TRANSITION AND DIFFRACTION RADIATION BY RELATIVISTIC ELECTRONS IN A PRE-WAVE ZONE

S.N. Dobrovolsky, N.F. Shul'ga, NSC “KIPT”, Kharkov, Ukraine

Abstract

The long-wavelength transition radiation from the ultrarelativistic bunched electrons on the thin metallic transverse-limited target has been considered. We have investigated the properties of transition radiation and whole electromagnetic energy flux in a pre-wave zone. The general formula for spectral-angular density of electromagnetic energy flux through the small-size detector in the "forward" and "backward" direction is obtained and analysed. We have showed that the angular distribution of electromagnetic energy flux in the "pre-wave zone" is strongly distorted in comparison to the case of transverse-unlimited detector.

1 INTRODUCTION

The long-wavelength transition radiation by relativistic electrons has various applications in the modern physics of accelerated particles. Coherent transition radiation emitted by short bunches of charged particles that passed through a thin plate, is a powerful tool for non-destructive beam diagnostics. On the basis of transition and Cherenkov radiation emitted by fast particles in the multilayer plates, the tabletop soft X-ray sources are developed. Transition radiation has been used for detection of high-energy particles and diagnostics in the LHC-based experiments. Recently, the coherent transition and Cherenkov radiation has been used for electromagnetic cascades detection in the number of experiments dealing with high-energy cosmic neutrino detection problem.

For long-wavelength region the transition radiation from relativistic particles is formed in the space region, which has macroscopic sizes. It can lead to the situation when both transversal and longitudinal sizes exceed the size of experimental setup. It is well-investigated situation when extended detector is placed in the limits of "coherent length" (longitudinal size of formation region), where the interference between radiation fields and electron’s own field is occurred. At typical angles $\vartheta \approx \gamma^{-1}$ of transition radiation the "coherent length" is determined by the relation $l_{coh} \approx 2\gamma^2\lambda$ [1], where γ is a Lorenz-factor of electron and λ is a radiated wavelength. For example, it is obvious that interference effects will happen within tens of meters for far-infrared transition radiation by 100-200MeV electrons. Recently, we called attention to the fact, that the transversal size of formation region $\gamma\lambda$ can reach macroscopic values too [2]. Therefore, the diameter of formation region may be comparable with size of target and may strongly affect the spectral-angular density of transition radiation. Transition radiation intensity in this case should be strongly depended on the transversal size of target in the millimeter and submillimeter waverange.

The sufficiently different situation can happen when we have small detector, which is placed not far from the target. Because the transition radiation is formed on the target's surface, the effect of interference between radiation from various elementary radiators is possible. Therefore, the conditions of near-field and far-field radiation will be determined by diameter of extended source of radiation. The longitudinal length of near-field radiation region has the same value as "coherent length" in the "forward” direction. But this interference effect will occur even for "backward" transition radiation alone. Longitudinal size of near-field region for "backward" direction will be much larger, than "backward coherent length", which has size nearly λ. The investigation of such effect in the case of infinite target was performed recently for "backward" radiation in small-angle approximation [3]. In present paper we will consider transversal spatial distribution of near-field "backward" transition radiation and "forward" electromagnetic energy flux through small detector, which is placed in the near-field region for various distances from the target and arbitrary angles of observation.

2 ELECTROMAGNETIC FIELDS AND RADIATION INTENSITY

We will consider distribution of electromagnetic fields after electron passage through a target. We will investigate the typical case of normal electron passage through the thin ideally conducted plate. We will consider the case when plate thickness is less than radiated wavelength λ but much more than electron's field "penetration length" into a metal. We assume that the electron is moving along the OZ axis and target is positioned in the plane $z = 0$. Let's write in space $z > 0$ and $z < 0$ the electric fields $\mathbf{E}^{+}(\mathbf{r},t)$ and $\mathbf{E}^{-}(\mathbf{r},t)$ in the form

$$\mathbf{E}^{+}(\mathbf{r},t) = \mathbf{E}^{(e)}(\mathbf{r},t) + \mathbf{E}'^{(+)}(\mathbf{r},t)\,, \quad \mathbf{E}^{-}(\mathbf{r},t) = \mathbf{E}'^{(-)}(\mathbf{r},t)\,, \tag{1}$$

