

# SPACE CHARGE LOSS MECHANISMS ASSOCIATED WITH HALF INTEGER RESONANCE ON THE ISIS SYNCHROTRON

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## Abstract

ISIS is the spallation neutron source at the Rutherford Appleton Laboratory in the UK. Operation centres on a 50 Hz proton synchrotron, which accelerates  $\sim 3 \times 10^{13}$  ppp from 70 to 800 MeV, corresponding to beam powers of 0.2 MW. Beam loss limits operational intensity, and a main contributing mechanism is the action of half integer resonance under high space charge. Progress on studies using particle in cell simulations to explore the evolution of envelope motion, associated 2:1 parametric halo, and growth of particles from the outer core is presented. Comparisons are made with relevant theoretical models by studying the simplified 4D coasting beam case.

## INTRODUCTION

The ISIS ring has a mean radius of 26 m and accelerates about  $3 \times 10^{13}$  protons per pulse (ppp) from 70 to 800 MeV in 10 ms. The beam is unbunched during the 150 turn charge-exchange injection process, where beam is painted over the transverse acceptances ( $\sim 400 \pi$  mm mr). Most beam loss occurs during the bunching process, when transverse space charge forces peak. The nominal tunes of the ISIS ring are  $Q_x=4.31$ ,  $Q_y=3.83$ , and during bunching peak incoherent tune depressions are  $\delta Q_{incoh} \sim 0.4$ . A major loss mechanism is the action of the half integer resonances ( $2Q_x=8$ ,  $2Q_y=7$ ).

Previous papers have looked at coherent envelope motion and resonance [1], and the related emittance growth as predicted by codes [2]. It was found that RMS emittance ( $\epsilon_{RMS}$ ) growth occurred beyond the incoherent limit, but before the coherent limit. The enhanced envelope motion near coherent resonance apparently drives emittance growth and loss (assuming an otherwise matched beam). The purpose of this paper is to study the behaviour of the beam near envelope resonance with simulations, and compare results with expectations from theory. This potentially gives useful insight into beam behaviour and increased confidence in simulation results. In particular, the development of halo, and its relationship to envelope driven parametric resonance are explored. The emphasis here is on studying the halo structure: understanding the processes that cause particles to diffuse into the halo are important future steps.

## