

SIMPLE CALCULATION METHOD OF DISTRIBUTED MOMENTUM ACCEPTANCE ALONG AN ELECTRON STORAGE RING

H. Tanaka[†], J. Schimizu, K. Soutome, M. Takao
 Spring-8/JASRI, Kouto, Mikazuki, Sayo Hyogo 679-5198, JAPAN

Abstract

Momentum acceptance (MA) plays an important role to enlarge beam lifetime under the Touschek-effect dominant condition. To optimize lattice parameters for large MA, it is crucial to estimate "effective MA" which can be used as a typical value for the estimation of Touschek lifetime. However, MA is not constant along a storage ring due to distributed energy dispersion and hence, it is not easy to calculate the MA distribution along the ring. We then developed a simple calculation method for local MA by using simulated phase space distortion, nonlinear dispersion up to 3rd order of relative momentum deviation $dp/p (= \delta)$, and dynamic aperture data at one point in the ring for the effective MA estimation. The calculated MA by this method well explains the experimental data in the SPring-8 storage ring. In this paper, we will present our calculation method and comparison between the calculation and experimental data.

1 KEY CONCEPTS OF THE METHOD

Here we limit the application of momentum acceptance (MA) to a Touschek effect in high-energy accelerators. Its elementary process is electron-electron scattering, which occurs near beam central orbit where the electron density is high. In this case local MA can be treated as a maximum momentum deviation that keeps the particle having zero initial amplitude stable. In order to calculate local MA, one needs to know (1) the relation between the momentum change and induced perturbation vector and (2) the dependence of dynamic aperture on the momentum variation at an arbitrary position. It is quite hard to calculate the above and estimate local MA every short step along the huge ring. We have solved this problem by introducing the analytical expression for higher order dispersion and propagation of transverse phase space. Our method only needs the dynamic aperture data at one point to estimate distributed MA along the ring.

1.1 Perturbation Vector

We assume that a pair of scattered electrons exchanges only longitudinal momentum in the scattering process. This means the perturbation vector describing the initial state of an excited betatron oscillation can be expressed by a polynomial of the energy dispersion and induced momentum deviation at the scattering position s .

We have already developed the expressions for nonlinear dispersion up to the 4th order of dp/p [1]. By the agreement between the calculation and measurement, it was confirmed that the calculation up to the 3rd order with ideal ring parameters is valid for the real ring of which orbit distortion is corrected. We then adopt the linear and nonlinear dispersion up to the 3rd order to calculate the

perturbation vector $X(s)$ as

$$\begin{aligned} X(s) &= (x(s), x'(s)), \\ x(s) &= \eta_0(s) \cdot \delta + \eta_1(s) \cdot \delta^2 + \eta_2(s) \cdot \delta^3 + \eta_3(s) \cdot \delta^4, \\ x'(s) &= \eta'_0(s) \cdot \delta + \eta'_1(s) \cdot \delta^2 + \eta'_2(s) \cdot \delta^3 + \eta'_3(s) \cdot \delta^4, \\ \delta &= \frac{dp}{p}, \quad \eta' = \frac{d\eta(s)}{ds}, \end{aligned} \quad (1)$$

where η_0 and η_{1-3} are respectively linear and 1st to 3rd order nonlinear dispersions at the position s . Here, we neglect the perturbation in vertical direction, because a median plane of the SPring-8 storage ring corresponds to the horizontal one and vertical dispersion caused by magnetic errors is quite small compared with the horizontal dispersion. Figure 1 shows the horizontal orbit shift induced by the nonlinear dispersion. The momentum deviation was set to 0.01. We see the orbit shift by the lowest nonlinear dispersion is comparable with that by the linear one.

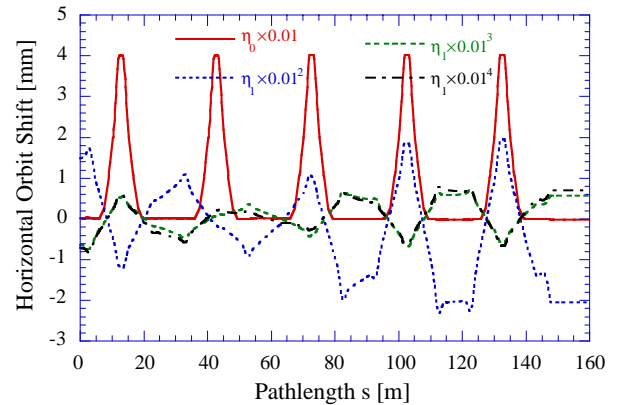


Figure 1: Horizontal orbit shift by nonlinear dispersion.

1.2 Phase Space Propagation

It is well known that normalized transverse phase space distorts from a circle due to sextupole fields and this distortion varies along the ring. To predict the transverse stability at an arbitrary point of the ring from the information at one point, we need to know the phase space propagation. As described in section 1, the perturbation vector induced by the scattering is horizontally polarized. And hence, only the propagation of horizontal phase space is necessary for the MA estimation. Figure 2 shows the normalized horizontal phase space distortion against horizontal betatron phase advance. The red line stands for the fitted result with a cosine function having a periodicity of one revolution.

