DETERMINATION OF LINEAR AND NON LINEAR COMPONENTS IN RHIC INTERACTION REGIONS FROM DIFFERENCE ORBIT MEASUREMENTS

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Abstract

A novel technique is presented to precisely measure the multipole components at each triplet inside the RHIC Interaction Regions (IRs). The technique is based on measurements of the strength of the overall kick that the orbit naturally experiences at a particular triplet due to the presence of linear and nonlinear errors. To measure the kick strength at the triplet, the action and phase before and after the triplet are measured. Action and phases before and after the triplet can be easily related to the strength of the overall kick at the triplet. Action and phase measurements are done on an orbit obtained from the subtraction of an orbit produced by turning on a dipole corrector and the baseline orbit (difference orbit). Application of this technique to difference orbits obtained during the proton RHIC 2001 run is shown to be very precise method to extract quadrupole components and there is experimental evidence that the technique could be equally precise to extract nonlinear multipole components.

1 INTRODUCTION

Under ideal conditions, the action J and phase φ of betatron oscillations of a particle should remain constant all around the ring. Magnetic errors in the different elements of the ring can lead to a change of these two constants of motion. These changes are used to determine the location of such errors and their strengths.

Action and phase associated with RHIC particle orbits at particular position in the ring are obtained from pairs of adjacent Beam Position Monitor (BPM) measurements. BPM measurements are converted into action and phase by inverting the equations:

$$x_1 = \sqrt{2J\beta_1}\sin(\psi_1 - \varphi) \quad (1)$$

$$x_2 = \sqrt{2J\beta_2}\sin(\psi_2 - \varphi) \quad (2)$$

where, x_1 and x_2 correspond to any two adjacent BPM measurements, β_1, β_2, ψ_1 and ψ_2 are their corresponding beta functions and phase advances.

Equations 1 and 2 are applied to all adjacent BPM measurements in the ring to obtain functions of action and phase with respect to s, the azimuthal location.

During the RHIC 2000 run, studies of action and phase indicated significant coupling errors at the RHIC IRs. A method based on first-turn orbit measurements and action and phase analysis was developed to find the magnitude of the coupling errors and to perform the corresponding correction [1].

The positive results obtained from the previous studies stimulate the development of a general method that would evaluate, from closed orbits (RHIC 2001 run), not only skew quadrupole errors but also gradient errors and nonlinear errors at RHIC IRs. These studies are described in the following paragraphs. The results obtained will be compared with the RHIC 2000 measurements and the triplet survey measurements performed directly inside the tunnel during the RHIC 2002 shutdown period.

2 EXPERIMENTAL PROCEDURE

Action and phase studies requires stable and defined closed orbit changes. Such changes are produced by changing a dipole corrector strength. The corrector must be chosen such that the difference of phase advance between the corrector and the IR under study is close to an odd multiple of $\pi/2$. This condition guarantees that the closed orbit change will be near its maximum when going through the IR. The strength of the corrector is chosen such to creates the largest possible closed orbit change but small enough to avoid beam losses. In RHIC, dipole corrector changes of tenths of mrads produces reasonable closed orbit changes without compromising the beam.

The closed orbit results not only from the applied dipole corrector change but also from dipole kick errors present in the ring. To eliminate these contributions, the baseline (orbit when the dipole corrector change is zero) is substracted from the original orbit, producing a so-called "difference orbit". These procedure also eliminates possible systematic offsets associated with the BPM measurements.

This experiment was repeated with four different strengths for each dipole corrector used and the resultant closed orbits were saved for analysis.

3 ANALYSIS

Action and phase analysis of the difference orbits obtained in the experiment indicates that action and phase remains roughly constant in the arcs while making significant jumps at the IRs (See Fig. 1). Indeed, it is possible to use the average action and phase of each arc to individually fit the BPM measurements of each arc to the betatron equation. Such fit corresponds to the red line seen in the top rectangle of Fig. 1: Since for this case the orbits to analyze are closed orbits, the resolution of the measurements is better than with the first turn orbits used in the initial experiments [1] . In particular, the 2 BPM's located in the center of each IR can be used to calculate action and phase

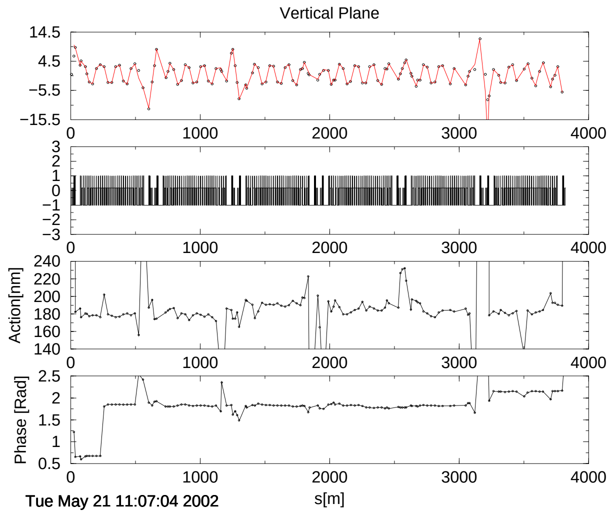

Figure 1: Action and Phase from difference orbit

inside that region. This extra information can be used to independently estimate errors at each triplet.

