

# LONGITUDINAL IMPEDANCES OF SOME SIMPLIFIED FERRITE KICKER MAGNET MODELS

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## Abstract

Two simplified models are presented in this paper in order to estimate the contribution of fast kicker magnets to the longitudinal impedance of synchrotrons. The first model is cylindrically symmetric with ferrite surrounding the beam aperture. The beam aperture is rectangular in the second model. Two opposing sides consist of ferrite and the others consist of perfect conductors. The analytical expressions of the longitudinal impedance for the two models are first derived. Subsequently, a numerical comparison between these expressions and simulation results obtained with the code HFSS are presented.

## 1 INTRODUCTION

Two very simplified kicker models are presented in this paper to estimate their contribution to the longitudinal coupling impedance. Analytical calculations for these models are given in [1]. In this paper, new simulation method with current sources [2] using HFSS [3] is described, and compared with the analytical calculations. Also, a technique to avoid the integration on the axis for the calculation of the longitudinal coupling impedance is shown.

## 2 MODEL 1

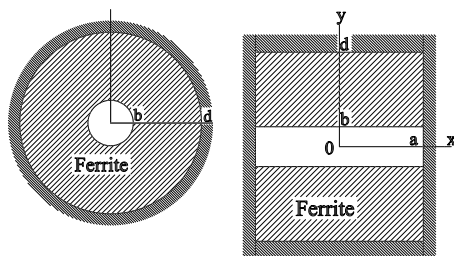


Figure 1: Model 1 (left): cross section for the coaxial geometry, and Model 2 (right): cross section for the modified, rectangular geometry. The beam moves along the  $z$  axis (out of the page).

Model 1 (Fig. 1, left) is a metal tube with inner radius  $d$ , which is homogeneously filled with a hollow ferrite cylinder with outer radius  $d$  and inner radius  $b$ . The beam is in the centre at  $r = 0$ . The length in axial direction is infinite for analytical calculations.

## 2.1 Analytical Calculation

By using the field matching technique, the longitudinal coupling impedance per unit length  $Z/L$  is derived as [1, 4]

$$\frac{Z}{L} = j \frac{Z_0}{2\pi b} \left[ FH \frac{k\epsilon_r}{k_r} - \frac{kb}{2} \right]^{-1},$$

$$FH = \frac{H_0^{(1)}(k_r d) H_1^{(2)}(k_r b) - H_0^{(2)}(k_r d) H_1^{(1)}(k_r b)}{H_0^{(1)}(k_r d) H_0^{(2)}(k_r b) - H_0^{(2)}(k_r d) H_0^{(1)}(k_r b)}, \quad (1)$$

where  $\epsilon_r$ ,  $\mu_r$ ,  $Z_0$ ,  $k$ ,  $k_r$ ,  $H$  are the relative permittivity, the relative permeability, impedance in vacuum ( $Z_0 = c\mu_0$ ),  $\omega/c$ ,  $k\sqrt{\epsilon_r\mu_r - 1}$ , and the Hankel function, respectively. The solid lines in Fig. 2 show the result, where  $b = 20$  mm,  $d = 80$  mm, and the length of the ferrite is  $l_F = 1.658$  m, to simulate the SPS MKE kicker [5]. From the figure we

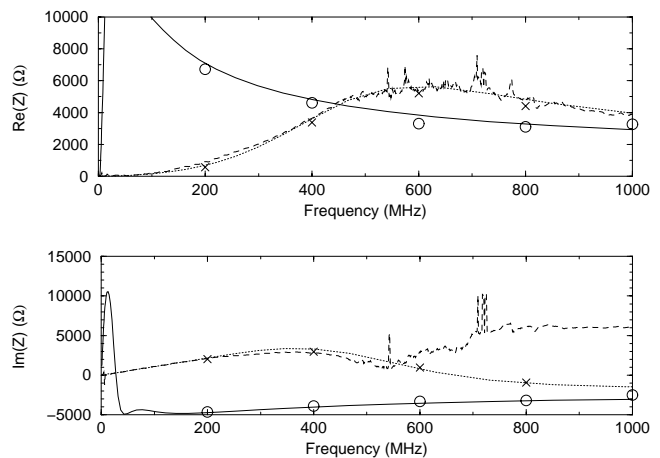


Figure 2: Analytical calculations and simulations of the longitudinal coupling impedance of the SPS MKE kicker. The solid and the dotted lines show the analytical results for Model 1 and 2, respectively. The circle and “x” symbols show the HFSS simulation results for Model 1 and 2, respectively. The dashed lines show the measured results by the coaxial wire method [5].

can see that Model 1 has a much larger real part than the measurement (dashed line) at low frequencies. This is because we did not take into account the effect of the hot and cold conductors, which modify the distribution of the TEM-like electromagnetic field of the beam. We will see this with Model 2.

