SPIN COUPLING RESONANCE STUDY* IN AGS

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Abstract

In the AGS spin resonances due to coupling may account for as much as a 50 percent loss in polarization at a reduced acceleration rate. The major source of coupling in the AGS is the solenoidal snake. In the past some preliminary work was done to understand this phenomena [1], and a method to overcome these resonances was attempted [2]. However in the polarized proton run of 2002 we sought to study more thoroughly the response of these coupled spin resonances to the strength of the solenoidal snake, skew quadrupoles and vertical and horizontal betatron tune separation. In this paper we present our results and compare them with those predicted by a modified DEPOL program [3].

1 OVERVIEW OF CONDITIONS FOR THE 2002 AGS RUN

The Brookhaven Alternating Gradient Synchrotron (AGS) is the third stage in a complex of accelerators that accelerate protons and Gold ions up to 250 GeV and 100 GeV respectively in the Relativistic Heavy Ion Collider (RHIC). In the AGS a partial solenoidal snake and RF dipole are employed to overcome imperfection resonances and strong intrinsic resonances respectively.

In past years the AGS was operated with an acceleration rate of $\frac{dG\gamma}{d\theta} = \alpha = 4.8 \times 10^{-5}$, where G the anomalous g-factor and θ the orbital bending angle. However during the 2002 run a backup power supply was used for the magnets resulting in a ramp rate of $\alpha = 2.4 \times 10^{-5}$. The lower acceleration rate made the use of a weaker partial snake possible since at a slower acceleration rate effective spin flipping due to the imperfection resonances is enhanced. Lowering the partial snake strength has the advantage of reducing the effective strength of the coupled spin resonances. In the past a 5% partial snake was used. During this run a hybrid partial snake ramp was found to be the most effective. The current control of the partial snake was set up to maintain a 3% snake from injection at $G\gamma = 7.5$ and ramp up to 5

2 COUPLING IN THE AGS

The primary source of coupling in the AGS is the partial solenoidal snake. In addition there exists a family of six skew quadrupoles. It has been observed that the bare AGS machine has a net skew quadrupole moment. It is believed that this is a result of systematic rolls due to the compensation of sags in the combined function AGS magnets. The direction of these rolls are correlated with the direction which the C-magnet is facing. The result is a net negative roll per magnet which has been estimated to have a magnitude of 0.5 mrad. Work with slow beam extraction has also shown that a family of skew quadrupoles needs to be powered at 50 amps in-order to alleviate the effects of coupling in the bare AGS. Using this value we can arrive at a lower bound estimate of magnitude the average roll of 0.15 mrad. Coupling studies from 15 years ago estimated that value to be 0.13 mrad [6]

Additionally, closed orbit error can contribute to coupling via feed down from the sextupole fields present in the AGS combined function magnets and sextupole magnets.

3 UPDATE ON MODIFICATION TO DEPOL PROGRAM

In previous papers [3] we reported on the modifications to the well established DEPOL code [4] to include the effects of coupling. We present now some additional modifications which have significantly improved the speed of this code. The central algorithm presented in [3] is created to evaluate the following Fourier integral,

$$\epsilon_K = -\frac{1}{2\pi} \oint \Big[(1+G\gamma)(\rho z'' + iz') - i\rho(1+G)(\frac{z}{\rho})'\Big] e^{iK\theta} d\theta \qquad (1)$$

Here ϵ_K is the spin resonance amplitude and K is the spin resonance tune. The solution, following the original DEPOL paper, was to break up the integral into a sum over all the lattice elements denoted with subscript m. The final closed solution for each element is given in Eq. 2.