In this case the action and phase jump is explained as if it were coming from a general magnetic kick $\Delta x'^{Trip}$ that can be obtained from measured quantities:

$$\Delta x'^{Trip} = \sqrt{\frac{\left(J_x^L + J_x^R - 2\sqrt{J_x^L J_x^R}\cos(\psi_x^L - \psi_x^R)\right)}{\beta_x^{Trip}}} \tag{3}$$

where J_x^L, J_x^R, ψ_x^L and ψ_x^R correspond to the action and phases a the left hand side and the right hand side of the particular triplet under study. The $\Delta x'^{Trip}$ obtained in this way is an effective kick that can be assumed to be all produced at a particular location s_0 within the triplet. s_0 can be chosen arbitrarily and it determines the beta function β_x^{Trip} to be used in the previous formula.

On the other hand, $\Delta x'^{Trip}$ corresponds to the sum all possible error present at a particular triplet either skew quadrupole errors, gradient errors or non linear errors. In general $\Delta x'$ can be written as [2]:

$$\begin{aligned} \Delta x' &= (A_1 y_0 - B_1 x_0 \\ &\quad + 2A_2 x_0 y_0 + B_2(-x_0^2 + y_0^2) + ...) \\ \Delta y' &= (A_1 x_0 + B_1 y_0 \\ &\quad + 2B_2 x_0 y_0 + A_2(x_0^2 + y_0^2) + ...) \end{aligned} \tag{4}$$

In the previous equations A_1 , B_1 ,A_2 , B_2 , etc are the different magnetic multipoles components present in the triplet. A_1 corresponds to the skew quadrupole error while B_1 corresponds to the gradient quadrupole error. x_0 and y_0 are the horizontal and vertical position of the beam at s_0. s_0 is chosen at the package corrector since A_1 will be then equal to the value to which the skew quadrupole corrector in the corrector package should be set to locally correct for linear coupling in the particular triplet under study.

It is possible to evaluate the different multipoles components in equations 4 if a set of measurements of the deltas versus the beam position in the horizontal and the vertical plane are available. The four difference orbits taken with different dipole corrector strengths will provide a set of four points for the function $\Delta x'(x_0, y_0)$ and $\Delta y'(x_0, y_0)$ that will allow the evaluation of at least the linear coefficients of equation 4

4 RESULTS

As was mention before values of $\Delta x'$ and $\Delta y'$ as function of the two variables x_0 and y_0 can be extracted from the difference orbits. x_0 and y_0 are no independent variables. There is a relation between these variables that depend on the amount of coupling present in the ring and that can be easily determined experimental. This means that $\Delta x'$ and $\Delta y'$ can be seen as function that depends only in one variable either, x_0 or y_0. Fig. 2 shows $\Delta x'$, $\Delta y'$ and x_0 as a function of y_0 at the right triplet of IR 2 for a set of four difference orbits taken by turning on the vertical dipole corrector bo7-tv13 at strengths -0.1 mrad, -0.05 0.05 mrad and 0.1 mrad (each point in the graph correspond to one strength). As can be seen in Fig. 2 the behavior of $\Delta x'$ and $\Delta y'$ as function of x_0 is almost linear pointing to the fact that the contribution of non linear errors to the action and phase jump at the right triplet of IR 2 is very small compared with the linear errors. The experiment is repeated with many other correctors and in different triplets with results that are summarized in table 1. The value of the skew quadrupole error reported on the table corresponds to the average of the values obtained with the different dipole correctors used in each case.

The measured skew quadrupole errors are slightly sensitive to the dipole corrector that is chosen to produce the betatron oscillations.This is a indication that the measurements are sensitive to the difference of phase advance between the dipole corrector and the IR. Even though the or-

