TUNE AND STABILITY OF HIGH INTENSITY BUNCH TRAINS IN THE CERN SPS AND LHC

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Abstract

The complex resistive wall impedance generates a non-uniform transverse focussing along a bunch train. The effect was first computed by V. Balbekov [1] for the UNK machine. The origin of the effect is discussed and then quantified for bunch trains in the CERN SPS and future LHC. Then the transverse resistive wall instability of bunch trains is examined and compared with the well known case of the uniformly filled machine. It is shown that the frequency of the most unstable mode is centred around the inverse of the length of the bunch train with a growth rate that is larger than the one expected from linear extrapolation of a full machine.

1 INTRODUCTION

V. I. Balbekov has computed the tune shift along a uniform train or batch of proton bunches for the high intensity UNK machine [1]. The origin of this effect is the complex transverse resistive wall impedance. It is computed here both for the SPS (fixed target and LHC beam operation) and for the LHC. It modifies the stability situation of the bunch train when compared to a machine which is uniformly filled. In the latter part of the paper it is shown that the driving force of the resistive wall instability depends critically on the batch length for constant local density.

2 THE ORIGIN OF THE TUNE GRADIENT ALONG A BUNCH TRAIN

Consider a train of identical bunches. The tune of the train and of the individual bunches will be modified by the so-called Laslett tune shifts [2]. Let us examine the various types of these shifts and their ability to impose a tune gradient along a batch or a train of bunches.

The *direct tune* shift acts on each bunch. The forces on each charged particle emanate from neighbouring charges and currents in the bunch. The force and the source are in perfect phase since there is no time delay between them. The effect will be the same on all bunches since they are assumed to be identical, hence it does not contribute to a tune gradient.

The *electrostatic tune* shifts are driven by the images created by the wall conductor and the electric field of the beam. As in the previous case there is no delay between the force and its source. The argument is valid both for the incoherent and the coherent electrostatic effect. Hence the electro-static tune shift does not contribute to a tune gradient across a uniform bunch train.

Two cases have to be considered regarding the *magnetic tune* shifts. The first one is produced by the image created by a magnetic surface. Only the DC magnetic field is involved, hence it acts on the bunch train in a uniform way and does not contribute to a tune gradient.

The second case concerns the image currents that drive the incoherent and coherent tune shifts. Again there is no delay between the source (beam and image current) and the force for the incoherent effect. Hence all bunches are perturbed in the same way such that the effect is uniform along the bunch. The same is no longer true for the coherent effect. A differential image current is set up that drives a wall voltage via the complex wall impedance (skin effect). The complex impedance causes a time delay between the source (the image current which is in phase with the beam current) and the electric field in the wall impedance that drives the tune shift. Consequently the focusing force will vary with time along the bunch train. The effect that was just described is nothing else but the resistive wall effect.

3 COMPUTATION OF THE TUNE GRADIENT ALONG A BUNCH TRAIN

The resistive wall impedance is proportional to $\sqrt{j/\omega}$. For convenience it is assumed that the thickness of the wall conductor is larger than the skindepth. The bunch train will be simply represented by a uniform current impulse of length τ. The bunch structure can be neglected since the resistive wall impedance falls off quickly with frequency from the lowest frequencies onwards. The revolution time of the machine is $T = 2\pi/\Omega$ and Ω is the angular revolution frequency. The spectrum of a uniform bunch train can be written as :

$$B(\omega) = \frac{\sin(\omega\tau/2)}{\omega/2}, \qquad (1)$$

which assumes a charge in the bunch train proportional to its length τ. The (de)focusing force will be proportional to the imaginary part of (only imaginary impedance causes a real tune shift) :

$$q(t) = \sum_{\omega=-\infty}^{\infty} e^{j\omega t} \frac{\sin(\omega\tau/2)}{\omega/2} \sqrt{\frac{j}{\omega}}. \qquad (2)$$

The summation is taken over the fundamental betatron frequency and its harmonics $\Omega_\beta=(n-Q)\Omega$ where Q is the transverse tune.

