PRINCIPLE OF CORRECTION OF ASYMMETRIC MAGNETIC FIELDS IN BENDING MAGNETS *

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Abstract

The generation of a high quality electron beam by a race-track microtron (RTM) requires highly precise magnetic fields in the two reversing magnets. At the RTM cascade MAMI (Mainz Microtron), a precision of 10^{-4} for the vertical field component B_y was achieved by symmetrical surface coils placed at the upper and lower pole surface in each of these magnets. For the Harmonic Double Sided Microtron [1], the fourth stage of MAMI, the correction must be extended to asymmetric field errors. The more complicated machining of the pole surfaces of its inhomogeneous end magnets [2] leads to a higher risk for the distortion of the mid plane symmetry. In addition, the correction of deflection errors by external dipoles is more difficult because the path length in the dispersion region decreases with the turn number. Therefore, a numerical method has been developed to calculate the complete set of symmetric and antisymmetric field components from a measurement of the B_y distribution on both sides of the midplane. From this the distribution of the field components parallel to pole surfaces is extracted and compared with those of the ideal magnet field configuration. The difference determines the necessary current distribution to correct the field errors. The method has been tested successfully in 3D-simulations by means of TOSCA.

1 PRINCIPLE

The fundamental equations of magnetostatics are given by [3]

$$\vec{\nabla}\vec{B} = 0 \tag{1}$$
$$\vec{\nabla}\times\vec{H} = \vec{j} \tag{2}$$

with the magnetic field $\vec{H}=\epsilon_0 c^2\vec{B}-\vec{M}$, magnetic induction $\vec{B}$ (in the following called field) , magnetization $\vec{M}$ and current density $\vec{j}$. From eq.(1) it follows that a vector potential $\vec{A}$ exists such that $\vec{B}=\vec{\nabla}\times\vec{A}$. In the case of hard ferromagnetic material ($\vec{j}=0$, $\vec{M}$ known) eq.(2) leads to the Poisson equation for $\vec{A}$ in the Coulomb gauge

$$\vec{\nabla}^2\vec{A}_M = -\frac{1}{\epsilon_0 c^2}\vec{j_M} \tag{3}$$

with the current density $\vec{j_M}=\vec{\nabla}\times\vec{M}$ caused by the magnetization. If the magnetized Volume V is bounded by the surface S eq.(3) is solved by [3]

$$\vec{A}_M(\vec{x}) = \int\limits_V \frac{\vec{\nabla}'\times\vec{M}(\vec{x'})}{4\pi\epsilon_0 c^2|\vec{x}-\vec{x'}|}d^3x' + \oint\limits_S \frac{\vec{M}(\vec{x'})\times\vec{n'}}{4\pi\epsilon_0 c^2|\vec{x}-\vec{x'}|}dS \tag{4}$$

For homogeneous magnetization of the volume V, as supposed in the following, the volume integral vanishes and $\vec{A}_M$ is given by the surface integral. Therefore, to correct the field $\vec{B}$, the magnetization $\vec{M}_E$ at the pole face must be found out and brought into coincidence with the design value $\vec{M}_D$. This requires a correction current density

$$\delta\vec{j} = \vec{\nabla}\times(\vec{M}_D-\vec{M}_E) = \epsilon_0 c^2\vec{\nabla}\times(\vec{B}_D-\vec{B}_E) \tag{5}$$

which is defined by the curl of the difference of the field at the pole face $\vec{B}_E$ and the design value $\vec{B}_D$. The field $\vec{B}_E$ can be calculated if $\vec{B}$ is known in some reference plane, e.g the midplane of the magnet, s. section 3.

2 IDENTIFICATION OF ANTISYMMETRIC FIELD COMPONENTS

Because of the strong B_y component of 10^4 G in the midplane of the HDSM bending magnets it is difficult to measure the horizontal components B_x and B_z which are expected to be in the range of a few Gauss. Therefore, a numerical method has been developed to calculate the distribution of $\vec{B}$ in the midplane from measurements [4]. In a current and material free region eq.(2) becomes

$$\vec{\nabla}\times\vec{B} = 0 \ . \tag{6}$$

Partial differentiation of eq.(1) with respect to x yields

$$\frac{\partial^2 B_x}{\partial x^2}+\frac{\partial^2 B_z}{\partial x\partial z} = -\frac{\partial^2 B_y}{\partial x\partial y} \ . \tag{7}$$

