ORBIT-RESPONSE MATRIX ANALYSIS AT HERA

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Abstract

The orbit-response matrix (ORM) of an accelerator is obtained by measuring the closed orbit deviation produced by an individual excitation of each correction coil. In theory, this matrix only depends on the beta functions and betatron phases at the beam position monitors (BPMs) and at the correction coils. By solving this nonlinear dependence by an iterative method, we have measured the Twiss functions of the HERA rings. Furthermore one can fit optical errors like quadrupole strength or roll angles to obtain the field deviations that produce a discrepancy between the measured and the theoretical ORM. Both methods are useful tools for finding errors of BPMs and of correction coils. While both methods, especially the second one, have been used at several circular accelerators, they have been found to become less useful and less accurate with an increasing size of the accelerator. Therefore they have never been intensively applied to an accelerator as large as HERA. Here we will report that the ORM analysis has nevertheless been found to be very useful for checking HERA's BPM system, for finding erroneous correction coils and for finding erroneous quadrupole settings.

1 INTRODUCTION

The 6335 m long circular storage ring HERA is the only high energy lepton-proton collider in the world. After having surpassed design luminosity in the year 2000 its two interaction regions for the particle physics detectors H1 and ZEUS were completely rebuilt between 9/00 and 7/01. The electron ring and the proton ring both have a new vacuum system 60 m to the right and the left of the interaction points. A total of 60 new magnets were installed, 4 of them superconducting, and the detectors both went through a major reconstruction. Now superconducting final focus quadrupoles reach inside the detector for an early separation of the electron and the proton beams and the detector solenoids no longer have a anti-solenoid, so that a skew quadrupole correction is required. After these severe changes during the upgrade, the magnet strength, the BPM system, and all optical features had to be checked. The ORM has turned out to be a useful tool for the required evaluations.

When the closed orbit position at M_x horizontal and M_y vertical BPMs is denoted by $\vec{x}$ and $\vec{y}$ and the K_x horizontal and K_y vertical corrector kicks are denoted by $\vec{\theta}_x$ and $\vec{\theta}_y$, then the ORM $\underline{A}$ relates changes of corrector kicks to the

changes of the closed orbit which they produce via

$$\left(\begin{array}{c} \Delta\vec{x} \\ \Delta\vec{y} \end{array} \right) = \underline{A} \left(\begin{array}{c} \Delta\vec{\theta}_x \\ \Delta\vec{\theta}_y \end{array} \right) . \qquad (1)$$

The matrix $\underline{A}^{\mathrm{meas}}$ relating corrector settings to BPM readings can easily be obtained by successively changing single corrector coils and measuring the closed orbit. This measured matrix will usually not agree with the ORM $\underline{A}^{\mathrm{mod}}$ of the optical model that is used to describe the accelerator. In turn it will also not correspond completely to the true ORM $\underline{A}$ due to errors in the BPM system or in the corrector coil settings. The difference between the model and the measured ORM can therefore reveal gauge errors in the corrector coils and in the BPMs as well as sources of optical errors.

This matrix consists of $(K_x + K_y) \cdot (M_x + M_y)$ data points. For HERA-e (M_x, M_y, K_x, K_y) is $(287, 287, 281, 277)$ and for HERA-p it is $(141, 141, 128, 126)$. The matrices therefore comprises 320292 and 71628 individual measurements. Since the $K_x + K_y$ different closed orbits sample the different field regions in all magnets quite well, the ORM contains a wealth of information about the fields in a circular accelerator.

In the HERA electron ring as well as in the HERA proton ring orbit-response matrices are being measured regularly during the ongoing commissioning process.