where $\mathbf{E}'^{(\pm)}(\mathbf{r},t)$ is a "forward" and "backward" radiation fields and $\mathbf{E}^{(e)}(\mathbf{r},t)$ is an electron’s own field**,** that uniformly moved with velocity $\mathbf{v}$ in the vacuum. The Fourier component of the $\mathbf{E}^{(e)}(\mathbf{r},t)$ field with respect to time is determined for relativistic electrons, with accuracy of γ^{-1}, by the expression

$$\mathbf{E}_{\omega}^{(e)}(\mathbf{r}) \approx \frac{\boldsymbol{\rho}}{\rho}\frac{e\omega}{v^2\gamma}2K_1\left(\frac{\omega\rho}{v\gamma}\right)\exp\left(iz\frac{\omega}{v}\right), \qquad (2)$$

where e - is an electron's charge, $v = |\mathbf{v}|$, $\boldsymbol{\rho}$ - transversal coordinate ($\mathbf{r} = \boldsymbol{\rho} + z\cdot\mathbf{e_z}$), K_1 – McDonald's function of first kind.

Target is an ideally reflecting screen, and therefore we can write in the plane $z = 0$ next conditions for "forward" and "backward" radiation fields $\mathbf{E}'^{(\pm)}_{\omega}(\mathbf{r})$

$$\mathbf{E}'^{(\pm)}_{\omega}(\mathbf{r}) = -\Theta(a-\rho)\mathbf{E}_{\omega}^{(e)}(\mathbf{r}) \;, z = 0, \qquad (3)$$

where $\Theta(x)$ - is a Heaviside step function: $\Theta(x)$=1 , if x≥0 and $\Theta(x)$=0, if x<0. The "forward" and "backward" radiation fields are propagating in positive and negative direction of OZ axis as a wave packet of free electromagnetic waves. To determine radiation fields in the whole "forward" and "backward" space we have to expand the bounded wave packets (3) to the full Fourier components and perform reverse Fourier transformation into space. By taking into account the (2) and (3) equations we will obtain the expression for the "forward" and "backward" radiation fields in the point $\mathbf{r}$

$$\mathbf{E}'^{(\pm)}_{\omega}(\mathbf{r}) = -\frac{2e}{v}\frac{\boldsymbol{\rho}}{\rho}\int_0^{\infty}\chi^2 d\chi \frac{J_1(\chi\rho)\cdot\exp\left(\pm iz\sqrt{\left(\omega/c\right)^2 - \chi^2}\right)}{\chi^2 + \left(\omega/v\gamma\right)^2} . (4)$$

Here J_1 is a Bessel function of first kind. This expression is correct for all distances greater than radiated wavelength from the target.

Now we can consider the electromagnetic energy flux, which crosses over the entire observation time through the small plate that is perpendicular to the vector $\mathbf{r}$ and placed at the distance r from the target. We should calculate the value of Poynting vector of electromagnetic flux

$$S = \frac{c}{4\pi}\int(\mathbf{E}\times\mathbf{H})\mathbf{n}\; d^2\sigma dt, \qquad (5)$$

where $\mathbf{n} = \mathbf{r}/r$ and $d^2\sigma$ is an elementary area with normal vector $\mathbf{n}$. Here the integration is over the surface of detecting plate and the time of observation. Now we can write "backward" transition radiation intensity and summary energy flux in the "forward" direction on a unit of frequency and on the unit of solid angle by the following equation

$$\frac{dS^{\pm}}{d\omega d\Omega} = \frac{cr^2}{4\pi^2}\left|\mathbf{E}_{\omega}^{e}(\boldsymbol{\rho},z)\cdot\Theta(z) + \mathbf{E}'^{(\pm)}_{\omega}(\boldsymbol{\rho},z)\right|^2 \;, \omega > 0, (6).$$

