FREQUENCY MAP ANALYSIS OF BEAM HALO FORMATION IN A HIGH INTENSITY PROTON LINAC

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Abstract

In this paper we describe the progresses achieved in the application of modern Hamiltonian dynamics tools, like frequency map analysis, to the problem of beam halo formation in a high intensity proton linac. In particular we discuss the extension of the approach introduced for a continuous beam in a FODO channel to a realistic linac, considering the problems connected with longitudinal dynamics and acceleration.

1 INTRODUCTION

One of the big challenges of the new generation of high power proton linacs is the control of beam losses down to a very low percentage. These losses are associated to the presence of a beam halo, populated by very few particles at large distance from the average beam dimensions. It is generally recognized that one of the main mechanism that generates beam halo are the nonlinear single particle resonances driven by the space charge.

These effects can be well described using the Frequency Map Analysis (FMA)[1] for the test particle and supposing that the core of the beam follows a known dynamics (Particle-core model). In a previous paper[2] we have simulated the 2D dynamics of a mismatched beam propagating in a FODO channel. One of the outcome of this analysis was that the dynamics of the test particle is better confined if the horizontal and vertical tunes are not equal (the average cross section of the beam is not round), since in this case the strong resonance $\nu_1 = \nu_2$ can be avoided.

In a linac the particle dynamics is intrinsically 3D, since the three tunes are comparable. We therefore extended our analysis assuming an ellipsoid uniformly populated, even if this case does not correspond to a regular self-consistent solution of the Poisson-Vlasov problem. As the first step we assumed cylindrical symmetry (solenoid focusing) and RF focusing without acceleration [3]; the space charge term for a bunch with cylindrical symmetry is calculated using the form factor $f(p)$[4].

In this paper we extended the space charge calculation a general ellipsoid (three different single particle tunes), introducing a generalized form factor, which allows to compute the envelope modes in the general case. One of the aims is to explore the advantages of using three different tunes. We therefore give an example of FMA for a mismatched bunched beam in a FODO channel (with RF focusing).

2 PARTICLE-CORE MODEL

We consider an elipsoidal bunch uniformely populated whose equation reads

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} \le 1 \tag{1}$$

where (x_1, x_2) are the transverse coordinates and x_3 is the longitudinal coordinate (distance respect to the reference particle). The beam is focused transversally by magnetic quadrupoles and longitudinally by RF cavities so that the axis a_j are functions of position s. The potential of the charge distribution is

$$\Phi = \frac{3Ne}{16\pi\epsilon_0} \int_\chi^\infty \frac{\left(1 - x_1^2/a_1^2 - x_2^2/a_2^2 - x_3^2/a_3^2\right) du}{\sqrt{(a_1^2+u)(a_2^2+u)(a_3^2+u)}} \tag{2}$$

where N is the total number of particles in the bunch and the variable χ is 0 if (x_1, x_2, x_3) is an internal point to the ellipsoid (1), and is otherwise defined as the positive solution of the equation

$$\frac{x_1^2}{a_1^2+\chi} + \frac{x_2^2}{a_2^2+\chi} + \frac{x_3^2}{a_3^2+\chi} = 1 \tag{3}$$

The single particle equations of motion can be written in the form

$$\begin{aligned}
x_1'' &= \tfrac{K_3 x_1}{kr} I_1(kr)\cos kx_3 \; - K_1 x_1 + \tfrac{\mu x_1}{\hat a_1 \hat a_2 \hat a_3} F\left(\tfrac{\hat a_1}{\hat a_2}, \tfrac{\hat a_1}{\hat a_3}\right) \\
x_2'' &= \tfrac{K_3 x_2}{kr} I_1(kr)\cos kx_3 \; - K_2 x_2 + \tfrac{\mu x_2}{\hat a_1 \hat a_2 \hat a_3} F\left(\tfrac{\hat a_2}{\hat a_1}, \tfrac{\hat a_2}{\hat a_3}\right) \\
x_3'' &= -\tfrac{K_3}{kr} I_0(kr)\sin kx_3 + \tfrac{\mu x_3}{\hat a_1 \hat a_2 \hat a_3} F\left(\tfrac{\hat a_3}{\hat a_1}, \tfrac{\hat a_3}{\hat a_2}\right)
\end{aligned} \tag{4}$$

