OBSERVATION OF NON-LINEAR EFFECTS IN THE SYNCHROTRON TUNE AT LEP

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Abstract

For the purpose of energy calibration of LEP 2, the synchrotron tune was measured as a function of total accelerating voltage. Since the synchrotron tune also depends on energy loss, this method can be used to extract the beam energy with a very good precision. In the last year of LEP running, the synchrotron oscillations were excited with RF phase noise during the measurements, and a frequency shift was observed. The generic description of the oscillation does not account for large amplitudes. A higher-order approximation reveals a frequency correction to be added to the model if the oscillation enters the nonlinear regime. This paper presents the observations and compares the measurements with the extended model.

1 INTRODUCTION

The precise measurements of the Z and the W boson masses require the knowledge of the centre-of-mass energy at the interaction points. The most precise average beam energy measurement is based on the technique of resonant depolarisation. However, this technique works only in the low energy region (40 – 60 GeV). For the LEP 2 phase extrapolation methods have to be used to obtain the beam energy [1]. The study and analysis of the dependence of the synchrotron tune Q_s on the total accelerating voltage provides a powerful way to extract the beam energy. Details on method and measurements can be found in [2].

In the last year of LEP running, the synchrotron oscillations were excited with RF phase noise during the measurements, and a frequency shift was observed. The generic description of the oscillation does not account for large amplitudes. A higher-order approximation reveals a frequency correction to be added to the model if the oscillation enters the nonlinear regime. In the following, the phenomenology will be described and measurements of the synchrotron tune with excitation with the framework of energy calibration will be compared to an extended model.

2 SYNCHROTRON OSCILLATIONS AT LEP

For a given machine optics the synchrotron tune depends mainly on the total RF voltage V_{RF} and the beam energy. The synchrotron tune is particularly sensitive to the energy for low RF voltages since the relation between Q_s and the total accelerating voltage V_{RF} is given by

$$Q_s^2 = \left(\frac{\alpha_c h}{2\pi E}\right) \sqrt{e^2\, V_{RF}^2 - U_0^2} \qquad (1)$$

where $U_0 = (C_\gamma/\rho)\ E^4$ is the energy loss, α_c the momentum compaction factor, h the harmonic number and ρ the average magnetic radius. Equation (1) is not suited for a high precision energy determination since it assumes that the RF voltage is homogeneously distributed along the ring, that the synchrotron oscillation amplitudes are small, and because it neglects a damping term related to the synchrotron radiation. At LEP 2 however, these assumptions no longer hold true: the RF cavities are concentrated in the four even straight sections and synchrotron radiation losses of the order of a percent of the beam energy are non-negligible. Obviously a more detailed description of Q_s is needed which takes all these effects into account. The contributions and corrections to Eq.(1) needed for an appropriate description of the synchrotron tune are discussed in detail in [2]. Taking all necessary corrections into account, Q_s can finally be expressed as

$$Q_s^4 = \left(\frac{\alpha_c h}{2\pi}\right)^2 \left\{\frac{g^2 e^2 V_{RF}^2}{E_c^2} + M g^4 V_{RF}^4 - \frac{1}{E_c^2}\,\tilde{U}_0^2\right\} \quad (2)$$

where g takes care of the effective voltage seen by the beam and M accounts for the RF distribution. The total energy loss is $\tilde{U}_0 = \frac{C_\gamma}{\rho} E^4 + K$ where K is the sum of all energy losses other than from dipole magnets, C_γ is Sand's synchrotron radiation constant and ρ is the bending radius. The corrected energy

$$E_c \;=\; E\,\left(1 - \frac{1}{\alpha_c}\,\frac{(f_{RF} - f_{RF}^c)}{f_{RF}^c}\right)$$

takes into account momentum offsets $\Delta p/p$ introduced by a difference between the operation frequency f_{RF} and the central frequency f_{RF}^c (tidal deformations of the earth,...) [3].