Here $d\Omega = \sin\vartheta d\vartheta d\varphi$ (ϑ and φ - is a polar and azimuthal angles) and the ϑ angle is counted out negative and positive directions of OZ-axis for "backward" and "forward" radiation accordingly. After substitution of electron's own field and radiation field from (2) and (4) into (6) the following expressions for spatial distribution of "forward" and "backward" electromagnetic energy flux are obtained

$$\begin{aligned}\frac{dS^{\pm}}{d\omega d\Omega} = \frac{e^2}{\beta^4 c\pi^2}y^2\cos\vartheta\times\Big|\gamma^{-1}K_1\left(y\sin\vartheta\gamma^{-1}\right)\cdot\Theta(z) - \\ -\beta\int_0^1 x^2 dx\frac{J_1(xy\sin\vartheta)}{x^2+\gamma^{-2}}\exp\left(iq\beta^{-1}(\beta\sqrt{1-x^2}-1)\right)\Big|^2\end{aligned}, (7)$$

where $y = \omega\cdot r\cdot c^{-1}$, $q = y\cos\vartheta$, $u = \omega\cdot a\cdot(c\cdot\gamma)^{-1}$, $\beta = v\cdot c^{-1}$.

The obtained expressions describe the energy flux in "forward" and "backward" directions through the small detector at the point. $\mathbf{r}$. It is obvious that "forward" energy flux is differing from "backward" energy flux by the presence of electron's own field term.

3 "BACKWARD" AND "FORWARD" ELECTROMAGNETIC ENERGY FLUX

Let's analyze the formula (7). We will consider the behavior of intensity of electromagnetic energy flux for various distances from the target.

First, we will analyze and simplify the value Φ under various distances from the target.

$$\Phi = \int_0^1 x^2 dx\frac{J_1(xy\sin\vartheta)\cdot\exp\left(iq\beta^{-1}(\beta\sqrt{1-x^2}-1)\right)}{x^2+\gamma^{-2}} . (8)$$

For distances $y >> \gamma$ (i.e. not closely to the target) we can perform the expansion in series for small x in the expression (8) and following

$$\Phi = \gamma^{-1}\exp\left(-i\xi\beta^{-1}\right)\cdot\int_0^{\gamma}u^2 du\frac{J_1(u\eta)}{u^2+1}\exp\left(-iu^2\xi\right), \quad (9)$$

where $\xi = \frac{y\cos\vartheta}{2\gamma^2}$, $\eta = \frac{y\sin\vartheta}{\gamma}$. It is necessary to note that ξ is a ratio between longitudinal distance and longitudinal "coherent length" $l = 2\gamma^2 c/\omega$ and η is a ratio between transversal distance and effective diameter of "formation zone" $\lambda\gamma$. Thus, the behavior of spatial distribution depends on "near-field" zone sizes.

Under conditions $\xi << 1$ and $\frac{\eta^2}{4\xi} \le 1$ we can obtain the final expression for the value Φ by some calculations

$$\Phi = \frac{\exp\left(-i\xi\beta^{-1}\right)}{\gamma\eta}2i\exp\left[i\frac{\eta^2}{8\xi}\right]\sin\left(i\frac{\eta^2}{8\xi}\right). \qquad (10)$$

Taking into account the formula (10), let's express the formula (7) for "backward" direction. Thus, under conditions $\vartheta << 1, r\omega\cos\vartheta(2c\gamma^2)^{-1} < 1$, we can write the distribution of "backward" transition radiation intensity

$$\frac{dS^-}{d\omega d\Omega} \approx \frac{4e^2}{c\pi^2\beta^4}\frac{\cos\vartheta}{\sin^2\vartheta}\sin^2\left(\frac{y\sin^2\vartheta}{4\cos\vartheta}\right). \qquad (11)$$

This formula gives the spatial distribution of radiation intensity in the pre-wave zone, not closely to the target. We can conclude from (11) that angular maximum of transition radiation intensity is located at angle $\vartheta \approx \sqrt{4y^{-1}}$. It is substantially different from well-known case of peak under γ^{-1}. This result is differing from the case of interference between electron's own field and radiation fields too. Here we are observing the interference between radiation from various elementary places of target's surface, i.e. the "transition radiation self-interference". This effect occurs even for "backward" direction. The near-field condition $2\gamma^2\lambda > z$ is determined by the size of effective diameter of formation zone $\lambda\gamma$ and characteristic angle of radiation from each elementary radiator $\vartheta_{eff} \approx \gamma^{-1}$. For far-field distances $2\gamma^2\lambda << z$ the whole surface can be considered as a pointed source of radiation and vice versa, but at near-field zone we must consider it as an extended source.