1.1 Obtaining Twiss Parameters

Since we have not yet analyzed the coupled ORM, we will refer to the Twiss parameters of one transverse plane as β and ϕ, so that the formula can be used for the vertical as well as for the horizontal plane. When the Twiss parameters and the tune $Q = \mu/2\pi$ of the accelerator were known at the monitors (index m) and the correctors (index k), the ORM could be computed as

$$A_{mk} = \frac{\sqrt{\beta_m \beta_k}}{2\sin(\mu/2)} \cos(|\phi_m - \phi_k| - \mu/2) . \qquad (2)$$

When the monitors have a gauge error of ξ_m and the correctors have a gauge error of ξ_k, then $A_{mk}^{\mathrm{meas}} = \xi_m \xi_k A_{mk}$. The gauge errors will therefore lead to an error $\sqrt{\xi_{m/k}}$ in the beta functions $\beta_{m/k}$, but even for very large gauge errors the phase can be determined accurately.

The measured matrix contains $M \cdot K$ data points, while only Twiss parameters at the monitors and correctors are unknown. The tune can be measured accurately, one of the phases can be set to zero, and one of the beta functions can be set to some model value. This leaves us with

$2(M + K) - 2$ unknowns, 1134 and 536 for the horizontal planes of HERA-e and HERA-p. The problem is very much over determined so that the free parameters can be found by minimizing

$$\sum_{k=1}^{K} \sum_{m=1}^{M} \left(\frac{A_{mk} - A_{mk}^{\mathrm{meas}}}{\sigma_m} \right)^2 \to \text{minimum} . \quad (3)$$

In order to give more weight to those monitors which have a smaller statistical error, we have first determined the standard deviation σ of 100 successive orbit measurements for each monitor. While we averaged over 64 single turn orbit measurements in HERA-e, the average standard deviation in the horizontal was $40\,\mu$m and in the vertical it was $20\,\mu$m.

Since A_{mk} is a nonlinear function of the Twiss parameters, we use an iteration procedure. In the nth step we solve in a least square sense

$$\frac{\sqrt{\beta_m^n}}{\sigma_m 2 \sin \mu} \left(\begin{array}{c} \cos(\phi_m^n \pm \mu/2) \\ \sin(\phi_m^n \pm \mu/2) \end{array} \right) \cdot \left(\begin{array}{c} f_k^{n+1} \\ g_k^{n+1} \end{array} \right) = \frac{M_{mk}^{\mathrm{meas}}}{\sigma_m} , \quad (4)$$

with $+$ for $\phi_m < \phi_k$ and $-$ for $\phi_m > \phi_k$. This is a very fast and simple two-dimensional least square problem for each corrector (k). For the next iteration one has $\beta_k^{n+1} = (f_k^{n+1})^2 + (g_k^{n+1})^2$ and $\phi_k^{n+1} = \arctan(f_k^{n+1}/g_k^{n+1})$. Similarly one obtains the $n + 1$st iteration of at the monitors (m). After about 50 iterations the fit converges. We arbitrarily chose the phase of the first monitor to agree to the phase in the optic model of HERA. One of the beta functions can be chosen arbitrarily due to the fact that only products $\beta_m \beta_k$ appear, so that a general scaling of all β_m with r can be compensated by a general scaling of all β_k with $1/r$. Since quadrupole errors lead to a harmonic beating of the beta function

$$\frac{\Delta \beta}{\beta} = \sum_i \frac{\beta_i \Delta k l_i}{2 \sin \mu} \cos(2|\phi - \phi_i| - \mu) , \quad (5)$$

this beating approximately averages out over 5 of the 72° and 2 of the 90° FODO cells in HERA-e and HERA-p. We therefore chose the free scaling factor so that

$$\sum \beta_m = \sum \beta_m^{\mathrm{mod}} , \quad (6)$$

where the sum goes over the FODO cells in the 4 HERA arcs.

