3D MULTI-FREQUENCY FEL SIMULATIONS WITH THE GENERAL PARTICLE TRACER CODE

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Abstract

The 3D General Particle Tracer (GPT) code has been extended to perform multi-frequency multi-pass free-electron laser (FEL) simulations. The new model is based on axisymmetric multiple Gaussian waves propagating in free space, without averaging over the undulator period. The field equations are integrated simultaneously with the equations of motion of the particles, making the model ideally suited to calculate electron beam phase-space distributions during and after FEL interaction. Due to the from-first-principles approach of the model, it is useful to a variety of radiation problems. The model and a proof-of-principle test are presented.

1 INTRODUCTION

The General Particle Tracer (GPT) [1,2] is a general package for the calculation of charged particle trajectories through electromagnetic fields in 3D. It employs a time-domain, adaptive stepsize fifth-order embedded Runge-Kutta method [3] and is suitable for a wide range of applications. The GPT code has the ability to solve user-defined additional differential equations while tracing the charged particles. This feature can now be used for the simulation of self-consistent FEL interaction, without averaging over a wiggler period.

The FEL radiation profile is modeled by the `G00mf` element as an orthogonal set of zero order Gaussian waves [4] between two curved mirrors at a distance L, as shown in Figure 1.
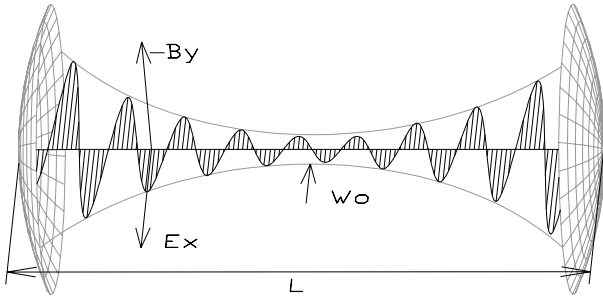

Figure 1: Electric field of two interfering Gaussian waves between two mirrors with a waist.

The amplitudes and phases of all modes, are calculated self-consistently in time-domain: The trajectories of the tracked particles are affected by the radiation field and the amplitude and phases of the radiation are affected by the particle trajectories. This approach produces accurate results for the radiation yield, while simultaneously

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tracking the 6D phase-space evolution of the electron beam. The model can be used for a variety of applications, and is especially useful for simulations where an electron beam needs to be transported after FEL interaction.

2 EQUATIONS

This section presents the electromagnetic field equations and the differential equations solved in the `G00mf` model. The input parameters of this GPT element are given in Table A.

Table A: Main parameters of the `G00mf` model.

Parameter	Description
w_o	Radius of waist [m].
L	Total cavity length [m].
`fmin`	Minimum frequency [Hz].
`fmax`	Maximum frequency [Hz].
`Nm`	Number of modes.
A_{in}	Initial field amplitude [V/m].
Δz	Initial translation of center pulse [m].

2.1 Electromagnetic field equations

The Gaussian waves included in the simulation are all assumed to have a polarization in the x-direction. The total electromagnetic field of all modes j, each with amplitude A_j and phase φ_j, is given by:

$$E_x = \sum_j A_j \frac{w_0}{w_j(z)} e^{-\frac{r^2}{w_j^2(z)}}$$

$$\times \cos\left(\omega_j t - k_j z - \frac{k_j r^2 z}{2(z^2 + z_{0,j}^2)} + \arctan\left(\frac{z}{z_{0,j}}\right) + \varphi_j \right)$$

$$B_y = E_x / c$$

where

$$w_j(z) = w_0 \sqrt{1 + z^2 / z_{0,j}^2}$$

$$z_{0,j} = \tfrac{1}{2} k_j w_0^2$$

$$\omega_j = k_j c$$

$$k_j = \left(n_j \pi - \arctan(2 L^2 / n_j \pi w_0^2) \right) / L$$

and n_j the longitudinal mode number, w_o the waist size, r the radial position of a particle and z the longitudinal position.

The total number of modes N_{all} in the specified frequency range `fmin` to `fmax` is given by:

$$N_{all} = \frac{2L}{c}(\texttt{fmax} - \texttt{fmin})$$

To save CPU time, only a specified limited number of modes Nm is included in the simulation. The mode spacing ΔN is defined as $\Delta N = Nall/Nm$. When $\Delta N=1$, all possible longitudinal modes are included in the simulation. When ΔN is 2, every other mode is included, such that every simulated mode represents not only itself but also its neighboring mode.