where $\mu = 3Ne^2/(4\pi\epsilon_0 mc^2\beta^2)$, βc is the particle velocity, k is the RF wave number,$I_0(kr)$ and $I_1(kr)$ are the modified Bessel functions, the symbol $'$ denotes the derivative with respect to s, $r = \sqrt{x_1^2 + x_2^2}$, $\hat a_j = \sqrt{a_j^2 + \chi}$ and we have introduced the form factor

$$F(p,q) = \int_0^1 \frac{v^2 d\,v}{\left(p^2 + (1-p^2)v^2\right)^{1/2} \left(q^2 + (1-q^2)v^2\right)^{1/2}} \tag{5}$$

Respect to the form factor defined in the literature [4] for an ellipsoid with a cylindrical symmetry, one has the relation $F(p,p) = f(p)$. We remark the other equalities coming from the symmetry and the gaussian theorem

$$\begin{aligned}
&F(p,q) = F(q,p) \\
&F\left(\tfrac{a_1}{a_2}, \tfrac{a_1}{a_3}\right) + F\left(\tfrac{a_2}{a_1}, \tfrac{a_2}{a_3}\right) + F\left(\tfrac{a_3}{a_1}, \tfrac{a_3}{a_2}\right) = 1
\end{aligned} \tag{6}$$

To complete the PC model, we have computed the envelopes equations by linearizing the single particle equations (4)

$$a_j''+\left(K_j-(1-\delta_{j3})\frac{K_3}{2}\right)a_j-\frac{\mu a_j}{V}F\left(\frac{a_j}{a_i},\frac{a_j}{a_h}\right)+\frac{\epsilon_j^2}{a_j^3}=0 \quad (7)$$

where j,i,h is any permutation of the indexes $(1,2,3)$, $V=a_1a_2a_3$, $\epsilon_j=5\sqrt{\langle x_j^2\rangle\langle {x_j'}^2\rangle-\langle x_jx_j'\rangle^2}$ and $\langle x_i^2\rangle=a_i^2/5$. In the case of a periodic magnetic lattice of length L it exists a periodic solution of the system (7) (matched case) and it is possible to introduce the Poincarè map of the system (4) and the phase advance per period ν_j.

3 ENVELOPE MODES

If the deviations Δa_j from the periodic envelopes are small, they can be calculated from the linearization of (7). In particular if the focusing is smooth ($\nu_j \ll 1/4$), one can directly calculate the equilibrium envelopes $a_j=\sqrt{\frac{\epsilon_j L}{2\pi\nu_j}}$, and the zero space charge tunes:

$$\nu_{0j}=\sqrt{\nu_j^2+\frac{\mu L^2}{4\pi^2 V}F\left(\frac{a_j}{a_i},\frac{a_j}{a_h}\right)} \quad (8)$$

The envelope modes are solutions of the system:

$$\Delta a_j''+\sum_{l=1}^{3}H_{jl}\Delta a_l=0. \quad (9)$$

where

$$H_{jj}=\frac{\nu_{0j}^2+3\nu_j^2}{L^2}+\frac{\mu}{V}\left(2F\left(\frac{a_j}{a_i},\frac{a_j}{a_h}\right)\right.$$
$$\left.-G\left(\frac{a_j}{a_i},\frac{a_j}{a_h}\right)-G\left(\frac{a_j}{a_h},\frac{a_j}{a_i}\right)\right)$$
$$H_{ji}=\frac{\mu}{V}\frac{a_j}{a_i}G\left(\frac{a_j}{a_i},\frac{a_j}{a_h}\right).$$

The function $G(p,q)$ is defined as:

$$G(p,q)=\int_0^1\frac{v^4 dv}{(p^2+(1-p^2)v^2)^{3/2}(q^2+(1-q^2)v^2)^{1/2}} \quad (10)$$

and it appears in the derivatives of the formfactor (5) according to

$$\frac{\partial F}{\partial p}=-\frac{1}{p}\left(F(p,q)+G(p,q)\right). \quad (11)$$

The function G has the properties

$$p^2G(p,q)=G(1/p,q/p)$$
$$(1-p^2)G(p,q)\quad +(1-q^2)G(q,p)=3F(p,q)-1 \quad (12)$$

The first equation (12) assures the Hamiltonian character of the system (7) and the matrix H is symmetric. The eigenvalues and the eigenvectors of the matrix H define the envelope frequencies and the envelope modes that influence the single particle dynamics in the unmatched cases.