Quadrupole errors $\Delta k l_i$ cause the phase to beat according to the formula

$$\Delta \phi = \sum_i \frac{\beta_i \Delta k l_i}{4 \sin \mu} \{\sin \mu + \sin(2\phi_i - \mu) +$$

$$\mathrm{sign}(\phi - \phi_i)[\sin \mu + \sin(2|\phi - \phi_i| - \mu)]\} . \quad (7)$$

When there is only one error field, the amplitude of the phase beat is just half as large as the relative amplitude of the beta beat.
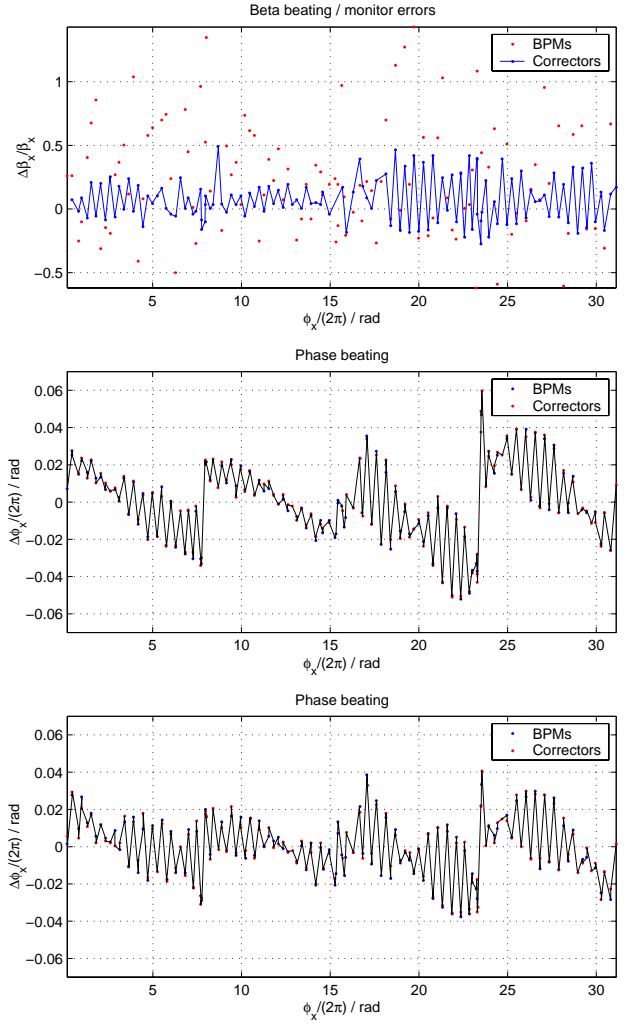

Figure 1: Beating of β_x (top) and ϕ_x (middle) for HERA-p at injection (40 GeV) before and after partial field corrections (bottom)

Figure 1 shows the so determined beta and phase beat in the HERA-p injection optics. Although gauge errors lead to errors in the beta functions, the phase measurements alone have already turned out to be very useful. From figure 1, for example, two magnets with incorrect remanence fields have been identified by phase jumps at $\phi^{\mathrm{mod}} \approx 7.7 \cdot 2\pi$ rad and $\phi^{\mathrm{mod}} \approx 23.5 \cdot 2\pi$ rad.

This Twiss parameter computation also gives a hint at gauge errors, $\xi_{m,k} = \beta_{m,k}^{\mathrm{fit}}/\beta_{m,k}^{\mathrm{mod}}$ is a rough estimate for gauge errors.

2 BPM RESPONSE MATRIX EVALUATION

Fitting system parameters like gauge errors and quadrupole strength, instead of Twiss parameters, to match the ORM can lead to a more accurate determination of the gauge errors. We have used the program CALIF [1] to fit these free parameters in order to minimize equation (3).
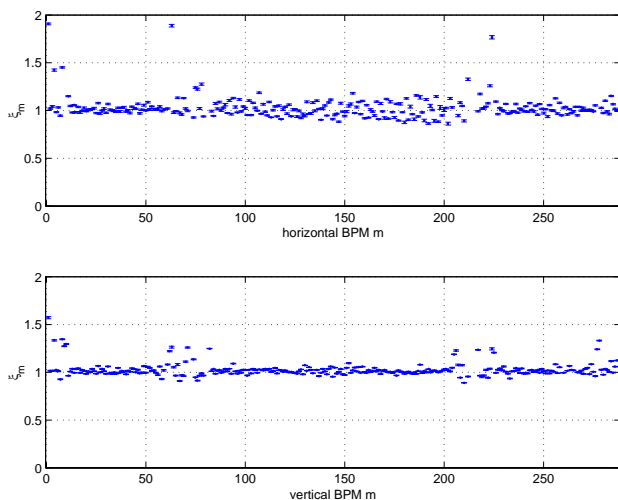
Figure 2: Gauge errors of HERA-e monitors at the beginning of commissioning