By Fourier analysis it can easily be shown that for any ΔN, there are ΔN identical optical pulses in a cavity with length L at any given time. As long as these pulses do not overlap, and as long as the electron bunch does not travel from one pulse into the other during the passage of the optical cavity, the simulation results will not be affected. Typical FEL's with high frequencies and large cavities allow a ΔN of well over a thousand without compromising the accuracy of the model.

The actual mode numbers n_j used in the simulation are uniformly distributed between the specified frequencies by:

$$n_j = \frac{2L\,\mathtt{fmin}}{c} + \left(j - \tfrac{1}{2}\right)\Delta N , \quad j = 1\ldots\mathtt{Nm}$$

2.2 Differential equations

The differential equations for the amplitudes and phases of all modes are derived by first separating each wave j in two orthogonal waves with amplitudes u and v:

$$E_{x,j} = u_j T_j \cos(\theta_j) - v_j T_j \sin(\theta_j)$$

where

$$u_j = A_j \cos(\varphi_j)$$

$$v_j = A_j \sin(\varphi_j)$$

$$T_j = \frac{w_0}{w_j(z)} e^{-\frac{r^2}{w_j^2(z)}}$$

$$\theta_j = \omega_j t - k_j z - \frac{k_j r^2}{2R_j(z)} + \arctan\!\left(z/z_{0,j}\right)$$

The total stored energy of mode j in length L is given by:

$$W_j = \tfrac{1}{4} A_j^2\, \pi\varepsilon_0 w_0^2 L$$

The energy lost by each macro-particle i, with charge Q_i, contributes to an increase in stored energy in all ΔN modes represented by mode j:

$$\frac{dW_j}{dt} = -\Delta N \sum_i Q_i\, \mathbf{v}_i \cdot \mathbf{E}_j$$

The differential equations solved by G00mf are based on the orthogonality of the modes and conservation of energy. They are solved for each mode individually and given by:

$$\frac{du_j}{dt} = -\sum_i \frac{2Q_i\,\Delta N}{\varepsilon_0 \pi w_{0,j}^2 L} v_{x,i} T_j \cos(\theta_j)$$

$$\frac{dv_j}{dt} = +\sum_i \frac{2Q_i\,\Delta N}{\varepsilon_0 \pi w_{0,j}^2 L} v_{x,i} T_j \sin(\theta_j)$$

The boundary conditions of the differential equations, the amplitudes and phases of all modes at $t=0$, are given by:

$$A_{j,t=0} = A_{in}\,\Delta N$$

$$\varphi_{j,t=0} = k_j\,\Delta z$$

The initial pulse is created in the center of the cavity. In order to overlap the radiation pulse with an electron bunch, the Δz parameter can be used to translate the initial pulse. For example, for a cavity between $z=0$ and $z=L$ the pulse at $t=0$ can be moved to $z=0$ by specifying $\Delta z = -L/2$.

2.3 Frequency domain output

The G00mf element allows detailed investigation of the simulations results, both in frequency- and time-domain, at user selectable simulations times. All output can be visualized directly, or analyzed with the standard GPT analysis tools like GDFA.

The GPT G00mf element writes the amplitude, phase, frequency and wavelength of each mode in the GPT outputfile. The amplitude, A00 [V/m], and phase, phi00 [rad], of each mode j are calculated from:

$$\mathtt{A00}_j = A_j / \Delta N = \sqrt{u_j^2 + v_j^2}\,/\Delta N$$

$$\mathtt{phi00}_j = \arctan\!\left(v_j / u_j\right)$$

The division by ΔN for the amplitude compensates the multiplication by ΔN in the interaction strength. This ensures that the amplitude of the output spectrum is independent of the number of modes. The total energy in the pulse, W00 [J], is given by:

$$\mathtt{W00} = \frac{\pi\varepsilon_0 w_0^2 L}{4\,\Delta N} \sum_j A_j^2$$

2.4 Time domain output

The time-structure of the optical pulse is calculated as the discrete Fourier transformation of the frequency spectrum. A mode spacing ΔN in frequency domain is equivalent to ΔN identical pulses in the round-trip time $2L/c$. As a result, all relevant time-values are constrained to the interval $\Delta t = 2L/(c\Delta N)$. The steps dt in this restricted window must be chosen well below the period of fmax. Centered around zero, the time-output t00 [s] is given by:

$$\mathtt{t00}_k = t_k = \left\{ -\tfrac{1}{2}\Delta t,\, dt - \tfrac{1}{2}\Delta t,\, 2\,dt - \tfrac{1}{2}\Delta t,\, \ldots,\, \tfrac{1}{2}\Delta t \right\}$$