[†] tanaka@spring8.or.jp

We see the distortion oscillates periodically along the ring by the phase advance. The blue diamonds also show the distortion calculated by the 1st order perturbation theory. The theory well explains the oscillation phase, but not the magnitude of the distortion. From these results we adopt the model that the phase space proceeds along the ring rotating by the betatron phase advance.

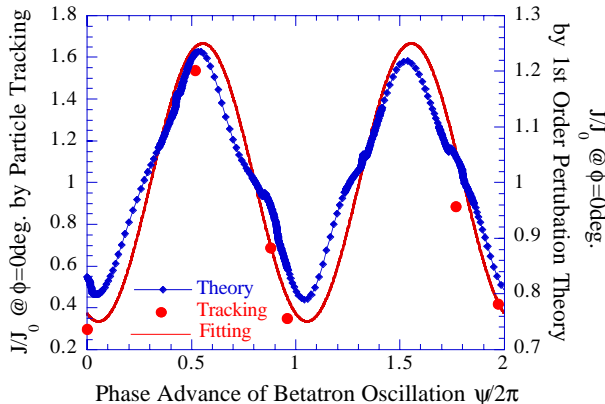


Figure 2: Propagation of phase space distortion.

1.3 Phase Space Distortion

In general, the phase space distortion depends on the amplitude of a betatron oscillation and momentum deviation from the nominal value. To precisely propagate the stability limit along the ring, it is important to consider the above effects in the manipulation. Figure 3 shows the phase space (shape) distortion at the injection point using the oscillation amplitude as a parameter. We see that the phase space approaches to the circle as the oscillation amplitude decreases.

2 CONSIDERATION IN CALCULATION

2.1 Dynamic Aperture

Dynamic aperture is calculated at the injection point changing δ by 0.01 step until the range where the aperture vanishes. In the calculation 6 by 6 tracking code based on the exact Hamiltonian [2] is used. As magnetic errors we adopt 236 normal and 132 skew integrated quadrupole fields estimated by analysis of the full transverse beam response [3]. Radiation excitation and damping are also included and sextupole magnets are treated as thick elements. In local MA calculation we use the averaged dynamic aperture data over 10 samples, each of which has a different random seed. In the calculation, the ratio of the vertical to horizontal tracking amplitudes is set to 1%.

2.2 Treatment of Phase Space Propagation and Distortion

The phase space propagation is performed in the program as follows. The rotation angle of the phase space at each calculation point is expressed by the sum of

global and local phase shifts. The global shift is just the phase advance from the injection to the calculation points and the local one is the angle value of the perturbation vector in the normalized phase space.

The phase space distortion is treated follows in the program as follows. Firstly the phase space distortion is simulated with the tracking code under the on-momentum condition changing the oscillation amplitude. From these data, a functional form of the phase space distortion is constructed including the emittance dependence. Parameters of this form are adjusted to meet each off-momentum dynamic aperture.

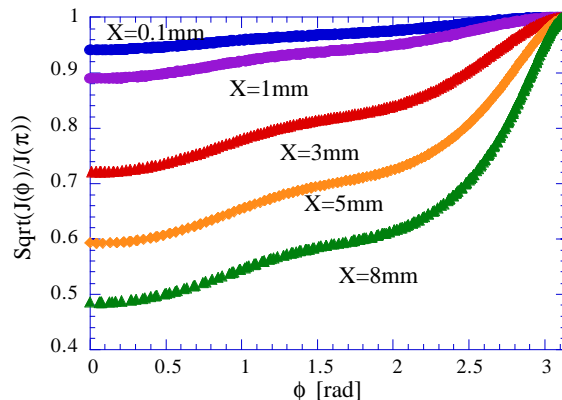


Figure 3: Shape distortion at injection point

3 APPLICATION RESULTS

We have investigated the effect of local chromaticity condition on the momentum acceptance. The SPring-8 storage ring has four magnet-free long straight sections (LSS's) of 27m long. Since phase matching condition over LSS's is easily broken for off-momentum particles, the stability of the off-momentum particles is sensitive to the condition of horizontal chromaticity at LSS's [4]. In order to adjust chromatic phase shifts in each LSS, two families of sextupole magnets, SF_LSS and SD_LSS were prepared in dispersion arc in the matching sections at both ends of each LSS. Fig. 4 shows the calculated MA distribution with the strength of SF_LSS as a parameter. In comparison with Fig. 1, we see the lowest order of nonlinear dispersion affects on local MA significantly. The difference between the MA distributions for positive and negative momentum deviations mainly comes from nonlinearity of the dispersion.

On the other hand, effective MA is obtained by integrating the calculated MA distribution weighting with the inverse product of horizontal and vertical RMS beam sizes. Fig. 5 shows the comparison between measured Touschek lifetime and the square of calculated effective MA. The measured dependence on the strength of SF_LSS is well reproduced by the calculation.