$$\begin{aligned}\varepsilon_m = \frac{1}{2\pi}\Bigg[&\frac{(1+K)(\xi_1 - i)}{\rho} z_1 e^{iK\theta_1} \\ &+ \frac{(1+K)(\xi_2 - i)}{\rho} z_2 e^{iK\theta_2} \\ &-(1+K)((z_2' - \frac{iK}{\rho} z_2) e^{iK\theta_2} - (z_1' - \frac{iK}{\rho} z_1) e^{iK\theta_1}) + \\ &(\frac{K(K^2+G)}{\rho^2})(\frac{1}{\sqrt{1+|r_e|}}\{(\frac{iK}{\rho} r_{e_{1,2}} - r_{e_{1,1}}) \\ &(\frac{(a_2\prime - \frac{iK}{\rho}a_2)e^{iK\theta_2} - (a_1' - \frac{iK}{\rho}a_1)e^{iK\theta_1}}{k_a - K^2/\rho^2}) - \\ &(\frac{(b_2' - \frac{iK}{\rho}b_2)e^{iK\theta_2} - (b_1' - \frac{iK}{\rho}b_1)e^{iK\theta_1}}{k_b - K^2/\rho^2}) \\ &+ r_{e_{1,2}}(a_2 e^{iK\theta_2} - a_1 e^{iK\theta_1})\Bigg]\end{aligned} \qquad (2)$$

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Here r_e is the rotation matrix which transforms from the x, x', z, z' coupled basis to the a, a', b, b' locally uncoupled basis (uncoupling each lattice element only). Since for intrinsic resonances K is not an integer Eq. 1 becomes an integral around the lattice an infinite number of times. Previously, a solution was derived by evaluating an appropriately large number of passes over the lattice.

However if we look closely at the behavior of the elements which make up the integral to be evaluated in Eq. 1 it appears that we can factor out the phase element which changes with each period around the lattice. The remaining elements in the sum remain constant for each pass. The factored phase elements can be evaluated analytically using the properties of a geometric series. The results are four separate enhancement functions,

$$\begin{aligned} E_u(N)_\pm &= \sum_{n=0}^{N} e^{i2\pi n(K\pm\mu_u[L_{max}])} \\ &= \pm e^{iN\pi(K\pm\mu_u[L_{max}])} \\ &\times \frac{\sin\left(\pi(N+1)(K\pm\mu_u[L_{max}])\right)}{\sin\left(\pi(K\pm\mu_u[L_{max}])\right)} \\ E_v(N)_\pm &= \sum_{n=0}^{N} e^{i2\pi n(K\pm\mu_v[L_{max}])} \\ &= \pm e^{iN\pi(K\pm\mu_v[L_{max}])} \\ &\times \frac{\sin\left(\pi(N+1)(K\pm\mu_v[L_{max}])\right)}{\sin\left(\pi(K\pm\mu_v[L_{max}])\right)} \end{aligned} \quad (3)$$

Here L_{max} indicates final μ phase function value in the lattice, N the number of passes around the lattice and u, v the gobally uncoupled basis. The function once evaluated can then be multiplied by the appropriate terms in the sum over one pass in the lattice.

Another issue concerns the measurement of emittance. Normally most machines are set up to evaluate the emittance provided there is no coupling. We developed a method to transform measurements taken in the AGS for the ϵ_x and ϵ_y values and transform them to measurements of ϵ_u and ϵ_v

In General we can define a sigma matrix.

$$\sigma_{xz} = \begin{pmatrix} \langle x^2\rangle & \langle xx'\rangle & \langle xz\rangle & \langle xz'\rangle \\ \langle x'x\rangle & \langle x'^2\rangle & \langle x'z\rangle & \langle x'z'\rangle \\ \langle zx\rangle & \langle zx'\rangle & \langle z^2\rangle & \langle zz'\rangle \\ \langle z'x\rangle & \langle z'x'\rangle & \langle z'z\rangle & \langle z'^2\rangle \end{pmatrix} \quad (4)$$

In the uncoupled u and v basis we can express the sigma matrix in terms of its twiss parameters.