Fig 1 shows two examples for different bunch train length. The time scale $t=0$ starts in the middle of the bunch train for convenience. The vertical scale is arbitrary but identical for the two plots. The time scale is for the LHC ($T = 2\pi/\Omega$ = 89 μs). The tune spread in the bunch train increases to a maximum for a length equal to half the circumference of the machine and decreases again when the whole machine gets filled progressively. The evolution of the spread Δq as a function of the fraction of the machine circumference τ/T occupied by the trains is shown in Fig 2.

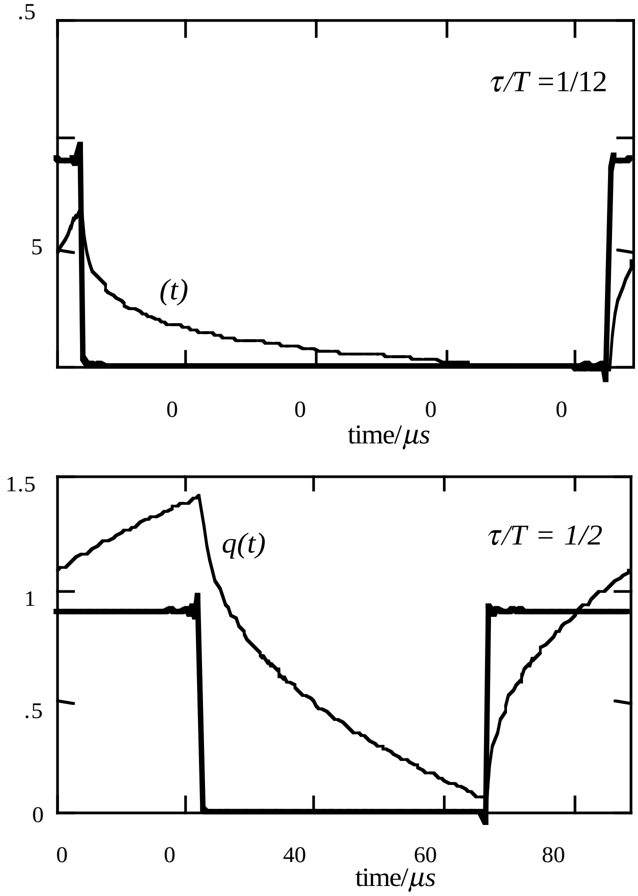

Figure 1: Tune evolution across a bunch train, *top:* train (heavy line) occupies τ/T=1/12 of circumference, *bottom :* train occupies τ/T =1/2 of circumference

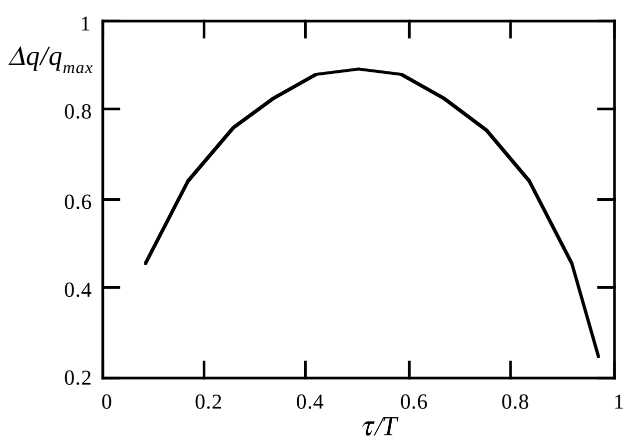

Figure 2: Tune spread in bunch train versus fraction of machine circumference τ/T occupied by the beam.

The normalising factor q_{max} is the tune shift induced by the imaginary part of the resistive wall for a full machine but with the same local density as the bunch train.

4 APPLICATION TO SPS AND LHC

The tune shift for a total beam current i_B , equal to the local beam current of the bunch train, is given by :

$$q_{\max} = \frac{1}{4\pi}\frac{R/Q}{E/e} Z_\perp i_B, \tag{3}$$

where $Z_\perp$ is the (imaginary part) transverse impedance, R the machine radius and E the beam energy. Table 1 summarises the effect of the tune gradient caused by the resistive wall in the SPS and the LHC for various bunch train situations.

