A Method for Calculating the "Undulator Spontaneous Emission" Produced in Waveguides, and Its Influence on Laser Start-up for Waveguided Free-Electron Lasers Such as "CLIO"

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Abstract

Several Free-Electron lasers (FEL), working in infrared, use a waveguide in place of the vacuum chamber of the undulator. We present here a new numerical method to calculate the amplitude and phase transverse distribution of the "undulator spontaneous emission" produced in this waveguide. The numerical process described here is quasi analytical. It is based on a standard method for wave propagation using the Fast Fourier Transform. This process is included in a numerical code to calculate the diffraction losses of the FEL cavity at the start-up of the laser oscillation. These losses are always larger than for the steady state regime. We present also the results obtained in the case of the "CLIO" infrared FEL.

1 INTRODUCTION

The Spontaneous Emission (SE) is the radiation produced by an electron beam passing through an undulator. It is involved in the operation of the Free-Electron Lasers (FEL). More precisely, the SE is the radiation source in the start-up of the FEL. This radiation is amplified, along the series of cavity round trips, up to saturation which leads to a TEM Gaussian mode (in simple cases). When diffraction occurs in the optical cavity, the transverse structure of the laser mode, at saturation, may be rather different as pure TEM Gaussian profile, and leads to large cavity losses. In any case, the total cavity losses, including diffraction, must be lower than the FEL gain in order to allow laser operation. The FEL operating in infrared range may produce a large amount of diffraction if some constraints are not respected : the vacuum chamber in the undulator must be replaced by a waveguide, and the size of cavity mirrors must be large enough in order to keep the laser beam clear.

The cavity losses and the intra-cavity laser mode can be calculated by numerical codes [1]. However, these codes are designed for steady-state regime, and the relevant losses correspond to the saturation regime. Such codes are not able to calculate the losses at laser start-up, which may be much larger than for saturation regime. Indeed, during the laser start-up, the transient mode does not fit the optimum mode of the optical cavity. If transient losses are larger than FEL gain, they will cancel the laser start-up (and laser will never reach saturation). The code which is presented here calculates the losses produces when the undulator Spontaneous Emission (SE) is stored in the cavity, i.e. at start-up of the FEL. It calculates first the SE produced in the waveguide, all along the undulator. This radiation is used as source in the start-up, and the cavity losses are computed after one round trip of the radiation in the cavity. The process used to calculate the wave propagation in the waveguide has been recently developed [2]. This method is based on the "Fast Fourier Transform" (FFT) ; it does not require TE or TM decomposition and is very simple in practice. The calculation of SE in the waveguide is also based on the same principle [3], and will be summarized below.

2 THE NUMERICAL CODE

The standard numerical code used here [1], which gives the saturation solution, starts from an initial wave front $\Psi_o(x,y)$, and calculates the propagation of the wave in the optical cavity. The round trip circulation of the wave in the cavity represents an iterative process, which is converging toward the steady-state solution. The wave front $\Psi(x,y)$ is sampled, with NxN points, in a given transverse area $[\Delta x, \Delta y]$. The initial wave front $\Psi_o(x,y)$ is generally a Gaussian transverse profile. This numerical code has been modified in order to give the start-up solution. The new process in not iterative, and gives the solution only after one cavity round trip. Also, the initial wave $\Psi_o(x,y)$ corresponds here to the SE radiation which is produced in the undulator waveguide. The calculation of $\Psi_o(x,y)$ involves a combination of the SE radiation process and the waveguide propagation. The various operators used, in the code, for the wave propagation, will be described below.

2.1 Free-Space Propagation

The calculation of wave propagation, in a free-space of distance L, is well known [1]: it is obtained by a convolution between Ψ and the Green function:

$\Psi'(x,y,L)=\Psi(x,y,0)*G(x,y)$, where
$G(x,y)=(-ik/2\pi L)*\exp(ikL)*\exp[ik*(x^2+y^2)/2L]$.

In the numerical code, this convolution is performed by a product in Fourier space, which is obtained using the FFT:

$$\Psi'(x,y,L)=[P]\,\Psi=[FFT^{-1}][xFTG][FFT]\,\Psi(x,y,0) \quad (1)$$

where [P] is the operator of propagation in free-space, and [xFTG] is an operator of product with the Fourier transform of G(x,y).