## 2.2 Simulation by HFSS

The well known coaxial wire method [6] for impedance measurement is prone to the simulation by HFSS. However, some care is required to use this method.

For our kicker case, the TEM-like waves attenuate quickly along the wire because of the high losses in the ferrite. Whereas with the real beam, the TEM-like waves do not attenuate along the beam direction because the beam feeds the energy to the waves. Therefore, the approximate ‘‘log’’ formulae [5] is used to take into account this effect. However, if there is a ‘by-pass’ [1], some portion of the electromagnetic field will go through, and the calculated coupling impedance would be different.

Nevertheless, another way to simulate the beam [2] in HFSS is to put current sources on the beam axis according to

$$I(t) = I_0 \exp(j\omega(t - z/c)). \quad (2)$$

The whole procedure is as follows:

1. In 3D Modeller, define planes around the axis.
2. In Boundary/Source Manager, assign current source boundary conditions on the planes.
3. Solve the problem.
4. In 3D Post Processor, assign magnitudes and phases of the current sources by using ‘Edit Source’ command and then plot the fields.

Since a variable current source (Eq. (2)) cannot be assigned in HFSS, we set many constant current sources of 2 cm length on the axis. The phase difference between adjacent current sources is  $2 \text{ cm} / \lambda \times 360^\circ$ . A problem of this method is that charges are created and annihilated at both ends of the current source. This is inconsistent with nature. This drawback may be neglected if the length of each source is small enough as compared to the wavelength.

Figure 3 shows the geometry used in the simulation. Since it has axial symmetry, we used a 1/36 model (i.e.



Figure 3: Model 1 geometry used for HFSS: only 1/36 of the structure is simulated. The inner and outer radii of the ferrite are 20 mm and 80 mm, respectively. The axial length is 1 m. The current sources are on the axis.

$10^\circ$  sector model). The length along beam direction is 1 m.

Figure 4 shows  $E_z$  along  $r = 20 \text{ mm}$  line at 600 MHz. Since  $E_z$  is independent of the transverse coordinate in the vacuum for Model 1, we can use this value to calculate the longitudinal coupling impedance. It is better to use the field far from the axis for the impedance calculation, in order to avoid noise by the current sources on the axis.

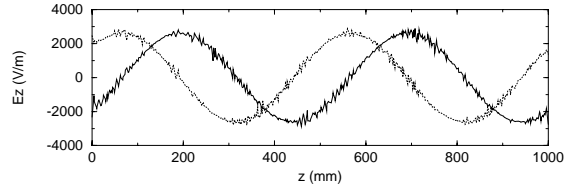


Figure 4:  $E_z$  along  $r = 20 \text{ mm}$  line corresponding to Model 1 at 600 MHz by HFSS simulation. Solid and dotted lines show the real and imaginary parts, respectively.

From the figure, one can see  $E_z \simeq -2000 + j2000 \text{ V/m}$ . Thus the coupling impedance is  $Z = -l_F E_z / I_0 = (3300 - j3300) \Omega/\text{m}$  at 600 MHz. The circle symbols in Fig 2 show the results; they agree well with the analytical calculation.

## 3 MODEL 2

Compared with the measurement, Model 1 gives a much larger coupling impedance at low frequencies, as shown in Fig. 2. This is because we neglected the metal electrode plates at the two sides. The electromagnetic field should be strongly deformed at low frequencies, so that most of the image current goes through the electrode plates. This lowers the impedance.

Electrodes were added at both sides in the Model 2 (Fig. 1, right), and the ferrite block were deformed in order to calculate the impedance easily. Model 2 is a metal tube with square cross section ( $|x| < a$ ,  $|y| < b$ ) and two ferrite slabs ( $|x| < a$ ,  $b < |y| < d$ ). The image current can find metallic by-passes left and right ( $|x| > a$ ). The length in axial direction is infinite for analytical calculations.

### 3.1 Analytical Calculation

By using a field matching technique, we obtain the longitudinal coupling impedance per unit length  $Z/L$  as [1]

$$\begin{aligned} \frac{Z}{L} &= j \frac{Z_0}{2a} \sum_{n=0}^{\infty} \left[ FX + FY - \frac{k}{k_{xn}} sh \ ch \right]^{-1}, \\ FX &= \frac{k_{xn} (1 + \epsilon_r \mu_r) sh \ ch}{k \epsilon_r \mu_r - 1}, \\ FY &= \frac{k_{yn} \mu_r sh^2 tn - \epsilon_r ch^2 ct}{k \epsilon_r \mu_r - 1}, \end{aligned} \quad (3)$$

where wave numbers  $k_{xn}$  and  $k_{yn}$  are  $(2n + 1)\pi/(2a)$  and  $\sqrt{(\epsilon_r \mu_r - 1)k^2 - k_{xn}^2}$ , respectively. The parameters  $sh$ ,  $ch$ ,  $tn$ , and  $ct$  are  $\sinh(k_{xn}b)$ ,  $\cosh(k_{xn}b)$ ,  $\tan(k_{yn}(b-d))$ , and  $\cot(k_{yn}(b-d))$ , respectively.