3 NON-LINEAR SYNCHROTRON OSCILLATIONS

The basic description of Q_s, Eq.(1), is derived from the following expression for the synchrotron frequency Ω

$$\Omega^2 \;=\; \omega_{\rm rev}^2 \left(\frac{\alpha_c h}{2\pi E_0}\right)\, e\, \frac{dV}{d\psi}(\psi_s) \qquad (3)$$

where $\omega_{\rm rev} = 2\pi\ f_{\rm rev}$, E_0 is the beam energy and ψ_s the stable phase angle. This implies, that $dV/d\psi$ is constant, which is valid only in the limit of small oscillation amplitudes. For a longitudinal excitation, this is no longer true, since the oscillation can no longer be considered "point-like" and the variation in $dV/d\psi$ has to

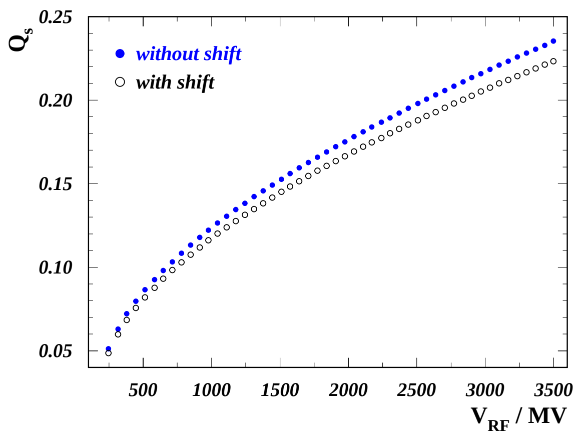

Figure 1: Simulation of synchrotron tune as a function of total accelerating voltage. The evolution of Q_s is shown with and without a frequency shift caused by large amplitude oscillations. The effect is exaggerated by about a factor 10 for illustration purposes.

be taken into account. The derivative of the RF potential, being proportional to $\cos\psi_s$ and therefore to Q_s^2 itself, can be viewed as a driving force of the longitudinal oscillation. A change in this driving force is given by its second derivative and therefore is proportional to $-\cos\psi_s \propto -Q_s^2$. As an illustration, the tangent representing $dV/d\psi$ for small amplitudes can be replaced by a chord for larger amplitudes. This can be viewed as taking the average of the maximal and minimal driving force defined by $\cos(\psi_s \pm \Delta\psi)$, where $\Delta\psi$ represents the amplitude of the phase oscillation induced by the excitation. The average force experienced by the bunch would therefore be $F \propto 1/2[\cos(\psi_s - \Delta\psi) + \cos(\psi_s + \Delta\psi)]$ with a corresponding change in force of $\Delta F \propto 1/2[\cos(\psi_s + \Delta\psi) - 2\cos\psi_s + \cos(\psi_s - \Delta\psi)] \propto \cos''\psi_s \propto -\cos\psi_s$. Therefore $\Delta Q_s^2 \propto -Q_s^2$ and the frequency shift due to the non-linear oscillation is $\Delta Q_s \propto -Q_s$. Figure 1 gives an example of this effect: a simulation of synchrotron tune as a function of total accelerating voltage is shown with and without a frequency shift caused by oscillation amplitudes of about 10 bunch lengths.

4 BEAM ENERGY FROM Q_S

The energy determination is based on a fit of the parameterisation Eq.(2) to the synchrotron tune Q_s measured as a function of the total RF voltage V_{RF}. With the exception of α_c and M which are taken from MAD [4], all parameters are allowed to vary in the fit. External knowledge is incorporated in the fit by introducing individual constraints of the type $(a - a_{\rm nom})^2/\sigma_a^2$ for all parameters in the form of contributions to the χ^2 of the fit, where a stands for a fit parameter and σ_a for its uncertainty. The value the parameter is constrained to is denoted by $a_{\rm nom}$. For very small values of σ_a, the parameter is fixed in the fit; for large values the contribution to the overall χ^2 is negligible and the parameter is free. The parameters representing additional energy losses, voltage calibration and M are constrained in

