INVESTIGATIONS OF BEAM INSTABILITIES IN PLS STORAGE RING

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Abstract

We examine the effect of wakefields generated by an interaction between circulating electron beam and rf cavities and components of vacuum chambers on the longitudinal beam distribution due to potential-well distortion, with particular emphasis on the PLS storage ring. In order to obtain equilibrium longitudinal beam distribution of a single beam, both numerical solution of the Vlasov equation and the multi-particle tracking method are utilized. Results show that the impedance due to the beam-cavity interaction affects less to the longitudinal beam deformation than the broad-band impedance in the ring does. It is shown that the results of the Vlasov method show good agreements with those of the multi-particle tracking method.

1 INTRODUCTION

When a beam circulates around a storage ring, wake force can be generated by an electromagnetic interaction between particles in the beam and the surrounding vacuum-chamber environment. The wake force may affect the longitudinal distribution of particles in the beam. Comparable studies on longitudinal beam distribution in electron storage rings have mainly carried out by utilizing two methods: (1) the Vlasov equation and (2) simulation by multi-particle tracking.

For the case of a constant RF gradient and no wakefield, the equilibrium particle distribution is a Gaussian in the longitudinal direction. However, the presence of the wakefield produces a distortion of the potential well and results in the distortion of the beam from Gaussian shape. The aim of this paper is to investigate the characteristic features of the longitudinal beam distribution under the effects of the wakefields.

We investiagte the longitudinal beam distributions due to the potential-well distortion (PWD) caused by the wakefields of RF cavities and components of vacuum chambers in the PLS storage ring. We consider the broad-band impedance model to obtain wakefield due to the components of vacuum chambers. Wakefield due to rf cavities is computed by using a computer code ABCI. With calculated wakefields, we obtain the longitudinal beam distribution by solving the Vlasov equation as well as using a multi-particle tracking.

2 POTENTIAL-WELL DISTORTION

The equations of motion of a particle are

$$z' = -\eta\delta, \delta' = V(z), \tag{1}$$

where a prime means taking derivative with respect to s. Hamiltonian $H(z,\delta)$ for the single particle motion in the bunch is :

$$H(z,\delta) = \frac{\eta^2 c^2 \delta^2}{2w_s} + \frac{w_s z^2}{2} - \int_0^z V_o(z')dz'. \tag{2}$$

The third term in the Hamiltonian is the potential-well term. The charge of the bunch induces the voltage V_o with the longitudinal wakefield $W(z)$ as

$$V_o(z) = k\int_z^\infty \rho(z')W(z-z')dz'. \tag{3}$$

Eq. (3) denotes the retarding voltage seen by a particle at longitudinal position z due to the wake force generated by preceding all particles; $\rho(z')$ is the particle density at location z'. k is a quantity which is related to the beam intensity. We assume that the wake is dissipated before the beam circulates one revolution.

We obtain the equilibrium longitudinal current distribution of a bunch.

$$I(t) = Kexp[-\frac{t^2}{2\sigma_z^2} + \frac{\int_0^t V_o(t')dt'}{\dot{V}_{rf}\sigma_z^2}], \tag{4}$$

$\dot{V}_{rf}$ is the slope of the rf voltage at the position of the bunch and is given by

$$\hat{V}_{rf} = w_{rf}\hat{V}_{rf}[1-(U_o/\hat{V}_{rf})^2]^{1/2}, \tag{5}$$

with w_{rf} the angular rf frequency, $\hat{V}_{rf}$ peak energy gain from the rf cavity and U_o average synchrotron radiation energy loss per turn. Here $t = z/c$ is the time displacement between a particle of the bunch and bunch center. In our notation a smaller value of t signifies an earlier point in time. The value of the normalized constant K in Eq. (4) is such that the complete integral of $I(t)$ is equal to the total charge in the bunch.

The induced voltage $V_o(t)$ is the sum of two terms : the sinusoidal rf voltage $V_{rf}(t)$, and a transient voltage $U(t)$ due to the self fields induced by the bunch

$$V_o(t) = V_{rf}(t) + U(t). \tag{6}$$

$V_{rf}(t) = \hat{V}_{rf}sin(w_{rf}t+\phi_s)$ is the accelerating voltage, taking into account sinusoidal beam loading. ϕ_s is synchronous rf phase. At low beam current the transient self fields are weak and $U(t)$ is negligible. Under such current, the instantaneous equilibrium current distribution has a Gaussian:

$$I(t) = Kexp(-t^2/2\sigma_z^2). \tag{7}$$

The transient voltage $U(t)$ results from interaction between the bunch and the elements of the ring such as accelerating cavities and vacuum chambers.

In next section, we obtain V_o due to the RF cavities and components of vacuum chamber in the PLS storage ring. Then we investigate the effect of V_o on beam shape and centriod position of the beam in the PLS storage ring.