$$\sigma_{uv} = \begin{pmatrix} \epsilon_u\beta_u & -\epsilon_u\alpha_u & 0 & 0 \\ -\epsilon_u\alpha_u & \epsilon_u\gamma_u & 0 & 0 \\ 0 & 0 & \epsilon_v\beta_v & -\epsilon_v\alpha_v \\ 0 & 0 & -\epsilon_v\alpha_v & \epsilon_v\gamma_v \end{pmatrix} \quad (5)$$

If we transform this to the coupled x-z basis.

$$\sigma_{xz} = R\sigma_{u-v}\overline{R} \quad (6)$$

We can obtain an expression for $\langle x^2\rangle$ and $\langle z^2\rangle$ in terms of the twiss parameters in the uncoupled basis and the rotation matrix elements.

$$\begin{aligned} \langle x^2\rangle = \epsilon_u & \left[\overline{R}_{1,1}R_{1,1}\beta_u - \overline{R}_{1,1}R_{1,2}\alpha_u \right. \\ & \left. -\overline{R}_{2,1}R_{1,1}\beta_u + \overline{R}_{2,1}R_{1,2}\gamma_u\right] \\ +\epsilon_v & \left[\overline{R}_{3,1}R_{1,3}\beta_v - \overline{R}_{3,1}R_{1,4}\alpha_v \right. \\ & \left. -\overline{R}_{4,1}R_{1,3}\alpha_v + \overline{R}_{4,1}R_{1,4}\gamma_v\right] \end{aligned} \quad (7)$$

$$\begin{aligned} \langle z^2\rangle = \epsilon_u & \left[\overline{R}_{1,3}R_{3,1}\beta_u - \overline{R}_{1,3}R_{3,2}\alpha_u \right. \\ & \left. -\overline{R}_{2,3}R_{3,1}\beta_u + \overline{R}_{2,3}R_{3,2}\gamma_u\right] \\ +\epsilon_v & \left[\overline{R}_{3,3}R_{3,3}\beta_v - \overline{R}_{3,3}R_{3,4}\alpha_v \right. \\ & \left. -\overline{R}_{4,3}R_{3,3}\alpha_v + \overline{R}_{4,3}R_{3,4}\gamma_v\right] \end{aligned} \quad (8)$$

Since $\langle x^2\rangle = \sigma_x^2$ and $\langle z^2\rangle = \sigma_z^2$ are what the IPMs physically measure it is then easy to solve for ϵ_u and ϵ_v.

We would also like to note a correction to the equation given in [3] for expressing the rotation matrix in terms of the elements of a 4×4 transfer matrix which was obtained from [5]. A corrected expression is given in Eq. 9.

$$\overline{r} = -\left(\frac{Tr(A-D)}{2} \pm \sqrt{|B+\overline{C}| + \frac{Tr^2(A-D)}{4}}\right) \times \frac{B+\overline{C}}{|B+\overline{C}|} \quad (9)$$

Here A,B,C and D represent 2×2 submatricies of the 4×4 transfer matrix, and the over bar indicates a symplectic conjugate. From r the full 4×4 rotation matrix can be developed as follows:

$$R = \frac{1}{\sqrt{1+|r|}} \begin{pmatrix} I & -\overline{r} \\ r & I \end{pmatrix} \quad (10)$$

4 COMPARISON OF RESULTS FOR COUPLING SPIN RESONANCES WITH DEPOL CALCULATIONS

During the 2002 polarized proton run, particular attention was paid to studying the impact of the coupling spin resonances during the $0+\nu$ resonance crossing since the analyzing power of the AGS polarimeter was sufficiently large at low energy to generate accurate measurements and the strength of the $0+\nu$ coupling spin resonance was large. For all DEPOL calculations we found it essential to include a Gaussian distributed net roll of -1.1 mrad per magnet to compare favorably with our measured results. A roll of -1.1 mrad is not unreasonable considering previous estimates. In Figs. 1 - 3 one can see the results of