We have also investigated the effect of horizontal and vertical (global) chromaticity on the momentum acceptance. In the SPring-8 storage ring, beam instability is presently suppressed by damping force due to chromatic phase shifts. The storage ring is thus usually operated

under the high horizontal and vertical chromaticity condition. It is therefore important to understand dependence of the beam lifetime on the chromaticity. Fig. 6 shows dependence of effective MA on the chromaticity. We see that the beam lifetime is not drastically reduced in the range where $0 \leq \xi_x \leq 4$ and $0 \leq \xi_y \leq 8$. In the calculation, the dynamic aperture is sensitive to the vertical chromaticity at around the horizontal chromaticity of ~ 8 . In the figure the measured two data are shown by crosses [5]. The measurement conditions are written in the figure. We see the calculation can explain the measured dependence. The calculation error is roughly estimated to $\sim 10\%$.

We emphasize here that effective MA becomes completely different when the linear dispersion is merely used. For the precise MA estimation, it is essential to consider nonlinearity of the dispersion.

4 SUMMARY

We presented the simple calculation method for local MA along the ring. Effective MA, which is obtained by the integration of local MA, has good agreement with the measured lifetime data in the SPring-8 storage ring. This result shows that our method is effective to estimate the momentum acceptance quantitatively.

REFERENCES

[1] H. Tanaka, M. Takao, K. Soutome, H. Hama, M. Hosaka, NIM **A431** (1999) 396.
 [2] J. Schimizu, K. Soutome, M. Takao, H. Tanaka, Proc. of the 13th Symp. on Accel. Sci. and Technol., Suita, Osaka, Oct., p.80 (2001).
 [3] H. Tanaka, K. Soutome, M. Takao, J. Schimizu, Proc. of the 13th Symp. on Accel. Sci. and Technol., Suita, Osaka, Oct., p.83 (2001).
 [4] H. Tanaka, K. Kumagai, N. Kumagai, H. Ohkuma, J. Shimizu, K. Soutome, M. Takao, Proc. of the 7th European Particle Accel. Conf., Vienna, June, p. 1086 (2000); H. Tanaka, K. Soutome, M. Takao, M. Masaki, H. Ohkuma, N. Kumagai, J. Schimizu, NIM **A** in press.
 [5] For example, W. Decking and D. Robin, Proc. of the 6th European Particle Accel. Conf., Stockholm, June, p. 1265 (1998).

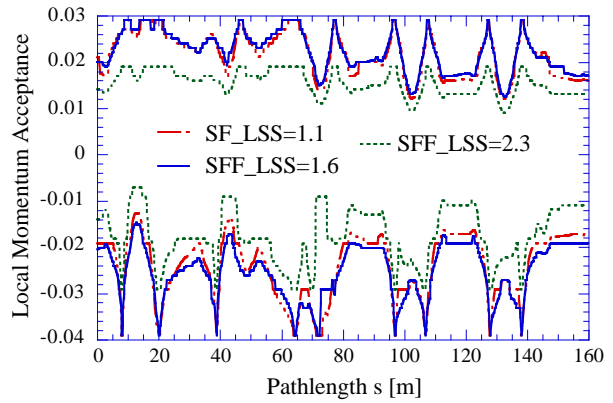


Figure 4: Calculated local MA distribution

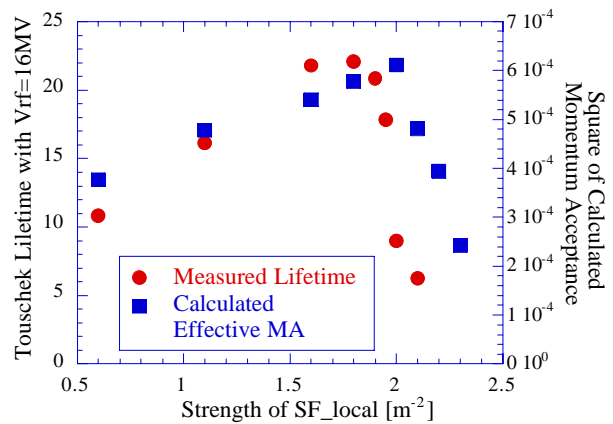


Figure 5: Comparison between lifetime and effective MA

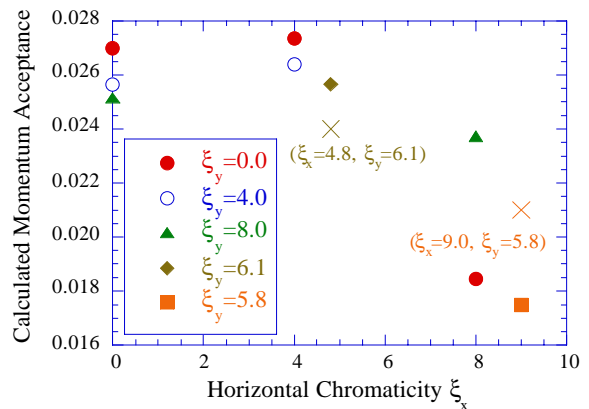


Figure 6: Dependence of momentum acceptance on chromaticity