2.2 Waveguide Propagation

The calculation of wave propagation, in a waveguide of distance L, is performed by the following series of operators [2] :

$$\Psi'(x,y,L) = [P] [Mx] \Psi (x,y) \qquad (2)$$

where [P] is the free-space propagation operator, which is described in §2.1 ; and [Mx] is called the "Mosaic operator" [2] of the laser field for horizontal polarization (we consider here the same case as the "CLIO" FEL [4] which is horizontally polarized). [Mx] corresponds to an operator of symmetry which takes into account the waveguide. This process for waveguide propagation is very simple. However, it requires an important condition : the size of the sampling domain $[\Delta x,\Delta y]$ must be exactly twice of the transverse size of the waveguide [a,b], with $\Delta x=2a$ and $\Delta y=2b$. Because of this condition, the numerical code must take into account two sampling domains, and must use resampling operators in the waveguide propagation process : [S+] for resampling from $[\Delta x,\Delta y]$ to [2a,2b], and [S-] for reverse resampling from [2a,2b] to $[\Delta x,\Delta y]$, where $[\Delta x,\Delta y]$ corresponds to the main sampling area. Therefore, the complete waveguide propagation process uses the following series of operators :

$$\Psi'(x,y,L) = [S-] [D] [P] [Mx] [D] [S+] \Psi (x,y) \qquad (3)$$

where the "Diaphragm operator" [D] erases the field outside of the waveguide entrance (or exit). This method of calculation of waveguide propagation is very simple and rapid because it uses the FFT and is not an iterative process.

2.3 Mirrors, Iris and Optical Gain

The operator for mirror reflection [1] is a simple product with a complex distribution, where the phase includes the mirror radius of curvature, and the amplitude includes the reflection coefficient and the mirror size. The iris or diaphragms in the cavity are represented by a product with an amplitude distribution, with 1 in the open area and 0 outside. The gain operator makes a product with the function $[1+g_o*\Gamma(x,y)]$, where $\Gamma(x,y)$ is a Gaussian distribution, and g_o is the gain coefficient. This gain coefficient g_o is calculated at the end of the cavity round trip, in order to keep a normalized energy for the wave front : $\|\Psi\|^2=1$.

2.4 Calculation of the Guided Spontaneous Emission for the Numerical Code

The initial wave $\Psi_o(x,y)$, which is used as source field in the code, corresponds to the SE radiation which is produced in the undulator waveguide. The instantaneous SE radiation, which is produced by an electron in the undulator, can be approximated by a $\delta(x,y)$ Dirac distribution in transverse plane. A TEMpq mode decomposition can be done using a rapid method [2] which comes from the waveguide propagation process described in §2.2. This method of calculation gives directly a matrix Cpq=[FFT][Mx] $\delta(x,y)$ which the elements represent the amplitude of TEMpq eigenmodes. The matrix [Mx] is described in §2.2. The obtained matrix Cpq has a simple structure : the elements C(p,q)=4 if p is even and q is odd, and 0 elsewhere. Obviously, as required by the condition introduced in §2.2, the size of the domain must be : $\Delta x=2a$ and $\Delta y=2b$. Now, the spectral distribution of the SE, emitted on the mode TEMpq by an electron all along the undulator, and observed at the undulator exit at position L, is : (4)

$$A_{pq}(\lambda) = \frac{L}{2\pi}.\mathrm{sinc}\left[\pi N\left(\frac{\Lambda_R}{\lambda}-1\right)\right].\exp\left[-i\pi N\left(\frac{\Lambda_R}{\lambda}-1\right)\right]$$

where N is the number of undulator magnetic periods, $\lambda=v/c$ is the observation wavelength and Λ_R is the resonance wavelength for the eignemode TEMpq :

$$\Lambda_R \cong \lambda_R + \frac{\lambda_o}{2}\lambda_R^2\left[\left(\frac{p}{2a}\right)^2+\left(\frac{q}{2b}\right)^2\right]$$

$$(5)$$

where $\lambda_R=\lambda_o/2\gamma^2.(1+K^2/2)$ is the standard resonance wavelength of the undulator in free-space. Using the matrix Cpq of mode distribution of SE, and integrating on the modes, we obtain the transverse distribution of amplitude of the spectrum of SE at exit of the waveguide, i.e. at the exit of the undulator :

$$A(x,y, \lambda) = [FFT^{-1}] \{Cpq.Apq(\lambda)\} \qquad (6)$$

This calculation is very simple in practice : it only requires one operator $[FFT^{-1}]$ applied to a matrix. This matrix is a product between the matrix Cpq and the analytical expression $A(x,y,\lambda)$ from (6). The calculation is limited here to the first harmonic of undulator, at any wavelength λ close to λ_R (only the first harmonic is involved in the FEL process). As for §2.2, a resampling operator [S-] is required afterwards in order to fit the main sampling area $[\Delta x,\Delta y]$.

