# THE IMPACT OF PSK TIMING ON ENERGY STABILITY OF e-BEAM AT FERMI@ELETTRA

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## Abstract

FERMI@elettra is a single-pass FEL user facility covering the spectral range 100-10 nm. It is based on the existing 1.0 GeV normal conducting Linac in operation at Synchrotron Ligth Source ELETTRA since the beginning of 1990. Seven of the existing Linac accelerating sections, S1-S7, are equipped with a Power Enhancement Network (PEN) system, normally known as SLED or RF pulse compression.

The main purpose of this technique is to manipulate the pulse shape supplied by the generator. This is done by using a couple of identical high Q resonators, together with a  $180^{\circ}$  phase flip of the RF pulse. The RF peak power is then increased over a small fraction of its original length.

The stringent requirements imposed by the FERMI project on the electron beam parameters require a detailed evaluation of the whole PEN process. Here the impact of the Phase Shift Keying (PSK), the timing of the RF phase flipping, is reviewed and analyzed to define a maximum budget for its time jitter.

A simulation model of the PSK timing system has been built up by use of MATLAB simulink. It concludes that a PSK time jittering of 10 ns will result in a beam energy jitter of 0.3 %.

#### **MOTIVATION**

The FERMI@ELETTRA FEL project requires a beam energy stability better than 0.1% [1]. Most of the beam energy gain is supplied by seven 6.1-m long S-band accelerating structures, powered by 45 MW klystrons (Thales TH2132A). These structures are  $3\pi/4$  mode Backward Traveling Wave (BTW), constant-impedance structures, each one equipped with its own PEN system.

As is known, with the use of an RF pulse compression scheme, the beam energy gain strongly depends on the timing of the  $180^{\circ}$  phase flip and the main RF characteristics of the PEN circuit.

Here we analyze the impact of the PSK time jitter on the energy stability of the Linac.

## FIELD AT THE OUTPUT OF THE PEN

Figure 1 shows a sketch of the power enhancement network (PEN) inserted between the klystron and its associated accelerating section, with waveforms in the time intervals A, B, C.

Assuming that the two PEN cavities (1,2) are identical, there will be no power reflected back to the klystron

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(assuming the load and the accelerating section are wellmatched).

The field *E* at the output of the PEN can be considered as the sum of an incident wave  $E_k$  from the generator (klystron) and an emitted wave  $E_c$  from the cavities,  $E=E_k+E_c$ .

We will calculate the field *E* during the three time intervals, denoted as A ( $0 < t < t_1$ ), B ( $t_1 < t < t_2$ ), and C ( $t > t_2$ ).



Figure 1: Sketch of power enhancement network and related waveforms.

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During time interval A, the PEN cavity fields and hence the emitted field vary as (assuming  $E_k(A)=-1$ , as shown in Figure 1),

$$E_{c}(A) = \alpha \cdot \left(1 - e^{-t/T_{c}}\right)$$

where  $\alpha = 2\beta/(1+\beta)$ , with  $\beta$  the cavity coupling coefficient and  $T_c$  the cavity filling time. In our case  $\beta = 10$ ,  $T_c = 2Q_L/\omega = 1.834 \ \mu s$ , with  $Q_0 = 190000$ ,  $f = 2998 \ MHz$ . At time  $t = t_1$ , the emitted field is

$$E_{c1} = \alpha \cdot \left(1 - e^{-t_1/T_c}\right)$$

The emitted field waveform during time interval A is sketched as in Figure 1.

The load field during pulse interval A is

$$E(A) = E_{c}(A) + E_{k}(A) = E_{c}(A) - 1, \text{ or}$$
  

$$E(A) = -1 + \alpha \cdot (1 - e^{-t/T_{c}}) = -\alpha \cdot e^{-t/T_{c}} + (\alpha - 1)$$
(1)

At time  $t=t_1-dt$ , the load field is,

$$E_{t1}^{-} = -1 + \alpha \cdot \left(1 - e^{-t_1/T_c}\right)$$

At time  $t=t_1+dt$ , the load field is obtained as

$$E_{t1}^{+} = E_{c1} + E_{k}(B) = E_{c1} + 1, \text{ or}$$
$$E_{t1}^{+} = \left[1 + \alpha \cdot \left(1 - e^{-t_{1}/T_{c}}\right)\right]$$
(2)

Note that  $\Delta E_{tl} = E_{tl}^+ - E_{tl}^- = 2$ . This discontinuity in the load field waveform at time  $t_l$  is shown in Figure 1.