3 EVALUATION OF THE WAKEFIELD

The induced voltage V_o in Eq. (4) is given by

$$V_o(t) = -\int_0^{\infty} W(t')I(t-t')dt' \tag{8}$$

with $W(t)$ the longitudinal wakefield, $W(t) = 0$ for $t < 0$. We see that Eq. (5) shows the self-consistent current beam distribution. Eq. (5) can be solved numerically to investigate the beam current distribution of the bunch in the presence of wakefields. Since $V_o(t)$ depends only on the current at more negative (earlier) times, the solution of Eq.(5) can be obtained if we begin at the head of the bunch (where V_o = 0) and proceed toward the bunch tail.

Taking the derivative of both sides of Eq. (5), we obtain the following relation:

$$\frac{\dot{I}}{I} = -\frac{t}{\sigma_z^2} + \frac{V_o(t)}{\dot{V}_{rf}\sigma_z^2}. \tag{9}$$

In what follows, all distances will be given in terms of σ_z. Thus the independent variables becomes $X = t/\sigma_z$. The normalized current is given by $Y = IZ_o/\dot{V}_{rf}\sigma_z$. The normalized charge which is the complete integral of Y equals $\frac{Z_oQ}{\dot{V}_{rf}\sigma_z^3}$ with $Z_o = 377\Omega$. Then Eq. (9) becomes to

$$Y' = -XY + \frac{YV_o(\sigma_z X)}{\dot{V}_{rf}\sigma_z}, \tag{10}$$

where a prime means taking derivative with respect to the X. The numerical solution of Eq. (10) for several values of beam current will be obtained in next section.

3.1 *Wakefield due to beam-cavity interaction*

The code ABCI is used to compute the longitudinal wakefield in the rf cavity. The short range wake of a bunch with $\sigma_z = 1\ mm$ was computed using a mesh step in the longitudinal and transverse directions of 0.1 mm. The longitudinal loss factor is shown to be 1.55 V/pC.

3.2 *Wakefield due to components of vacuum chambers*

The longitudinal wake force generated by a beam interacting with discontinuities of components in the ring is approximated by a broad-band impedance with the quality factor of the order of unity.

In a broad-band impedance model, the longitudinal wakefield $W(z)$ is given by

$$W(z<0) = 2\alpha R_\mathrm{s} e^{\alpha z/c}\left(\cos\frac{\bar{\omega}z}{c} + \frac{\alpha}{\bar{\omega}}\sin\frac{\bar{\omega}z}{c}\right), \tag{11}$$

$$W(z=0) = \alpha R_\mathrm{s}, \tag{12}$$

where $\alpha = \omega_\mathrm{R}/2Q$ and $\bar{\omega} = \sqrt{\omega_\mathrm{R}^2 - \alpha^2}$, Q=$R_\mathrm{s}\sqrt{C/L}$ is the quality factor, $\omega_\mathrm{R} = 1/\sqrt{LC}$ is the resonant frequency and $R_\mathrm{s} = C_\mathrm{R}/(2\pi b)Z_{||}/n_\mathrm{h}$ is the shunt impedance. Here, C_R is the ring circumference and n_h is the harmonic number. b is the effective radius of the vacuum-pipe. Z/n=1 Ω and w_r= 6.67 GHz are used in the simulation.

4 CALCULATED LONGITUDINAL BEAM DISTRIBUTION

We investigate the effects of beam-cavity interaction on the longitudinal beam distribution with the Eq. (10). Fig. 1 shows the calculated bunch shapes as a function of a beam current due to the PWD that is caused by four rf cavities in the PLS storage ring. Fig. 1 displays the bunch shapes for the beam current of 2 mA, 5 mA, 10.5 mA and 14 mA. The horizontal axis is $X = t/\sigma_z$. The vertical axis gives $Y = IZ_o/(\dot{V}_{rf}\sigma_z)/10^8$ with $Z_o = 377\Omega$. As can be seen, the bunch shapes are shifted forward than Gaussian. Due to energy conservation, centroid shift multiplied by $-\dot{V}_{rf}\sigma_z$ gives the higher mode losses.

Fig. 2 shows the calculated bunch shapes as a function of a beam current due to the PWD that is caused by components of vacuum chambers in the PLS storage ring. Fig. 2 presents the bunch shapes for the beam current of 2 mA, 5 mA, 10.5 mA and 14 mA. As can be seen, the bunch shapes are also shifted forward than Gaussian.

One noteworthy feature of Figs. 1 and 2 is that the beam distribution leans more and more forward ($z < 0$) as the beam intensity increases. This effect is caused by the parasitic loss of the beam which can be compensated by the RF voltage. The beam distribution is Gaussian at low beam currents and it shows growing deformation of the beam shape from Gaussian with increasing beam current. The beam distribution shows a steeper slope in front of beam and thus the beam tail undergoes large wakefields. It is shown that the bunch centroids are more shifted forward due to wakefield of the broad-band impedance than that of rf cavities.