Table 1: Vertical tune spread in beam batches for various situations in the SPS and the LHC.

	i_B	τ/T	$Z_\perp/j$	E/e	q_{max}	Δq
PS	*A*		*MΩ/m*	*GV*	10^{-3}	10^{-3}
ixed target	0.27	1/2	200	14	13	11.7
batch	0.67	1/12	"	26	17	7.8
batch	"	1/6	"	"	"	11
batch	"	1/4	"	"	"	13
HC						
batch	"	1/12	52	450	0.46	0.2
batch	"	1/2	"	"	"	0.4
1.7 batch	"	11.7	"	"	"	0.114
1.7 batch	"	11.7	116	7000	0.067	0.016

It may be useful to compare the magnitude of this effect with the direct space charge tune shift which plays an important role both in the SPS and the LHC. The direct space charge tune shift can be written as [2]:

$$q_{sc} = \frac{1}{4\pi}\frac{R/Q}{E/e}\frac{\varepsilon_0 Z_0 Q}{\varepsilon_y \gamma}\langle i_b \rangle. \tag{4}$$

The ratio of the tune shifts given by Eq. 3 and 4 is :

$$\frac{q_{\max}}{q_{sc}} = \frac{Z_\perp \gamma \varepsilon_y}{\varepsilon_0 Q Z_0}\frac{i_B}{\langle i_b \rangle} = \frac{Z_\perp \gamma \varepsilon_y}{\varepsilon_0 Q Z_0}\frac{t_{bb}}{\sqrt{2\pi}\sigma_t}, \tag{5}$$

where Z_0 is the impedance of free space, ε_y the normalised transverse emittance, γ the relativistic factor, ε_0 the Laslett coefficient, t_{bb} the bunch spacing, σ_t the rms. bunch length and $\langle i_b \rangle$ the average bunch current. The tune shift ratio in the SPS for nominal LHC type beam parameters is 0.45 while it is 0.3 in the LHC, both at injection. The tune spread of a bunch train as a whole is equal to the tune spread of its constituent bunches augmented by tune differences between them. Clearly, the tune gradient along the LHC batches is a non-trivial component of the tune spread of a batch, especially for the leading one, which helps slightly for stability but it reduces the decoherence time which makes the transverse emittance conservation a more difficult task.

5 TRANSVERSE STABILITY OF BUNCH TRAINS

The beam envelope spectrum of a bunch train that occupies the full circumference consists of a single *DC* component. It couples with the lowest slow wave mode and drives the transverse resistive wall instability. The situation for a partially filled machine is very different. The resistive wall impedance is proportional to a factor (see also above) that is called $z(\omega)$:

$$z(\omega) = \sqrt{j/\omega}\,. \qquad (6)$$

The basic bunch train envelope spectrum is given by Eq. 2. The frequencies ω to be considered are $n\Omega$, where n takes positive and negative integer values. Notice that for $\tau = T$, only the DC component is different from zero. The instability will set up a coupled bunch oscillation in the bunch train. The central angular frequency of this oscillation is $\omega_i=(n-Q)\Omega$. The spectrum of the oscillation is simply the basic bunch train spectrum shifted by ω_i:

$$B_i(\omega) = \frac{\sin\left((\omega-\omega_i)\tau/2\right)}{(\omega-\omega_i)/2}. \qquad (7)$$

Another way of describing the phenomenon is to say that the instability drives a higher mode of the bunch train spectrum. The driving force of the instability can be derived from the convolution of $z(\omega)$ and $B_i(\omega)$. The frequency ω_i is such that the convolution maximises the level of anti-damping, considering the fact that positive frequencies (fast waves) are damped and negative frequencies (slow waves) are anti-damped. Hence, the maximum of the real part of following function yields ω_i:

$$\sum_{\omega=-\infty}^{\infty} B_i(\omega) z(\omega) = \sum_{\omega=-\infty}^{\infty} \frac{\sin\left((\omega-\omega_i)\tau/2\right)}{(\omega-\omega_i)/2}\sqrt{\frac{j}{\omega}}. \qquad (8)$$

A plot of Eq. 8 (real part) is shown in Fig 3 for a full machine (lower curve) and a partially filled machine (upper curve).