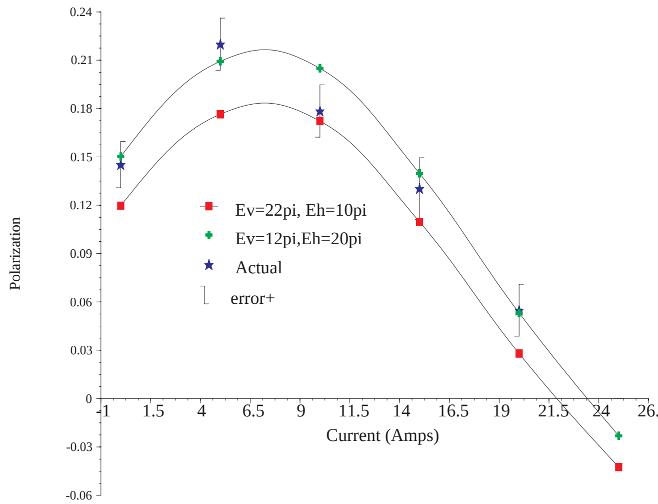

Figure 1: Polarization after crossing the $0+\nu_x$ and $0+\nu_z$ resonances with fixed vertical tune and horizontal tune ($\nu_z = 8.8$, $\nu_x = 8.78$). Scanning through skew quadrupole input currents from 0 to 25 Amps. The vertical and horizontal emittances were measured at $(11 \pm 1)\pi$ and $(21 \pm 1)\pi$ mm-mrad. In addition a distributed roll of -1.1 mrad was included.

our tune scans, snake scans and skew quadrupole scans, respectively. All calculations assume a 70% polarization at injection into the AGS.

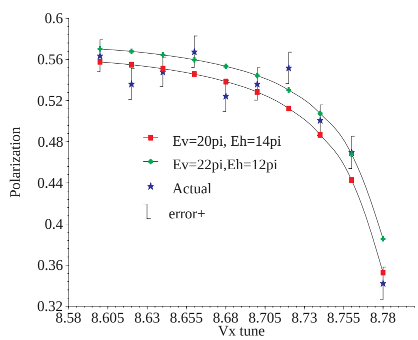

Figure 2: Polarization after crossing the $0 + \nu_x$ and $0 + \nu_z$ resonances with fixed vertical tune ($\nu_z = 8.8$) scanning horizontal tunes. Vertical and horizontal emittances were measured at $(13 \pm 1)\pi$ and $(21 \pm 1)\pi$ mm-mmrad respectively for DEPOL calculations. In addition a roll of -1.1 mrad was included.

5 CONCLUSION

We see very good agreement between our DEPOL calculations and measurements when a -1.1 mrad roll was included. Based on the sensitivity of our DEPOL calculations this figure should have an error of ± 0.2 mrad. We are currently examining data from crossing the three weak intrinsic resonances ($24 - \nu$, $24 + \nu$ and $48 - \nu$). Our preliminary results suggest that a more accurate assessment of net roll per magnet may be possible since changes in the net roll on the order of ± 0.01 mrad can have significant effect on the fine structure of these resonance crossings.

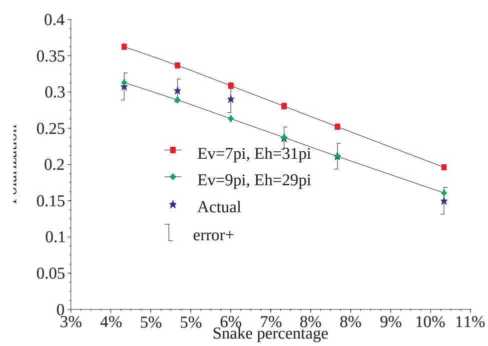

Figure 3: Polarization after crossing the $0 + \nu_x$ and $0 + \nu_z$ resonances with fixed vertical tune and horizontal tune ($\nu_z = 8.8$, $\nu_x = 8.7$ scanning from 4 to 10% partial snake strength. Vertical and horizontal emittances were measured at $(8 \pm 1)\pi$ and $(30 \pm 1)\pi$ mm-mrad for DEPOL calculations. In addition a roll of -1.1 mrad was included.

6 REFERENCES

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