significant difference between the wakes calculated for the first and the second module, indicating that the transition length to the steady state exeeds one module length.

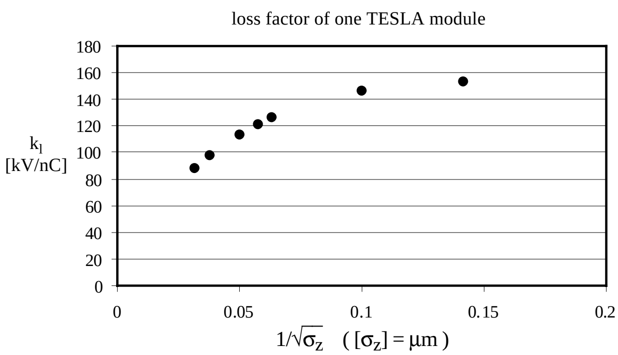

Figure 2: Calculated loss factor per TESLA module versus $1/\sqrt{\sigma_z}$ (σ_z =50...1000 μm)

In Figure 2 the calculated loss factor per module (results for second module in a string of two) as a function of bunch length is shown. One can clearly recognize that the dependence of σ_z is much weaker than $1/\sqrt{\sigma_z}$, in contrast to the numerical result of a single-cell calculation, so that the condition of an infinite periodic structure seems to be relatively well fulfilled for the TESLA linac. From the numerical results an analytical approximation to the (point charge) wake potential per module has been derived [10]

$$w(s) = 315 \cdot \frac{V}{pC} \cdot \left(1.165 \cdot exp\left(-\sqrt{\frac{s}{3.65\ mm}}\right) - 0.165\right) \qquad (6)$$

Fourier transformation of the wake potential yields the impedance $Z(\omega)$. From $Z(\omega)$ we obtain the relevant high-frequency contribution to the loss factor, $k_{loss}(\omega_l)$, by integration together with the bunch spectrum and the lower integration boundary ω_l as a variable. The resulting $k_{loss}(\omega_l)$ is shown in Figure 3 for the case of σ_z=25 μm. We find that the contribution to the loss factor above the Niobium gap frequency amounts to only about 3 %, in comparison to 32 % in the diffraction model estimate. The total loss factor, extrapolated to σ_z=25 µm from the numerical result (Figure 2), amounts to 165 kV/nC per module. In comparison the diffraction model amounts to 152 kV/nC per module and is thus in a good agreement with the numerical result. For the average wakefield power we get P_{tot}=1.17 W per 9-cell cavity, considerably less than for the first 9-cell cavity. The power dissipated in the cavities at the 2 K level is then only about 0.04 W, on average or less than 10 W during the beam pulse, which does not lead to a significant increase of the niobium surface temperature.

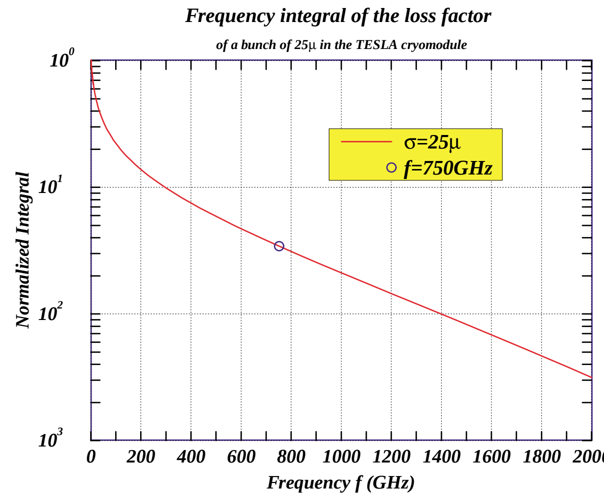

Figure 3: Fractinal part of the loss factor above a lower frequency boundary.

4 CONCLUSION

The results of the diffraction and the numerical model have clearly shown, that the TESLA cavities are not quenched by the wake fields of ultrashort bunches, but that there is a significant increase in the heat load on the cryogenic system. The total loss factors in the quasi equilibrium state calculated by the diffraction and by the numerical model are in very good agreement, the difference is lower than 10 %. However, the amount of energy radiated in frequencies above 750 GHz amounts to 32 % of the total energy loss within the diffraction model for a single cell and amounts to only 3 % within the numerical model, which is an order of magnitude less.

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