5 MULTI-PARTICLE TRACKING

The initial distributions of macro-particles in the phase space are given with Gaussian. Each macro-particle i is tracked in phase space of position and energy coordinates (z_i, δ_i) with equations of motion which include radiation damping, radiation excitation, rf voltage and wakefield.

The longitudinal motion of the particle i is advanced on each turn according to the equation:

$$\Delta\delta_i = -\frac{2T_o}{\tau_d} + 2\sigma_\delta\sqrt{\frac{T_o}{\tau_d}}r_i + \dot{V}_{rf}z_i + V_o(z_i), \quad (13)$$

$$\Delta z_i = \frac{\alpha c T_o}{E_o}(\delta_i + \Delta\delta_i), \quad (14)$$

with T_o the revolution period, τ_d the longitudinal damping time, α the momentum compaction factor, E_o the nominal energy and r_i is a random number from a normal set with mean 0 and rms 1. For simulations we take E_o=2 GeV, f_{rf}=500 MHz, α=0.00181, σ_δ=6.8 $\times$ 10^{-4} and τ_d=8 ms. We choose $\hat{V}_{rf}$=1.6 MV, σ_z=5 mm and the synchrotron frequency f_{so}=11 kHz. We track the macroparticles for five longitudinal damping times. When we examine the dependence of the simulation results on the number of macroparticles, it is shown that there is no great difference in the results if we track around 40000 macro-particles. To calculate λ_z in each turn we use 50 bins extending over $5\sigma_z$.

The effect of the wakefield due to four RF cavities in the PLS storage ring is investigated. Tracking result of the longitudinal beam distribution for the beam of 14mA after five longitudinal damping times is compared with that of the Vlasov method in Fig. 3. In this figure, solid curve and dots show the results obtained by the Vlasov method and the multi-particle tracking, respectively. They show the agreement is quite good.

The effect of wakefield due to the broad-band impedance in the PLS storage ring is also investigated. Tracking result of the longitudinal beam distribution for the beam of 14 mA after five longitudinal damping times is compared with that of the Vlasov method in Fig. 4. Curve and dots show the results that are obtained by the Vlasov method and the multi-particle tracking, respectively. They show a good agreement. It is shown that the calculated bunch shapes agree well with that obtained using the multi-particle tracking method.

6 CONCLUSION

We investigated the effect of longitudinal wakefields on a distortion of the equilibrium longitudinal particle distribution in a beam. The equilibrium longitudinal beam distribution in the PLS storage ring is investigated by using the Vlasov method and a multi-particle tracking method. The results of two methods showed good agreements. It is shown that broad-band impedance more affects the longitudinal beam distribution than the cavity-beam interaction in the PLS ring.

7 REFERENCES

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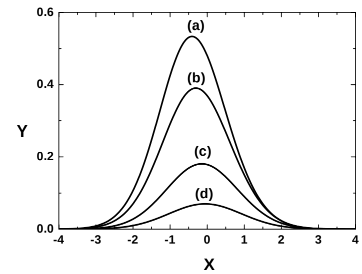

Figure 1: Calculated beam distribution under the effect of the wakefield of the rf cavities in the PLS ring.

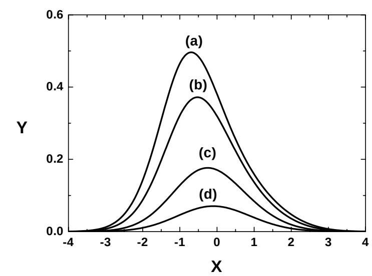

Figure 2: Calculated beam distribution under the effect of wakefield of the broad-band impedance in the PLS ring.

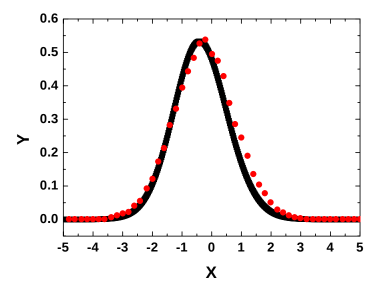

Figure 3: Longitudinal beam distributions due to the rf cavities for the 14 mA beam. Shown are tracking result (dots) and the Vlasov method result (curve).

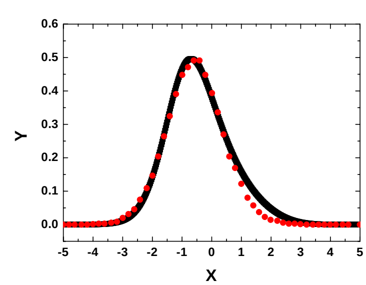

Figure 4: Longitudinal beam distributions due to the broadband impedance for the 14 mA beam current. Shown are tracking result (dots) and the Vlasov method result (curve).