Using $\partial B_x/\partial z=\partial B_z/\partial x$ from eq.(6) one gets

$$\frac{\partial^2 B_x}{\partial x^2}+\frac{\partial^2 B_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial x\partial y} \tag{8}$$

and in the same way

$$\frac{\partial^2 B_z}{\partial x^2}+\frac{\partial^2 B_z}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z\partial y} \ . \tag{9}$$

Inspecting eq.(8) and (9) in an area Ω in the midplane of the magnet (and filled by the field of the magnet), eq.(8) and (9) describe a Dirichlet boundary value problem if B_x and B_z are known on the boundary $\partial\Omega$. In the following it is assumed that $B_x(x,0,z)=0$, $B_z(x,0,z)=0$ on $\partial\Omega$.

* Work supported by Deutsche Forschungsgemeinschaft (Graduiertenkolleg “Physik und Technik von Beschleunigern”)
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The solutions of eq. (8) and (9) can be found numerically. For that, the midplane is covered with a mesh with $N_x \times N_z$ points which are also measuring points for B_y. The mesh size is $h_x = h_z$ and $x_i = i \cdot h_x$, $z_j = j \cdot h_x$. The discretization of the left side of eq.(8) is given by

$$\frac{\partial^2 B_x(x_i,0,z_j)}{\partial x^2} + \frac{\partial^2 B_x(x_i,0,z_j)}{\partial z^2} \approx \frac{1}{h_x^2}\Big[B_x(x_{i-1},0,z_j) + B_x(x_i,0,z_{j+1}) + B_x(x_{i+1},0,z_j) + B_x(x_i,0,z_{j-1}) - 4B_x(x_i,0,z_j)\Big] \tag{10}$$

and analogous for B_z in eq.(9).
The second derivative $\partial^2 B_y/\partial x \partial y$ in the midplane on the right side of eq.(8) can be obtained from the measured field component B_y in a distance h_y above and below the midplane on the mesh points:

$$\frac{\partial^2 B_y(x_i,0,z_j)}{\partial x \partial y} \approx \frac{1}{4h_x h_y}\Big[B_y(x_{i+1},h_y,z_j) - B_y(x_{i+1},-h_y,z_j) + B_y(x_{i-1},-h_y,z_j) - B_y(x_{i-1},h_y,z_j)\Big] \tag{11}$$

and analogous $\partial^2 B_y/\partial z \partial y$ for eq.(9). Taking into account the mentioned boundary conditions the discretization of eq.(8) results in a system of simultaneous linear equations

$$\mathbf{D}\vec{b_x} = \vec{\zeta_x} \tag{12}$$

with (using $\partial_{xy} := \frac{\partial^2}{\partial x \partial y}$)
$\vec{b_x}^T = (B_x(x_1,0,z_1), ..., B_x(x_{Nx},0,z_1), ..., B_x(x_1,0,z_{N_z}), ..., B_x(x_{N_x},0,z_{N_z})),$
$\vec{\zeta_x}^T = -h_x^2(\partial_{xy}B_y(x_1,0,z_1), ..., \partial_{xy}B_y(x_{N_x},0,z_1), ...\partial_{xy}B_y(x_1,0,z_{N_z}), ..., \partial_{xy}B_y(x_{N_x},0,z_{N_z}))$
and a $(N_x \cdot N_z) \times (N_x \cdot N_z)$ block tridiagonal matrix

$$\mathbf{D} = \begin{pmatrix} \mathbf{D_{N_x}} & -\mathbf{1_{N_x}} & \cdots & \mathbf{0} \\ -\mathbf{1_{N_x}} & \mathbf{D_{N_x}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\mathbf{1_{N_x}} \\ \mathbf{0} & \cdots & -\mathbf{1_{N_x}} & \mathbf{D_{N_x}} \end{pmatrix} \tag{13}$$

wich contains the $N_x \times N_x$ identity matrix $\mathbf{1_{N_x}}$ and the $N_x \times N_x$ tridiagonal matrix

$$\mathbf{D_{N_x}} = \begin{pmatrix} 4 & -1 & \cdots & 0 \\ -1 & 4 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ 0 & \cdots & -1 & 4 \end{pmatrix} . \tag{14}$$

The unknown field component B_x in the midplane is found by multiplication of eq.(12) with the inverse matrix $\mathbf{D}^{-1}$. In the same way one gets the field component B_z.