CALIF contains also a LOCO-type fit [2, 3]. CALIF expands the orbit matrix's dependence on the free parameters in a 1st order Taylor expansion, and then optimizes in a least square procedure using singular value decomposition (SVD). Again iteration is needed due to a nonlinear dependence on the fit parameters. The least square fit involves very large matrices in the case of HERA. We write all $M \cdot K$ matrix elements into one vector $\vec{A}$, expand the model ORM as a linear expansion of the fit parameters,

$$|\vec{M}^{\mathrm{mod}} + \partial_{\vec{f}^T}\vec{M}^{\mathrm{mod}}\Delta\vec{f} - \vec{M}^{\mathrm{meas}}|^2 \rightarrow \text{minimum} . \quad (8)$$

For all gauges and all $N_q = 576$ quadrupoles, the involved matrix has $M \cdot K \cdot (M + K + N_q) = 92260168$ elements for HERA-e's horizontal plane. At other accelerators it has been possible to fit a sufficient number of parameters to achieve an agreement between the model and the measured ORM on the level of the monitor precision [2]. HERA is relatively large and therefore we have never managed to work with a sufficient number of suitable fit parameters to bring this minimum below 7 times the monitor precision.

Figure 2 shows the fitted BPM gauge errors of HERA-e at the beginning of the commissioning. Many of these errors were due to incorrect cables or unusual vacuum chambers which are now being taken into account.

2.1 Obtaining Quadrupole Errors from the measured betatron phase

While fitting a model to match the ORM has turned out to be a useful tool for finding monitor errors in HERA, it is very slow and requires several GBytes of main memory. Since the described method of obtaining betatron phases $\phi_{m/k}$ does not need to fit gauge errors, is extremely fast and we have extended it to fit quadrupole strength to match the betatron phases of the model to the measured phases. Such a phase correction is routinely used at CESR [4], where the phase is however measured in a more direct fashion from

multi turn data, which are not accessible with the HERA-e monitor system. For the fit we bring equation (7) in the form $\phi_{m/k} + \sum_i B_{m/k,i}\Delta kl_i = \phi_{m/k}^{\mathrm{mod}}$ and solve it with SVD in a least square sense.

3 HERA RESULTS

The results obtained with these methods during the on-going commissioning of HERA were

1. Several incorrectly wired monitors were found.
2. Four correctors and 12 BPMs with incorrect longitudinal positions in the optics database were found.
3. Many monitors with nonstandard vacuum chambers and therefore incorrect gauge factors were found.
4. Four corrector coils with incorrect gauges due to stray fields were identified.
5. An extremely large beta beat in HERA-p had been observed [5] at 920 GeV and this error was analyzed and corrected in the injection and the luminosity optics.
6. An error in the strength of final focus magnet was found and corrected.
7. Inconsistencies in the settings of the HERA-p tune correction quads were found.

These tools for analysis of the optics have not only shown useful in correcting the monitor system and in finding strong optical errors during commissioning of the large accelerator HERA, they also serve as a routine check for making sure that no slowly evolving magnet errors disturb routine operation.

4 CONCLUSION

Measuring an ORM is a relatively fast procedure, the HERA-e matrix with 320292 entries can be measured in 60 minutes. The analysis obtaining the phases and an estimate of the gauge errors only takes a few seconds on a standard PC. A full analysis of the ORM is however a more involved job and needs some dedicated work with CALIF. While both methods were applied to HERA, the fast phase determination has evolved into a standard tool to quickly see larger errors in the rings.

5 REFERENCES

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