The dotted lines in Fig. 2 show the result. The measurements agree well with the calculation, except for the imaginary part above 500 MHz. Figure 5 shows the transverse magnetic field vectors. At 200 MHz (left plot), the magnetic field is modified so that most of the image current

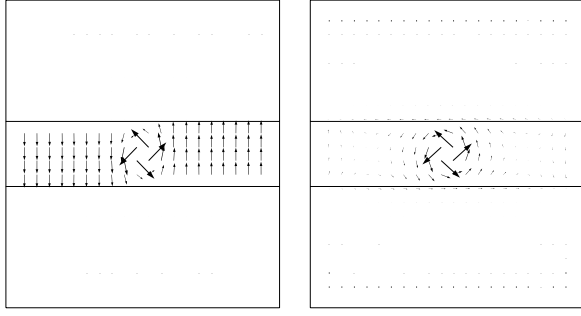


Figure 5: Transverse magnetic field vectors at 200 MHz (left) and 800 MHz (right).

goes through the metal plates ( $|x| = a$ ). As the frequency increases (right plot), some amount of field penetrates into the ferrite blocks, which induces energy loss.

### 3.2 HFSS Simulation

The geometry is shown in Fig. 6. Since it is symmetric with

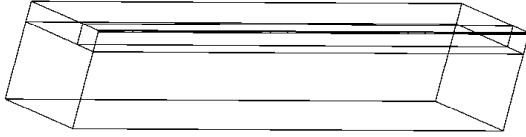


Figure 6: Model 2 geometry used for HFSS.

respect to the  $x = 0$  and  $y = 0$  planes, a 1/4 model is used in the simulation. Much more volume is needed to simulate Model 2 than for the Model 1. The number of mesh points, due to the computer resource problem at CERN should not exceed about 50000; with this volume, we can achieve a minimum mesh size (with 'Manual mesh' option in HFSS) of 10 mm which is insufficient. Therefore the actual length of the model was reduced from 1 m to 50 cm.

Since Model 2 does not have axial symmetry,  $E_z$  depends on the transverse coordinates. In this case, we can use the following technique to calculate impedance. From Maxwell's equations, we obtain

$$-\frac{\partial E_z e^{jkz}}{\partial r} + \frac{\partial (E_r + Z_0 H_\phi) e^{jkz}}{\partial z} = \frac{1}{r} \frac{\partial Z_0 H_z e^{jkz}}{\partial \phi}. \quad (4)$$

By averaging the above equation with  $\phi$ , we obtain

$$-\frac{\partial \langle E_z \rangle_\phi e^{jkz}}{\partial r} + \frac{\partial (\langle E_r \rangle_\phi + Z_0 \langle H_\phi \rangle_\phi) e^{jkz}}{\partial z} = 0, \quad (5)$$

where  $\langle \rangle_\phi = \int d\phi / 2\pi$ . Thus the following integral from ( $r = 0, z = z_{min}$ ) to ( $r = 0, z = z_{max}$ ) does not depend on the path:

$$Z(\omega) I_0 = - \int_{z_{min}}^{z_{max}} dz E_z(r = 0, z) e^{jkz}$$

$$= - \int_L \{ dz \langle E_z \rangle_\phi + dr (\langle E_r \rangle_\phi + Z_0 \langle H_\phi \rangle_\phi) \} e^{jkz}, \quad (6)$$

where  $L$  is an arbitrary path in vacuum region connecting ( $r = 0, z = z_{min}$ ) and ( $r = 0, z = z_{max}$ ). In our case, since the electromagnetic fields are proportional to  $\exp(-jkz)$ , integration can be done easily and we obtain the coupling impedance  $Z$  as

$$Z(\omega) = - \frac{l_F}{2\pi I_0} \int_0^{2\pi} d\phi E_z(r, \phi, z) \exp(jkz). \quad (7)$$

We used this equation to calculate the coupling impedance. The result is shown as "x" symbols in Fig. 2, which agree with the analytical results (dotted lines).

## 4 CONCLUSIONS

A new impedance calculation method using HFSS is presented. The result agrees with the analytical value. The metal plates in Model 2 change the impedance to lower value at low frequencies. Model 2 gives good estimate of the broad-band longitudinal coupling impedance at least at low frequencies.

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