

# NONLINEAR-RESONANCE ANALYSIS OF HALO-FORMATION EXCITED BY BEAM-CORE OSCILLATION

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## Abstract

The emittance growth and halo formation for a mismatched beam of a 1-D Gaussian distribution in a uniform focusing channel were examined by means of a macro-particle simulation and an isolated nonlinear resonance theory. Nonlinear fields in an actual particle distribution have been shown to significantly affect both the halo's location and size. For further application of the isolated resonance Hamiltonian analysis, the 2-D simulation code based on the Hybrid Tree code method has been developed. The results manifested the nonlinear resonance excited by beam core oscillation similar to that in the 1-D case.

## 1 INTRODUCTION

One of the major issues in high-power accelerators is the activation of accelerator components due to beam loss. The beam loss must be reduced to a sufficiently low level to allow hands-on-maintenance. In order to produce an acceptable design, it is important to understand the mechanisms of emittance growth and halo formation that result in beam loss.

An understanding of halo-formation mechanisms in circular rings seems to be quite difficult, because numerical calculations over a sufficient number of turns require unrealistic CPU times and memory and repeated betatron oscillations through a huge number of lattice elements takes a key role in the resonant interaction. We have pursued a strategy to develop a useful analytic model capable of predicting the position of the halo as a function of the beam and machine parameters for a realistic beam distribution. As the first step of this strategy, halo formation in a 1D Gaussian distribution in a uniform focussing channel has been numerically examined, and a second-harmonic nonlinear resonance excited by the rms core oscillation has been identified to be a driving mechanism of halo formation. This view has been confirmed by an analytic approach based on isolated nonlinear resonance theory [1,2]. The simulation and theory have shown that highly nonlinear components in a real distribution strongly affect the halo location. The highlight of the first step [3] is summarized in this paper.

As the second step, halo formation mechanisms in the 2-D model are explored. For this purpose, the 2-D simulation code in a more realistic lattice has been developed. Nonlinear resonances were excited by core oscillation, which inherently originates from the lattice.

Essential features of the 2-D simulation are described in this paper.

## 2 NONLINEAR-RESONANCE ANALYSIS OF HALO-FORMATION EXCITED BY BEAM-CORE OSCILLATION

We chose to apply the current study to the 12GeV proton synchrotron (KEK-PS). Most of the calculation parameters were taken from the KEK-PS, where the injection energy is 500MeV and the bare tunes are  $\nu_x = 7.15$  and  $\nu_y = 6.23$ . In order to manifest the key role of the space-charge effects in halo formation, an extremely high current, where the maximum incoherent tune shift is 1.85, was studied.

A 1-D simulation was used to understand the detailed and dynamic processes involved in the physical phenomena. In the simulation, a beam distribution is assumed to be both infinite and uniform in the horizontal and longitudinal planes and finite and non-uniform in the vertical plane. Space-charge fields affect the betatron motion of the beam in the vertical direction. In addition, it is assumed that the beam propagates through free space so that the effect of the image charge is ignored.

The simulations were carried out for three cases of mismatched beams with Gaussian, waterbag and square-cosine distributions. The square-cosine distribution is defined as  $f = f_0 \cos^2[\pi(y^2+y'^2)^{1/2} \{2R(y,y')\}^{-1}]$ , where  $R(y,y')$  is the distance from the origin to the outer edge through  $(y,y')$  in phase space. All initial distribution functions have the same total current and the same rms emittance as the matched beam. The rms emittance growth of Gaussian and square-cosine distributions quickly arrive at the steady state after less than a few tens of turns, whereas the rms emittance of the waterbag beam still grows over 1200. The beam density of a beam with an initial square-cosine distribution approaches a Gaussian distribution in the steady state. On the other hand, a beam with a waterbag distribution tends to become flat because of redistribution towards the beam edge. The phase-space projections suggest that particles escaping from the core are responsible for the growth of rms emittance. In addition, it is remarkable that there are two resonance islands. Since there are no external nonlinear fields in the uniform focusing channel, the sources driving the nonlinear resonances have been identified to be the space-charge self-fields. The simulation results also show a notable oscillation of the

rms beam, which is simply induced by mismatching. A parametric resonance between the single-particle motion and the rms beam core oscillation can be excited when the depressed betatron tune  $\nu_\beta$  and the rms core oscillation tune  $\nu_c$  satisfy  $\nu_\beta/\nu_c = i/j$ , where  $i$  and  $j$  are integers. The lowest dominating resonance is obviously a second-harmonic resonance which is capable of creating two resonance islands. Certainly, the FFT of the core oscillation exhibits a single sharp peak at  $\nu_c = 10.45$ . The results strongly suggest that the major source of the second-harmonic resonance is the rms beam core oscillation.