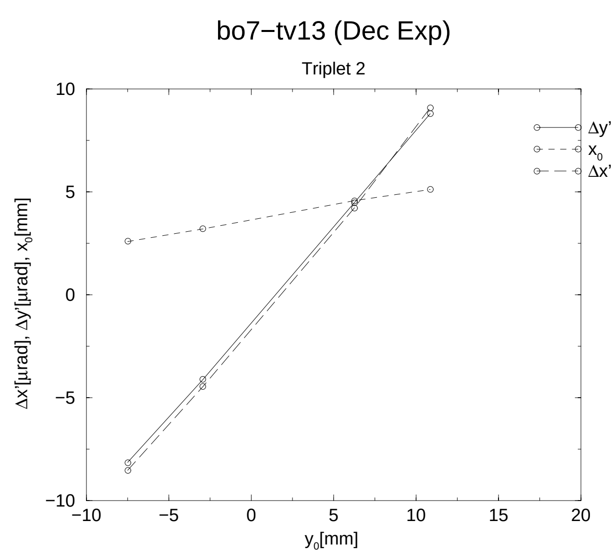

Figure 2: Relation between Magnetic kicks and Beam Position

bits used to find the errors at a particular IR were chosen with optimal phase advanced in one of the planes it was not always possible to meet the same condition in the other plane. It is possible to have a complete control over this problem if a horizontal and vertical dipole corrector are used simultaneously to produce the betatron oscillation.

Table 1: Measured Skew Quadrupole Errors (2001 Run) and skew quadrupole corrector values (all values are given in 10^{-3} 1/m)

Triplet	**Skew Error**	**Corr.**	**Total Triplet**	**Total IR**
7	(-0.001 ± 0.01)	-0.8	-0.8	0.6
8	(0.1 ± 0.07)	1.3	1.4	
9	(-0.04 ± 0.03)	0.35	0.32	1.1
10	(0.13 ± 0.03)	0.65	0.78	
1	(-1.11 ± 0.03)	1	-0.1	0.9
2	(1)	0	1	

Another possible source of error is the slight difference in tunes between the model used to do the analysis and the real tune of the machine. This difference creates a slight slope in the graphs of phase vs “s” in the arcs. A re-tuning of the lattice model would hopefully reduce this tilt in the phase. Doing all previous correction it is not unreasonable to expect measurements of the skew errors with significant figures up to 10^{-5} 1/m.

5 COMPARISON WITH 2000 MEASUREMENTS AND ROLL ANGLES MEASUREMENTS

The 2001 measurements were performed with skew quadrupole correctors set to the values shown in table 1 under the label “Corr.”. The following column of the same table show the errors that would be measured at the triplets if the correctors were off. The next column “Total IR” is just the sum of the triplet error at each IR from the previous column.

Since the RHIC 2000 run skew error measurements were done with the skew quadrupole corrector off, these two last columns are the appropriate one to do comparisons and has been reproduced again in table 2. The other two columns of table 2 show the measurements done from first turn difference orbits in RHIC 2000 run using two different methods: the Orbit Bump method and the Action-Phase Jump method [1]. The Orbit Bump method can give an estimate of the skew error for each triplet while the studies of action and phase performed in the RHIC 2000 run only could give an estimate for the whole IR. There is an excellent agreement between the two runs data within the 10% uncertainty of the measurements.

Table 2: Skew Error Measurements Comparison for Blue Ring 2000 Run vs 2001 Run (All values are given in $10^{-3} 1/m$)

Triplet	**Orbit Bump (2000)**	**Total Triplet (2001)**	**Action and Phase Jump (2000)**	**Total IR (2001)**
7	-0.84	-0.8	0.67	0.6
8	1.32	1.4		
9		0.32	1	1.1
10		0.78		
1	-0.22	-0.1	0.99	0.9
2	1.23	1		

The skew errors have their origin in the quadrupole rolls of the triplets at the IR. During the 2002 shutdown period the rolls of sector 8 quadrupoles (triplets) in Blue and Yellow ring were measured with the results that range between 5 mrad and -1.6 mrad. All 3 roll angles of each triplet can be combined to find a equivalent skew quadrupole error:

$$(kl)_{sc} = \frac{\sum_{i=1}^{3}(-2\frac{\phi_i}{f_i})\sqrt{\beta_x^i\beta_y^i}}{\sqrt{\beta_x^{Trip}\beta_y^{Trip}}} \tag{5}$$

where f_i, ϕ_i, β_x^i, β_y^i correspond to the focal lengths, roll angles, beta functions (in both planes) of each of the quadrupoles that make up the triplet. β_x^{Trip}, β_y^{Trip} are the beta functions at the place where the equivalent skew quadrupole error want to be calculated, in this case at the position of the skew quadrupole corrector.

The equivalent skew quadrupole error for triplet 8 in Blue Ring calculated with formula 5 is 1.6e-3 1/m compared to 1.4e-3 (action-phase jump value in table 2) and 1.3e-3 (Orbit Bump value in table 2). Similarly, for Yellow Ring the calculated value is -1.13e-3 1/m compared to the measured value of 1.1e-3 1/m.Taking into account that errors in the measurements are about 10% there is good agreement between the strength derived from the measured roll angles and the strength derived from orbit based measurements.

It is also possible to study higher order errors with this technique (see equation 4) and experiments were performed with this purpose in the RHIC 2001 run. The results of the analysis of such experiment can be found in [2]

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7 REFERENCES

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