We explicitly compute the envelope frequencies $(\alpha_1,\alpha_2,\alpha_3)$. For a symmetric circular elipsoidal bunch $(a_1=a_2)$, the matrix H simplifies according to

$$H_{11}=H_{22}=\frac{\nu_{01}^2+3\nu_1^2}{L^2}+\frac{\mu}{V}\frac{2p^2-5+5f(p)+4p^2f(p)}{8(p^2-1)}$$
$$H_{33}=\frac{\nu_{03}^2+3\nu_3^2}{L^2}+\frac{\mu}{V}\left(2f(p)+\frac{3f(p)-1}{p^2-1}\right)$$
$$H_{32}=H_{31}=H_{13}=H_{23}=-\frac{\mu p}{V}\frac{3f(p)-1}{2(p^2-1)}$$
$$H_{12}=H_{21}=\frac{\mu p}{V}\frac{2p^2-3+3f(p)}{8(p^2-1)}$$

and there exists a quadrupole mode $\Delta\vec{a}=(-1/\sqrt{2},1/\sqrt{2},0)$ whose frequency is $\alpha_1=\sqrt{H_{11}-H_{12}}$ and the other two modes have cylindrical symmetry.

4 A SIMPLE MODEL

For the numerical simulation we have chosen a FODO cell where two thin lens RF cavities are inserted. We have considered a stationary beam in the nonrelativistic case. The main parameters of the FODO cell are reported in the table 1. The spacecharge parameter μ is fixed at $10^{-9}m$ that

Table 1: Nominal Case

Quadrupole (m^{-2})	emitt. (m)	tunes
$K_f=14$	$\epsilon_1=10^{-6}$	$\nu_{01}=0.1943$
$K_d=13$	$\epsilon_2=10^{-6}$	$\nu_{02}=0.1585$
$K_z=25$	$\epsilon_3=10^{-6}$	$\nu_{03}=0.1131$

corresponds to a tune depression $\simeq 90\%$, k is $6\pi m^{-1}$, and $L=1m$. In the table 2 we compare the single particle tunes and the envelope frequencies computed in the smooth approximation and the FFT transform of the numerical orbits. In the figure 1 we show the FFT transform of the projection on the plane (x_1,x_1') of a single particle orbit in a mismatched case.

Table 2: Frequencies and modes

Mode	Smooth	FFT
ν_1	0.1747	0.1746
ν_2	0.1364	0.1374
ν_3	0.0902	0.0902
α_1	0.3674	0.3679
α_2	0.2945	0.2947
α_3	0.2039	0.2027

We have shown [2] that the FMA can be applied to study the phase space in the mismatched case. The basic idea of

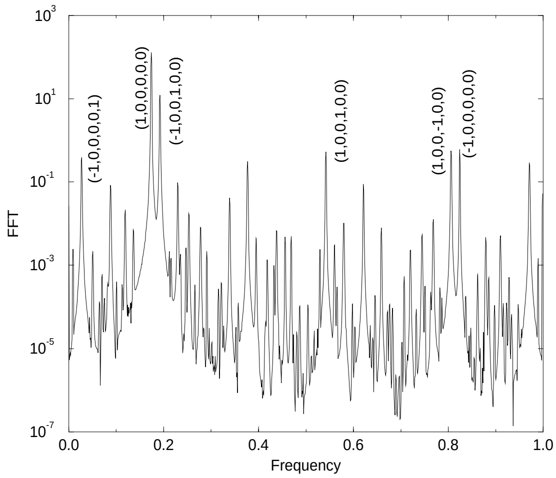

Figure 1: FFT of a single particle orbit projected in the (x_1, x_1') plane in the mismatched case; the integer vectors give the linear combinations $n_1\nu_1 + n_2\nu_2 + n_3\nu_3 + n_4\alpha_1 + n_5\alpha_2 + n_6\alpha_3$ which define the Fourier spectrum.

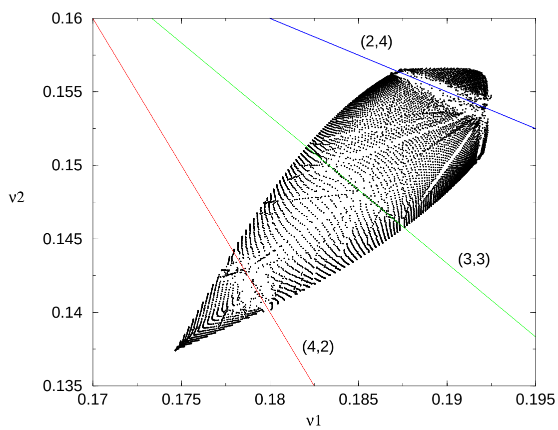

Figure 2: FM of the transverse phase space for the matched case; the straight lines correspond to the resonances $n_1\nu_1 + n_2\nu_2 - n = 0$

the FMA is to compute a transformation between the orbits and the frequency space whose regularity properties are directly related with the geometry of the phase space. We have considered a section at $x_3 = x_3' = 0$ of the bunch and we have distributed an uniform grid of points on the plane (x_1, x_2) to explore the phase space structure outside the bunch up to $3a_j$ emittances. In the figure 2 we plot the FM of the orbits whose initial point is one of the grid points in the matched case. The regular distribution of the points in the frequency space is due to the regularity of the dynamics and the low order resonances (straight lines) are well separated in the phase space. In the figure 3 we plot the same FM in the mismatched case. We remark that a large chaotic region appears in the phase space space due to the crossing of the resonances $2\nu_1 - \alpha_1 = 0$ and $2\nu_2 - \alpha_2 = 0$, but no resonance is excited between the tunes and the third envelope frequency α_3. Indeed a mismatch on the horizontal envelope does not excite the third envelope mode as it can be check from the smooth ap-

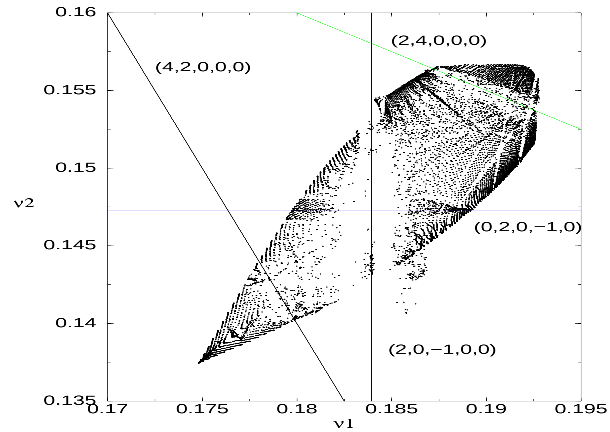

Figure 3: FM of the transverse phase space for the unmatched case (10% of mismatched on the horizontal envelope); the straight lines correspond to the resonances $n_1\nu_1 + n_2\nu_2 + n_3\alpha_1 + n_4\alpha_2 + n_5\alpha_3 - n = 0$

proximation (the eigenmodes are $(0.996, 0.0848, 0.0413)$, $(-0.0875, 0.994, 0.0689)$, $(-0.0352, -0.0722, 0.997)$).

5 CONCLUSION

This preliminary study show that the FM can be applied to analyze the phase space of the 3 dimensional Particle in Core model in the mismatched case. The FMA gives informations of the nonlinear resonances and the chaotic regions that dominate in the phase space. A 3 dimensional plot of the FMA and a stability study of the chaotic region by means of a tracking program is a work in progress.

REFERENCES

[1] J.Laskar, Physica D, **67**, p. 257 (1993).

[2] Bazzani A., Comunian M., Pisent A., “Frequency Map Analysis of an intense mismatched beam in a FODO channel”,*Particle Accelerators* , Vol. 63, pp 79-103.

[3] Bazzani A., Comunian M., Pisent A., “Frequency map analysis for beam halo formation in high intensity beams”, proceedings of Particle Accelerator Conference 1999, pp 1773-1776.

[4] T. Wangler,“Principles of RF linear accelerators”, Wiley series in beam physics and accelerator technology, pp 276-278.