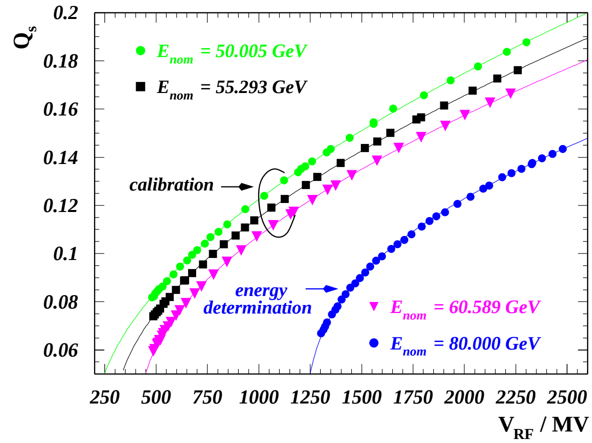

Figure 2: Measured Q_s is shown as a function of V_{RF} for different beam energies. Calibration range and energy determination range are indicated.

the described way. A constraint implemented in this fashion modifies the effective number of degrees of freedom in the fit. A detailed description of this effect can be found in [5]. The uncertainty of each individual Q_s measurement is determined from the scatter of the fitted residuals.

The effective voltage seen by the beam can be significantly different from the sum of all individual cavity voltages due to voltage calibration, phasing and longitudinal alignment errors. A crucial correction to Eq.(1) is therefore the "voltage correction factor" g which translates $V_{RF} \rightarrow g\, V_{RF}$. This factor is considered a constant as function of energy since for all datasets a similar voltage range was used. It can be determined by fits over a given voltage range of separate datasets at beam energies which have been previously measured very precisely with resonant depolarisation. To this end, a scan in g is done, associated with a fit as described above for each value of the voltage calibration factor (g is fixed in this case). The"true" g is found if the energy resulting from the fit corresponds to the reference energy known from resonant depolarisation. The average of the g values obtained in this way for the low energy datasets with its scatter as uncertainty is then used to extract the beam energy of the high energy dataset. Obviously, this energy measurement is strongly correlated to g.

The procedure of g determination and energy extraction is illustrated in Fig. 2, where the measured Q_s is shown as a function of V_{RF} for different beam energies. Calibration range and energy determination range are indicated.

5 IMPACT OF NON-LINEARITIES ON ENERGY DETERMINATION

A shift of Q_s proportional to $-Q_s$ such as the one in the example given in Fig. 1 will be absorbed by the voltage calibration factor g in the analysis procedure, if not taken into account properly. Since Q_s is a function of the beam energy, the shift is also energy dependent and will become smaller with increasing energy (and therefore decreasing Q_s) for a given RF voltage. This unavoidably leads to an

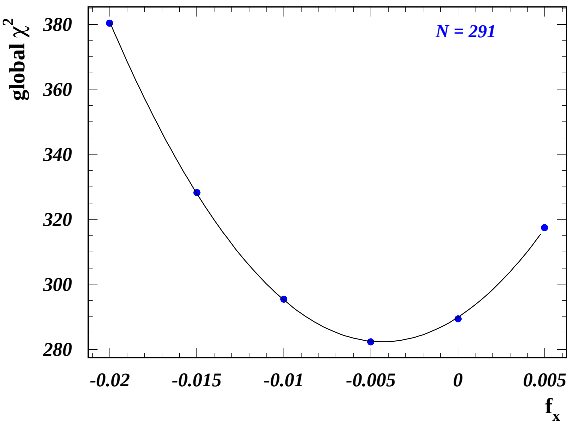

Figure 3: Global χ^2 as a function of the proportionality factor of the frequency shift f_x determined from all low energy datasets with RF phase noise. The total number of individual Q_s measurements in these datasets is 291.

apparent dependence of the voltage calibration factor g on the beam energy.

An oscillation amplitude of the order of one bunch length, for example, would generate a frequency shift of about $-0.005 \cdot Q_s$ and would lead to an increase of g of about 10^{-4}/GeV.