3 CORRECTION OF ANTISYMMETRIC FIELD ERRORS

In a charge and material free region $\vec{B}$ can be derived from a magnetic scalar potential ψ that must fulfil the Laplace equation $\Delta\psi = 0$ according to

$$\vec{B} = -\vec{\nabla}\psi . \tag{15}$$

Usually ψ is expressed in form of a power series

$$\psi(x,y,z) = \sum_{k,l \geq 0} a_{k,l}(z) \frac{x^k}{k!}\frac{y^l}{l!} . \tag{16}$$

Taking into account only terms up to second order and expanding also the coefficients $a_{k,l}(z)$ into a power series, the potential ψ in the neighbourhood of a mesh point $\vec{p} = (x_i, 0, z_j)$ is given by

$$\begin{aligned}\psi(x,y,z) \;=\; & a_{100}(x-x_i) + a_{010}y + a_{001}(z-z_j) \\ & + \frac{1}{2}a_{200}(x-x_i)^2 + \frac{1}{2}a_{020}y^2 + \frac{1}{2}a_{002}(z-z_j)^2 \\ & + a_{110}(x-x_i)y + a_{101}(x-x_i)(z-z_j) \\ & + a_{011}y(z-z_j)\end{aligned} \tag{17}$$

with (using $\partial_x := \frac{\partial}{\partial x}$, $\partial_z := \frac{\partial}{\partial z}$)
$a_{100} = B_x(\vec{p})$, $a_{010} = B_y(\vec{p})$, $a_{001} = B_z(\vec{p})$
$a_{200} = \partial_x B_x(\vec{p})$, $a_{020} = -a_{200} - a_{002}$, $a_{002} = \partial_z B_z(\vec{p})$
$a_{101} = \partial_z B_x(\vec{p})$ $a_{110} = \partial_x B_y(\vec{p})$, $a_{011} = \partial_z B_y(\vec{p})$.
The partial derivatives can be approximated by difference quotients of the calculated (B_x, B_z) and measured field components (B_y) in the midplane. The required $\vec{B}$ at the pole surface (x_i, y_{pole}, z_j), cf. section 1, is calculated by inserting eq.(17) into eq.(15). According to eq.(5) the necessary surface current density to correct the field errors is given by

$$\sigma_x = \epsilon_0 c^2 (B_{z,D} - B_{z,E}) \quad \sigma_z = -\epsilon_0 c^2 (B_{x,D} - B_{x,E}) \tag{18}$$

for the upper and with opposite signs for the lower pole face. Here the curvature of the pole surface is neglected for simplification. For the current distribution one gets

$$I(x,z) = \int_0^x ds\sigma_z(s,z) - \int_0^z ds\sigma_x(0,s) . \tag{19}$$

4 SIMULATIONS WITH TOSCA

In order to check the method an asymmetrical field error in the HDSM magnet was simulated with TOSCA by a small deformation of the upper pole surface, see Fig. 1.

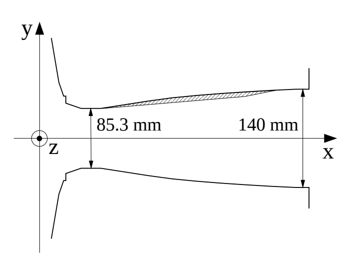

Figure 1: Scheme of deformation. The upper pole surface was lowered by adding a 0.1 mm thick iron layer tapered in z-direction (indicated by the hatched area).