During time interval B, the fields in the PEN cavities and hence the emitted field vary exponentially between  $E_c=E_{c1}$  and  $E_c=-\alpha$ , which is the level the field would eventually reach if the  $E_k(B)=+1$  was to last indefinitely. That is,

$$E_{c}(B) = (E_{c1} + \alpha) \cdot e^{-(t-t_{1})/T_{c}} - \alpha$$

At time  $t_2$  the emitted field is

$$E_{c2} = (E_{c1} + \alpha) \cdot e^{-(t_2 - t_1)/T_c} - \alpha$$

Since  $E_k(B) = +1$ , the load field is given by

$$E(B) = E_c(B) + 1, \text{ or}$$
$$E(B) = (E_{c1} + \alpha) \cdot e^{-(t-t_1)/T_c} - (\alpha - 1)$$

Expressing the field in terms of  $E_{tl}^{+} = E_{cl} + 1$ ,

$$E(B) = \left(E_{t_1}^+ + \alpha - 1\right) \cdot e^{-(t-t_1)/T_c} - (\alpha - 1)$$
(3)

At time  $t=t_2$ -dt,  $E(B)=E_{t_2}$  where

$$E_{t2}^{-} = \left(E_{t1}^{+} + \alpha - 1\right) \cdot e^{-(t_2 - t_1)/T_c} - \left(\alpha - 1\right)$$
(4)

During time interval C the generator field is zero. The load field is then equal to the emitted field, and both vary as

 $E(C) = E_{c}(C) = E_{c2} \cdot e^{-(t-t_{2})/T_{c}}$ 

Since 
$$E_{c2} = E_{t2}^{+} = E_{t2}^{-} - 1$$
,  
 $E(C) = (E_{t2}^{-} - 1) * e^{-(t-t_2)/T_c}$ 
(5)

Note again that the discontinuity in the load field waveform at time  $t=t_2$  is equal to the discontinuity in the incident waveform,  $\Delta E_k$ . This is shown in Figure 1.

Equations (1), (3) and (5) show how the output field from the PEN varies during time intervals A, B and C in terms of the basic cavity and pulse parameters and the fields  $E_{t1}^{+}$  and  $E_{t2}^{-}$ . In turn, the fields  $E_{t1}^{+}$  and  $E_{t2}^{-}$  are given by equations (2) and (4).

#### FIELD IN ACCELERATING STRUCTURE

In the following we'll exam the field distribution and propagation in the accelerating structure.

The accelerating section is a BTW constant-impedance structure. The measured attenuation factor of the sections is  $\tau$ =0.641 Neper, and the measured filling time of the sections is  $T_a$ =0.757  $\mu$ S. [1]

The field at any point along the structure can be obtained from the field at the RF input. Because it is a backward travelling wave structure we define the downstream end (the RF input port) as z=0 and the upstream end of the section as  $z=z_L$ .

$$E_{s}(z,t) = E_{s}(0,t-\Delta t) \cdot e^{-\frac{\tau}{z_{L}}z}$$
(6)

where  $z=v_g \Delta t$ ,  $v_g$  is group velocity, and  $\Delta t$  is the time elapsed during the propagation of the RF pulse from the input port to position *z*.

Notice that  $z_L = v_g \cdot T_a$ ,  $z_L$  is length of the section (6.15 m),  $T_a$  is the filling time of the section, thus

$$E_{s}(z,t) = E_{s}(0,t-\Delta t) \cdot e^{-\frac{t}{T_{a}}\Delta t}$$
(7)

So, the field inside the accelerating structure due to the waveform giving by equations (1), (3), (5) will be expressed in forms:

$$E_{s,A}(z,t) = E_{s,A}(0,t-\Delta t) \cdot e^{-\frac{\tau}{T_a}\Delta t} =$$

$$= \left[-\alpha \cdot e^{-(t-\Delta t)/T_c} + (\alpha-1)\right] \cdot e^{-\frac{\tau}{T_a}\Delta t}$$

$$E_{s,B}(z,t) = E_{s,B}(0,t-\Delta t) \cdot e^{-\frac{\tau}{T_a}\Delta t} =$$

$$= \left[\left(E_{t_1}^+ + \alpha - 1\right) * e^{-(t-\Delta t-t_1)/T_c} - (\alpha-1)\right] \cdot e^{-\frac{\tau}{T_c}\Delta t}$$

$$E_{s,C}(z,t) = E_{s,C}(0,t-\Delta t) \cdot e^{-\frac{\tau}{T_a}\Delta t} =$$

$$= \left(E_{t_2}^- - 1\right) \cdot e^{-(t-\Delta t-t_2)/T_c} \cdot e^{-\frac{\tau}{T_a}\Delta t}$$
(9)
(10)

# **ENERGY GAIN CALCULATION**

The energy gain V would be the integral of equation (6) over the entire section. Again, notice that in our case the

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RF propagating direction is opposite to beam moving direction,

$$V = \int_{z_{ups}}^{z_{dwns}} E_s(z,t) d(-z)$$
<sup>(11)</sup>

Here,  $z_{ups}$  denotes upstream z,  $z_{dwns}$  downstream z. d(-z) is due to that z=0 is the input port (the downstream end),  $z_L$  is the upstream end of the section.