In order to confirm the speculation that the rms beam core oscillation is capable of driving the second-harmonic nonlinear resonance, we have developed an analytic approach using an isolated resonance Hamiltonian. Here, the beam distribution is assumed to be a Gaussian distribution with the rms beam size oscillating at a single frequency. This is  $\sigma(s) = \sigma_0\{1 + \delta\cos(\omega_c s)\}$ , where  $\sigma_0$  is the averaged rms beam size,  $\delta$  is the maximum deviation from  $\sigma_0$  and  $\omega_c$  is the frequency of the beam core oscillation. Then the Gaussian distribution in the rest frame is given by  $n(y,s) = N_0\exp[-y^2/\{2\sigma(s)^2\}]/\{(2\pi)^{1/2}\sigma(s)\}$ , where  $N_0$  is the total number of particles per unit length in the rest frame. The electric field of this beam associated with the charge density is written in the form of a Taylor expansion. Introducing action-angle variables  $(\phi, J)$ , the Hamiltonian is expressed as

$$H = \omega_\beta J - A_0 \sum_{n=0}^{\infty} \frac{(-1)^n F_n(\phi, s)}{n!(2n+2)(2n+1)} \left( \frac{J}{\sigma_0^2 \omega_\beta} \right)^{n+1}, \quad (1)$$

where  $A_0 = 2\sigma_0 e^2 N_0 / \{\gamma^2 \epsilon_0 m v^2 (2\pi)^{1/2}\}$  and  $F_n(\phi, s) = [1 - (2n+1)\delta\cos(\omega_c s)]\cos^{2n+2}\phi$ . The second-harmonic resonance is excited in the case that the phase of Eq. (1) slowly varies with  $s$  [5]. Because the second-harmonic resonance is excited in the case of  $\nu_\beta/\nu_c = 1/2$ , it is found that the slowly oscillating phase is  $2\phi - \omega_c s$ . The rapidly oscillating terms, except for the terms including  $2\phi - \omega_c s$ , disappear after averaging the Hamiltonian of Eq. (1) over many turns. The averaged Hamiltonian is called the isolated resonance Hamiltonian. A canonical transformation from  $(\phi, J)$  to  $(\psi, J)$ , where  $\psi = \phi - \omega_c s/2$  is physically a rotation in phase space, is made to remove any time-dependence from the isolated resonance Hamiltonian. The isolated resonance Hamiltonian,  $H_{iso}$ , is written as

$$H_{iso} = \alpha_0 J - A_0 \sum_{n=0}^{\infty} \beta_n J^{n+1} \left[ \frac{1}{n+1} - \frac{2n+1}{n+2} \delta \cos 2\psi \right], \quad (2)$$

where,  $\alpha_0 = \omega_\beta - \omega_c/2$  and  $\beta_n = (-1)^n (2n)!(n!)^3 (n+1) (4\sigma_0^2 \omega_\beta)^{n-1}$ . In order to obtain a necessary and sufficient limit in summation of Eq. (2), the values of the resonance width were calculated as a function of limit. The limit must be more than 11. A lower limit gives the wrong results. Here, 15 has been applied. A contour plot of the Hamiltonian and the simulation result for the case of an initial square-cosine beam are shown in Figs.1. The

values of  $\omega_c$ ,  $\sigma_0$  and  $\delta$  at the steady state are chosen based on the simulation results. The calculated locations of the resonance islands are in good agreement with the simulation. We have reached the conclusion that the second-harmonic nonlinear resonance is driven by the beam-core oscillation of the nearly Gaussian distribution. In addition, Fig. 1 clearly indicates that the outer edge of the resonance corresponds to the location of the halo.

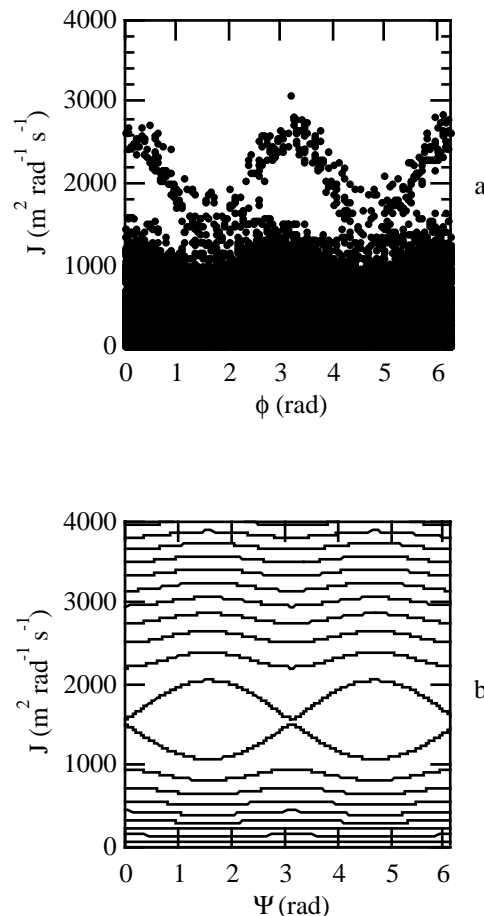


Figure 1:(a) Simulation result exhibited in the action-angle space. (b) Contour plot of the Hamiltonian (2).  $\sigma_0 = 4.38$  mm corresponds to  $J = 251.2$  m<sup>2</sup>rad<sup>-1</sup>s<sup>-1</sup> at  $\phi = 0$  rad.