To extract the proportionality factor of the frequency shift, f_x, from the measurements done with longitudinal excitation in the year 2000, a global fit over all low energy datasets ("calibration range") is performed. The total number of individual Q_s measurements in these datasets is 291. The full procedure, including the determination of the voltage calibration factor as described in the previous section, is applied to the low energy datasets for different values of the proportionality factor of the frequency shift f_x. The correction for the shift in Q_s is applied in the form $Q_s^{\mathrm{true}} = Q_s^{\mathrm{meas}}/(1 + f_x)$. The global χ^2_{glob} as a function of f_x is defined as the sum over the individual χ_i^2 obtained from the voltage calibration fits using Eq.(2) of all m=11 low energy datasets:

$$\chi^2_{\mathrm{glob}}(f_x) = \sum_{i=1}^{m} \chi_i^2(f_x). \quad (4)$$

Figure 3 shows the $\chi^2_{\mathrm{glob}}(f_x)$ determined in this way for different values of f_x. From this dependence, the global proportionality factor of the frequency shift can easily be determined to be

$$f_x^{\mathrm{glob}} = -(0.0043 \pm 0.0016).$$

This shift factor corresponds roughly to an oscillation amplitude of one bunch length. From the three high energy datasets acquired with excitation by RF phase noise, the beam energy can be extracted and compared with the reference energies (determined by calibrated NMR measurements, see [1]). The error on this energy difference is composed of the statistical uncertainty and the dominant uncertainty imposed by the determination of f_x. The latter is obtained by performing the energy fit at $f_x + \Delta f_x$ and $f_x - \Delta f_x$ and assigning half of the difference of the resulting beam energies as uncertainty. Under the assumption of fully correlated errors, the average energy shift relative to the reference energy is $\Delta E_{\mathrm{beam}} = -(45 \pm 37)$ MeV.

Measurements in earlier years without excitation are not subject to a frequency shift correction. This is because the energy uncertainty due to "natural" longitudinal oscillation (which is much smaller than the excited oscillation as indicated by streak camera observations) is absorbed in the uncertainty of the determination of the voltage calibration factor. As described previously, the average g is obtained from the set of low energy datasets belonging to one high energy dataset. In this case, g does not show an apparent dependence on beam energy as for measurements with longitudinal excitation.

6 SUMMARY

In the framework of energy calibration of LEP 2, the synchrotron tune was measured as a function of total accelerating voltage. From these measurements, the beam energy can be extracted with a fit of a Q_s model adapted for the conditions at LEP 2. Since the results of these fits depend strongly on the effective voltage seen by the beam, a voltage calibration is mandatory. In the last year of LEP running, the synchrotron oscillations were excited with RF phase noise during the measurements, and a frequency shift was observed. Oscillation amplitudes of the order of about one bunch length, however, produce a shift in synchrotron frequency proportional to Q_s that will appear in the analysis as an apparent dependence on beam energy of the otherwise constant voltage calibration factor. The proportionality factor of the frequency shift could be determined and was found to correspond to an oscillation amplitude of one bunch length. Taking the shift into account, beam energies were determined and compared to the reference energy measurement.

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8 REFERENCES

[1] A. Blondel et al. Evaluation of the LEP centre-of-mass energy above the W-pair production threshold. CERN-EP 98-191, CERN, 1998.

[2] A.-S. Müller. *Precion Measurement of the LEP Beam Energy for the Determination of the W Boson Mass*. PhD thesis, Universität Mainz, Shaker Verlag, 2000.

[3] L. Arnaudon et al. Effects of Terrestrial Tides on the LEP Beam Energy. In *NIM A357*, pages 249–252, 1995.

[4] H. Grote and F. C. Iselin. The MAD Program, User's Reference Manual. SL Note 90-13 (AP) (Rev. 4), CERN, March 1995.

[5] A.-S. Müller. Degrees of Freedom Determination in Accelerator Physics Optimisation Problems Subject to Global Constraints. In *Proceedings of the 9th European Particle Accelerator Conference*, 2002.