3 SIMULATION FOR THE "CLIO" INFRARED FEL

A simulation of cavity losses as a function of vertical radius of curvature for the cavity mirrors of CLIO [4] is shown in figures 1 and 2. The parameters are displayed in §4. The losses are mainly due to the small diameter of cavity mirrors and to the small size of waveguide entrance. The undulator waveguide on CLIO is flat : a=30mm and b=15mm. In first approximation, we can consider that the laser mode is guided on vertical axis and propagates in free-space on horizontal axis. Therefore, in simulations, the horizontal radius of curvature remains constant (RcX=3m). A comparison in shown here between the losses for start-up regime (top of figures), and saturation regime (bottom). Two wavelengths are displayed : λ=70 and 130μm on figure 1 and 2 respectively. At 70μm, the losses are larger than 100% at start-up, and below 30% at saturation. This difference exists also at 130μm, but is less important. Also, the choice of radius of curvature {RcY1,RcY2} is important,

because some set of mirrors would give low losses at saturation, but will not allow laser start-up. These points are principally located on left hand side of the figures. The point giving the best compromise is noted on figures with a label "B", and corresponds to a pair of toroïdal mirrors with RcY1=RcY2=1.9m (still with RcX=3m). At start-up for 70µm, this point gives about 100% of losses, and at saturation it gives less than 20%. At large wavelength, the limit of laser operation is close to λ =130µm, which gives losses of 130% at start-up and 100% at saturation.

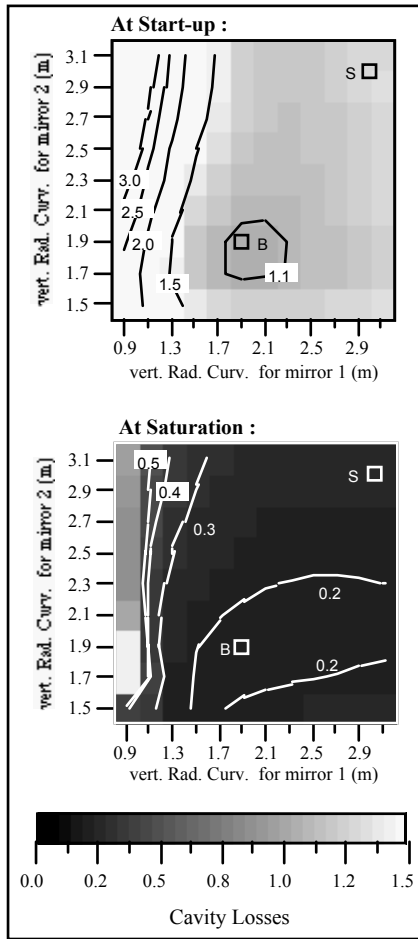Figure 1 : Cavity losses on CLIO for λ=70µm

The point corresponding to spherical mirrors (RcX=RcY=3m) is noted with a label "S". At 70µm, it gives about 120% at start-up, and allows laser operation. At 130µm, the point S forbids laser operation in any case. The experiments obtained with the CLIO FEL are in good agreement with these simulations. The standard cavity, with spherical mirrors (point S), allows laser operation up to λ=90µm. A pair of toroïdal mirrors (RcY1=1.1 and RcY2=1.7m) has also been used, and it did not allow laser start-up, at any wavelength. This last configuration of mirrors was chosen from the previous result of simulation at large wavelength, which was only performed for saturation regime. The present simulation, at start-up, explains clearly the reason why the laser did

not work : the operating point was in "white area", on left hand side of figures.
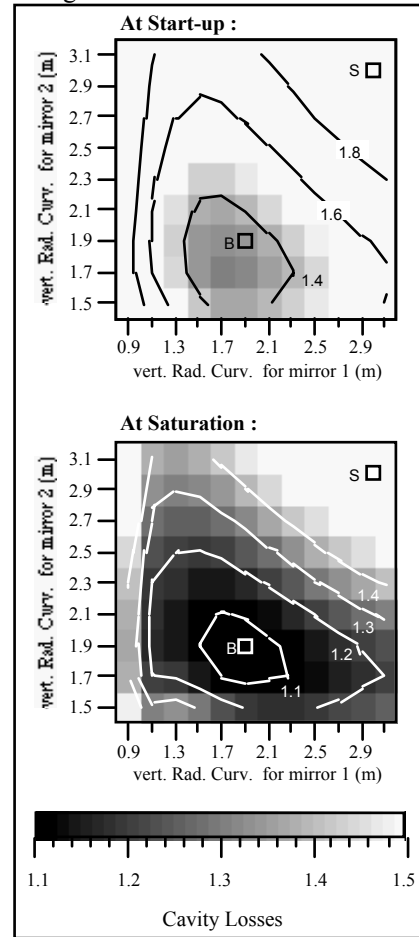

Figure 2 : Cavity losses on CLIO for λ=130µm

4   PARAMETERS OF CLIO

cavity length : 4.8m
mirrors diameter : 38mm
undulator length : 2m
waveguide length : 2m
waveguide aperture : 30mm in H, and 15mm in V
electron beam size : 2mm in H and V

5   REFERENCES

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