### 3 2-D SIMULATION RESULT

As the second step, the 2-D case has been under consideration. In order to delineate halo formation mechanisms in the 2-D case, a 2-D simulation code in more realistic focusing channels has been developed. The electric field originating from the beam space charge is calculated by the Hybrid Tree code method. The dense core region is assigned by PIC-style charge in the similar way to that in Ref.4. Then, the Tree code method [5] is applied over the total region of interest. Effects of the image charge are ignored. The space charge forces are included as delta-function-like kicks in orbit tracking,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s+\Delta s} = M \begin{pmatrix} x \\ x' \end{pmatrix}_s + \begin{pmatrix} 0 \\ \frac{eE_x(x_{s+\Delta s})}{\gamma^2 m v} \end{pmatrix}, \quad (3)$$

where  $M$  is the transfer matrix of linear focusing system and  $\Delta s$  is the longitudinal step. For choosing  $\Delta s$ , the saturation of the rms beam size  $\sigma$  of the simulation result was monitored as a function of  $\Delta s$ . As a result,  $\Delta s = 10\text{cm}$  was applied.

To justify the code, the simulation results were compared with the results of ACCSIM [4]. The same beam parameters and machine conditions were assumed for this bench mark test. Both results have been confirmed to be in excellent agreement with each other.

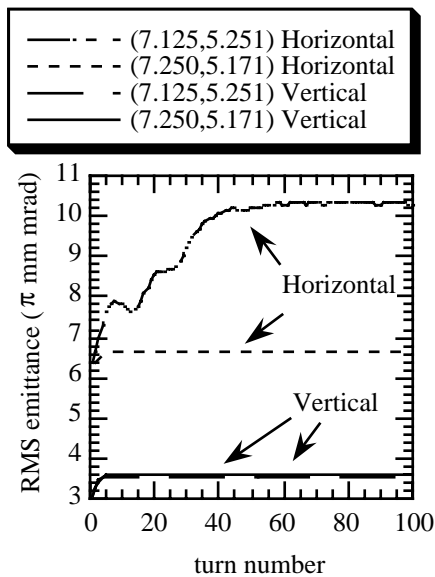


Figure 2: RMS emittance growth

The simulations were carried out for mismatched beams with Gaussian distribution in a typical FODO lattice (KEK 12Gev PS). The bare tunes  $(\nu_x, \nu_y)$  were chosen as  $(7.125, 5.251)$  and  $(7.250, 5.171)$ . Here, the momentum spread was assumed to be 0%. The maximum incoherent tune shifts were 0.25 in the horizontal plane and 0.45 in the vertical plane. The rms emittance and phase space projection were measured at the position of the flying wire monitor [6]. The horizontal and vertical rms emittance growth of the  $(7.250, 5.171)$  beam quickly arrive at the steady state after 5 turns (see Fig.2) because of the filamentation caused by mismatching. The vertical rms emittance growth of the  $(7.125, 5.251)$  beam quickly arrive also, whereas the horizontal rms emittance grows until 40 turns because of the nonlinear resonance as shown in Fig.3. Since any nonlinear magnet component are not included in these calculations, a driving source of the nonlinear resonances is attributed the space-charge self-fields. This resonance seems to slowly oscillate with phase of  $4\phi - \omega_s$ . This is understandable from the fact that the depressed tune in the horizontal plane is 7 and

the beam core oscillates 28 times per 1 turn because of the lattice consisting of 28 cells. This will be proved by the 2-D isolated resonance Hamiltonian analysis.

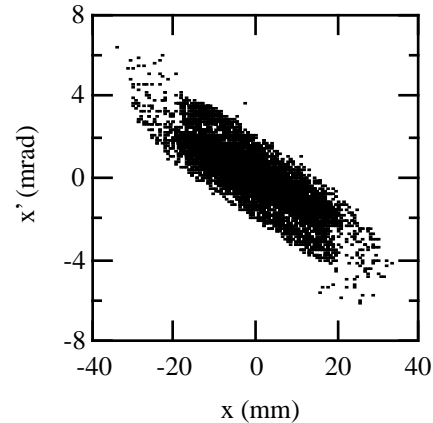


Figure 3: The phase space projection in the horizontal plane

## 4 CONCLUSION

Parametric interactions between a core oscillation and a highly nonlinear motion of individual particle drive the second-harmonic resonance for a 1D Gaussian distribution. The second-harmonic resonance is a source of emittance growth and results in beam halo which is created as an outer edge of the resonance islands. The location of halo is analytically tractable using the canonical equations derived from the isolated resonance Hamiltonian. Nonlinearity in the particle motion is crucial to determine the location of halo; the second-harmonic terms down-fed from higher-order nonlinear terms are included in order to accurately estimate the halo location.

The 2-D simulations have suggested driving mechanism of halo similar to that in 1-D case. The nonlinear parametric resonance theory which is under developing seems to be applied there. A whole analysis will be given in the coming paper.

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