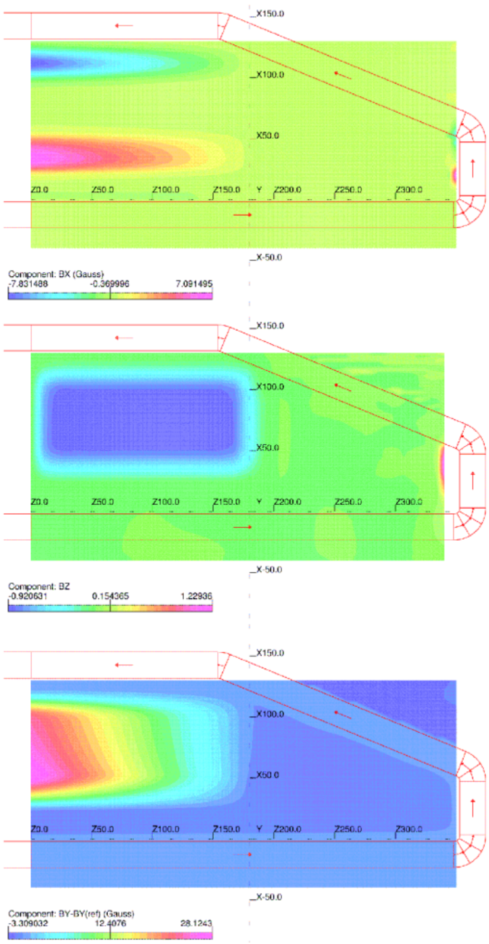

Figure 2: Field components B_x, B_z and $B_y - B_{y,ref}$ in the midplane caused by the pole face deformation shown in Fig. 1.

It results in field components B_x, B_z in the midplane with max. values of 8 G resp. 1 G. The difference between B_y and the reference field $B_{y,ref}$ (ideal symmetry) amounts up to 28 G, see Fig.2. For the calculation $N_x = 200$, $N_z = 700$ and a mesh size of $h_x = h_z = 1$ cm was chosen. The B_y component in the midplane as well as $h_y = 2$ cm above and below the midplane was taken from the TOSCA-simulation. With the numerical procedure explained in section 2 the components B_x and B_z were calculated in the midplane. The results of the TOSCA-simulation could be reproduced within an accuracy of 5%. As pointed out in section 3 the field $\vec{B}$ at the pole face, the surface current density and finally the correction current were calculated. Figure 3 shows the resulting correction coil for the upper

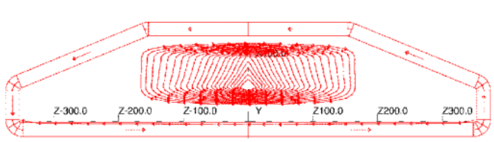

Figure 3: Main coil and calculated correction coil for compensation of simulated asymmetric field errors.

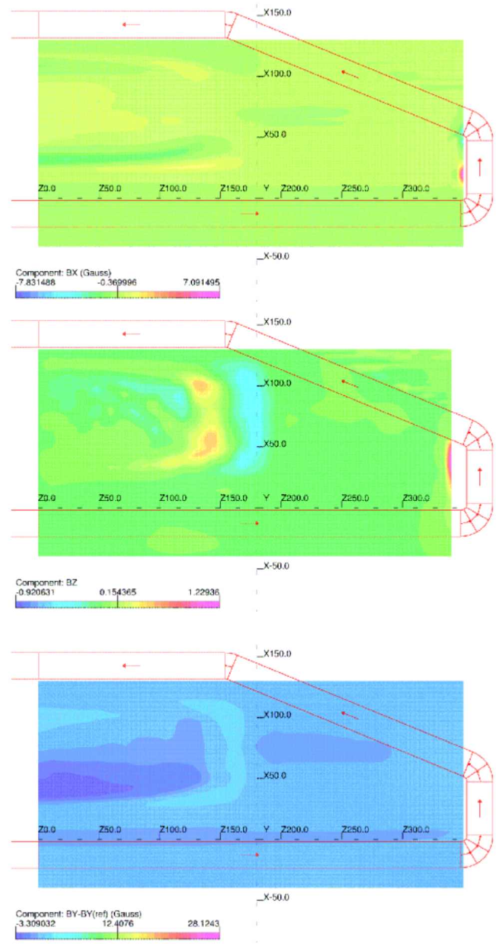

Figure 4: Field components B_x, B_z and $B_y - B_{y,ref}$ in the midplane using the correction coil shown in Fig. 3.

pole for the asymmetric field errors. Within the accuracy of the calculations no correction coil is necessary for the lower pole. In Fig.4 the corrected components B_x, B_z and $B_y - B_{y,ref}$ are presented as a result of a TOSCA simulation with correction coil (switched on). The field errors in the midplane could be reduced by about a factor of 10.

5 REFERENCES

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