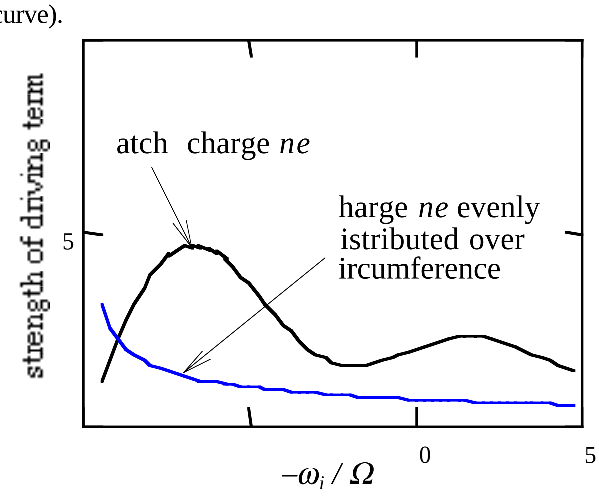

Figure 3 : Strength of instability driving term and frequency of most unstable mode for a full machine and for a bunch train with the same charge packed in 1/4 of the circumference.

The single spectral line of a full machine is shifted at the lowest slow wave mode and yields the frequency of the instability. The spectrum of a partially filled machine is much wider and contains both fast (damped) and slow (anti-damped) waves. The situation resembles in some way single bunch head-tail modes apart from the fact that chromaticity shifts the single bunch mode spectrum and modifies the instability driving term. It can clearly be seen that the driving force for the bunch train is larger than for the uniform beam with the same charge, and that it occurs, in this particular case, at a mode n-Q around -3 and not close to 0. The frequency and the instability driving force will change progressively when a machine is filled with a sequence of identical batches. Fig 4 shows the relative driving force and the frequency of the most unstable mode for constant local beam density as a function τ/T. It turns out that the frequency of the most unstable mode is about $1/\tau$.

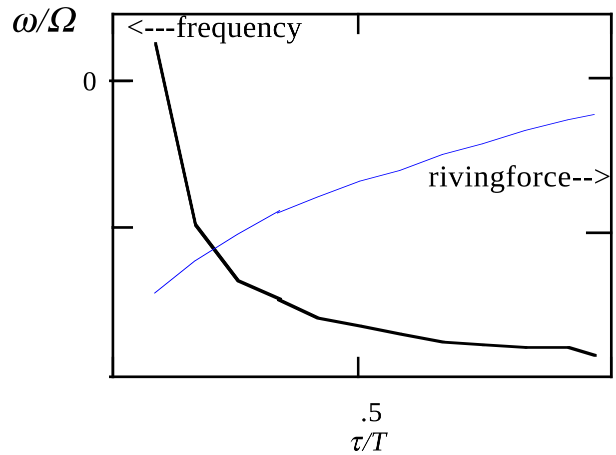

Figure 4: Frequency and relative driving force of resistive wall instability for a partially filled machine.

6 CONCLUSIONS

The most unstable mode of the resistive wall instability of a beam in a machine that is being filled with a sequence of batches is centred around a frequency approximately given by $f = 1/\tau$, i.e. the inverse of the length of the bunch train. The driving force increases more than linearly with the total charge in a train. Furthermore it is accompanied by an increase in tune spread due to the complex nature of the skin effect impedance. This effect is not negligible since it culminates at 45% of the space charge tune shift (the main contributor to the tune spread) in the SPS and at 30% in the LHC for bunch trains of nominal bunch intensity at injection energy. Landau damping is slightly better but its benefit is lost by the reduction of the decoherence time. Consequently the requirements on the transverse feedback are enhanced in view of the conservation of the transverse emittance especially for the leading batches of an injection sequence.

REFERENCES

[1] V.I. Balbekov, The Effect of Vacuum Chamber Wall Conductivity of an Accelerator on the Value of Coulomb Tune Shifts, EPAC, Berlin 1992.

[2] L.J. Laslett, On Intensity Limitations imposed by transverse Space-charge Effects in circular Particle Accelerators, BNL 7534, 1963.