The Fourier transform of the frequency response gives the amplitude At00 [V/m] and power Pt00 [W] in time domain:

$$At00_k = \sum_j A_j \sin(\omega_j t_k + \varphi_j)$$

$$Pt00_k = \tfrac{1}{4}\pi\varepsilon_0 w_0^2 c \, At00_k^2$$

3 MODEL TEST

In this section the G00mf model is tested by evaluating the output spectrum and time domain response of a single charged particle through the example set-up of Table B. The set-up consists of two undulators with the same period, but different number of periods and strengths. There is a drift between the undulators. The macro particle consists of electrons amounting to a total charge of −1 pC. The interaction is calculated with 1000 longitudinal cavity modes, equally spaced between 0 and 100 THz. Output is generated when the particle is 30 mm behind the second undulator.

Table B: Simulation parameters used to test the model.

Parameter	Value
Undulator period	30 mm
Number of periods	5 and 10
Undulator strength	0.5 and 1
Length of optical cavity	1 m
Rayleigh length	1 m
Drift length between undulators	100 mm
Initial particle energy	γ=50
Bunch charge	-1 pC

Figure 2 gives the output amplitude in frequency domain of all 1000 cavity modes included in the simulation. Figure 3 shows the corresponding time domain signal. At the output position, the radiation arrives before the particle. The time-domain response has the precise structure of the static undulator field and even the matching periods of the undulator are reproduced in the plot. This is a direct visual proof that the model is correct.
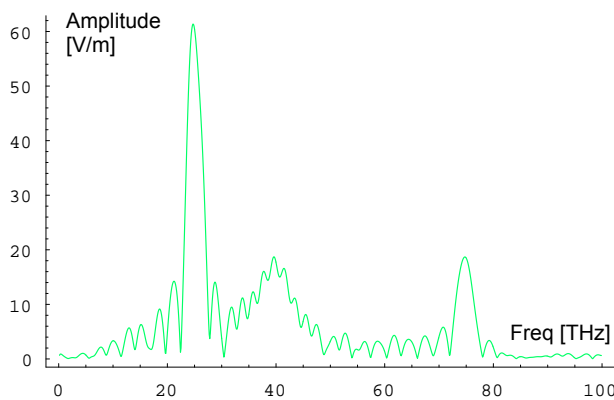

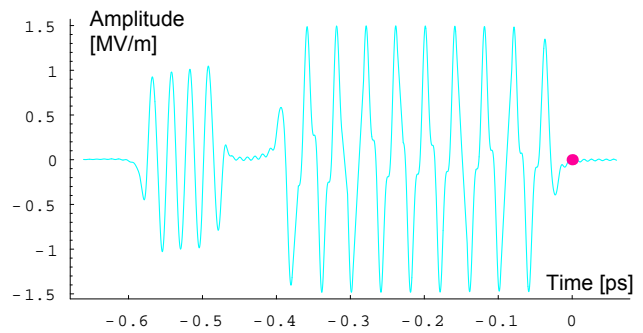Figure 2: Spectrum radiated by the test particle after one pass through the set-up of Table B.

Figure 3: Corresponding wave train in time domain. The red dot indicates the position of the macro particle.

An example of GPT calculations with the G00mf element is shown in Figure 4, where the gain in a single undulator is plotted as function of frequency. The undulator consists of 30 periods of 30 mm. The different lines represent different values for the initial emittance.
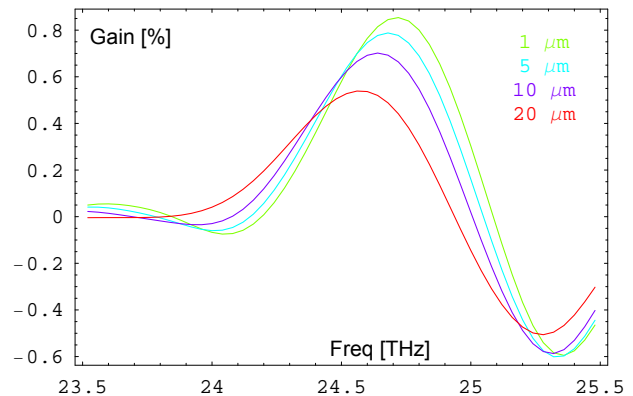

Figure 4: Single pass gain versus frequency in a FEL with one undulator. The different lines represent different emittances.

4 CONCLUSION

The G00mf element of the GPT code allows self-consistent, non-wiggler averaged, FEL simulations. Emphasis is distributed equally between the produced radiation and the phase-space evolution of the electron beam. This model is very useful for the non-linear regime.

The current status of the GPT project can be found on the web at *www.pulsar.nl/gpt*.

5 REFERENCES

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