Spectral-angular electromagnetic energy distribution in the "forward" direction for the distances $2\gamma^2\lambda > z > \gamma\lambda$ is described by the following formula

$$\frac{dS^+}{d\omega d\Omega} \approx \frac{e^2}{c\pi^2\beta^4}y^2\cos\vartheta \times$$
$$\times\left|\gamma^{-1}K_1\left(y\sin\vartheta\gamma^{-1}\right) - \beta\frac{2}{y\sin\vartheta}\sin\left(\frac{y\sin\vartheta}{4\cos\vartheta}\right)\right|^2. \qquad (12)$$

The expression (12) describes the "forward" spectral-angular distribution of energy flux at the whole observation time through the dot-like detector that is placed near-field region. Given density is combined by the energy flux from electron's own field and by the transition radiation energy flux. Fig.1 illustrates the spatial distribution of "forward" energy flux for different distances from the target. For small-angle region (i.e., when detector is placed closely to the electron trajectory) the term that corresponds to electron's own field contribution will dominate. For large-angle region (i.e., when detector is located farther than $\lambda\gamma$ from electron trajectory) the radiation field contribution for the summary energy flux will determine the distribution behavior. At intermediate angles we can observe sufficient distortion, which is caused by both the interference of radiation from elementary radiators on the radiated surface (target) and the interference between own and transition radiation fields within the "coherent length".

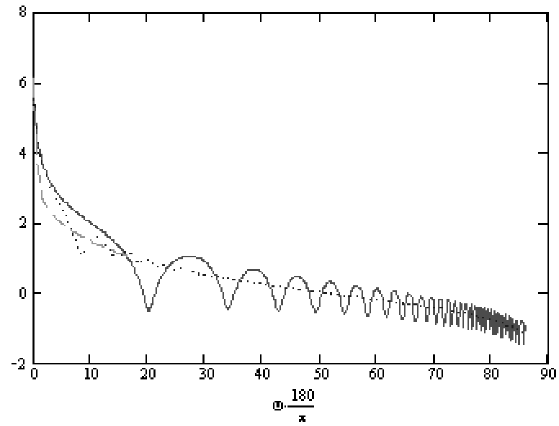

Fig.1 Spectral-angular density of electromagnetic energy flux $W(\Theta)=c\pi^2e^{-2}(dS/d\omega d\Omega)$ in the "forward" direction for electron with $\gamma = 30$. Solid curve corresponds to the case when $y = 50$, dotted curve - $y = 300$ and dashed curve - $y = 3000$.

4 CONCLUSIONS

The long-wavelength transition radiation from the ultrarelativistic electrons on the thin metallic transverse-limited disk has been considered. The spectral-angular distribution of "forward" electromagnetic energy flux through the small detector is analyzed. It is shown that interference between radiation fields from various elementary radiators and interference between electron's own field and radiation fields are occurred. We have showed that the angular distribution of electromagnetic energy flux in the "pre-wave zone" is strongly distorted as compared to the case of transverse-unlimited detector.

A number of experiments concerning long-wavelength transition radiation of relativistic electron bunches were performed in the last years [4-5]. For the typical experimental conditions (electrons with $\gamma \approx 10 \div 100$, long-wavelength transition radiation with $\lambda \approx 1 \div 10$ *cm.*) both the longitudinal and transversal sizes of "pre-wave zone" are the macroscopic values and can be compared with distance to the detecting setup. Thus, the considered interference effects can play very important role in such experiments. Therefore, it is necessary to take into account this effect under experimental data analysis.

5 REFERENCES

[1] V.L. Ginzburg, V.N. Tsytovich, "Transition radiation and transition scattering" Adam Hilger, Bristol, 1990.

[2] N.F. Shul'ga, S.N. Dobrovolsky, "Theory of relativistic electron transition radiation in a thin metal target", JETP, 90, pp.579-583, 2000.

[3] V.A. Verzilov, "Transition radiation in the prewave zone", Phys. Lett. A 273, pp.135-140, 2000.

[4] X.Artru et al. "Resolution power of optical transition radiation: theoretical considerations" NIM B145, pp.161-168, 1998.

[5] Y.Shibata, et al. "Coherent transition radiation in the far-infrared region", Phys. Rev. E49, pp.785-793, 1994.