Normalizing z with  $z_L$ , that is,  $z'=z/z_L$ ,  $z_{ups}'=z_{ups}/z_L$ ,  $z_{dwns}'=z_{dwns}/z_L$ , (11) is written in form,

$$V = \int_{z'_{ups}}^{z'_{dwns}} E_s(z',t) dz'$$
(12)

Considering the field distributions given by (8), (9), (10) and denoting that  $t'=t/T_a$ ,  $\Delta t'=\Delta t/T_a$ ,  $u=T_a/T_c$ , the integral (12) could be written in forms

$$V_{A} = \int_{z'_{dwns}(A)}^{z'_{uys}(A)} E_{s,A}(z',t') d(\Delta t')$$
(13)

$$V_{B} = \int_{z'_{dwns}(B)}^{z'_{ups}(B)} E_{s,B}(z',t') d(\Delta t')$$
(14)

$$V_{c} = \int_{z'_{dwns}(C)}^{z'_{ups}(C)} E_{s,C}(z',t') d(\Delta t')$$
(15)

Where, from (8), (9), (10)

$$E_{s,A}(z',t') = E_{s,A}(0,t'-\Delta t') \cdot e^{-\tau \cdot \Delta t'} = = \left[ -\alpha \cdot e^{-u(t'-\Delta t')} + (\alpha - 1) \right] \cdot e^{-\tau \cdot \Delta t'}$$
(16)

$$E_{s,B}(z',t') = E_{s,B}(0,t'-\Delta t') \cdot e^{-\tau \cdot \Delta t'} = \\ = \left[ \left( E_{t_1}^+ + \alpha - 1 \right) \cdot e^{-u(t'-\Delta t'-t_1')} - (\alpha - 1) \right] \cdot e^{-\tau \cdot \Delta t'}$$
(17)

$$E_{s,C}(z',t') = E_{s,C}(0,t'-\Delta t') \cdot e^{-\tau \cdot \Delta t'} = = (E_{t2}^{-}-1) \cdot e^{-u(t'-\Delta t'-t_{2})} \cdot e^{-\tau \cdot \Delta t'}$$
(18)

The remaining task is to determine the limits of integration:  $z_{dwns}'(A)$ ,  $z_{ups}'(A)$ ;  $z_{dwns}'(B)$ ,  $z_{ups}'(B)$ ;  $z_{dwns}'(C)$ ,  $z_{ups}'(C)$ . Taking into account the discontinuities (which occur at  $t_1$  and  $t_2$ ) and the propagation of the field inside the structure, we can determine the limits of integration as shown below.

For  $z_{dwns}'(A)$ ,  $z_{ups}'(A)$ :

at	$z_{dwns}'(A)$	$z_{ups}'(A)$
$t' \leq l$	0	t'
$1 < t' \leq t_1'$	0	1
$t_1' < t' \le t_1' + 1$	$t'-t_1'$	1
$t_1' + 1 < t'$	1	1

at	$z_{dwns}'(B)$	$z_{ups}'(B)$
$t' \leq t_I'$	0	0
$t_1' < t' \le t_1' + 1$	0	$t'-t_1'$
$t_1' + 1 < t' \le t_2'$	0	1
$t_2' < t' \le t_2' + 1$	t'-t <sub>2</sub> '	1
$t_2' + 1 < t'$	1	1

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at	$z_{dwns}'(B)$	$z_{ups}'(B)$	
$t' \leq t_1'$	0	0	
$t_1' < t' \le t_2'$	0	$t'-t_1'$	
$t_2' < t' \le t_1' + 1$	t'-t <sub>2</sub> '	$t'-t_1'$	
$t_1' + 1 < t' \le t_2' + 1$	$t'$ - $t_2'$	1	
$t_2' + 1 < t'$	1	1	

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at	$z_{dwns}'(C)$	$z_{\mu\nu\sigma}'(C)$
$t' \leq t_2'$	0	0
$t_2' < t' \le t_2' + 1$	0	t'-t <sub>2</sub> '
$t_2' + 1 < t'$	0	1

Using the above tables and Eq. (13), (14) and (15), we can obtain the energy gain versus time:

$$V = V_A + V_B + V_C \tag{19}$$

Figure 2 shows the results obtained for different choices of PSK times. The y-axis shows the ratio of energy gain with SLED to non-SLED operation. Form this we conclude that a 10 ns PSK time jitter results in 0.3% beam energy jitter.

#### CONCLUSION

A simulation model is built up using MATLAB simulink to calculate the energy gain of the PEN system installed on the ELETTRA Linac and its sensitivity with respect to PSK time jitter. The results show that the allowed PSK time jitter should be about 2 ns or better in order to meet the requirement of FERMI project.



Figure 2 : PEN energy gain versus beam injection time.

#### REFERENCES

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- [3] P. B. Wilson, SLAC-TN-73-15.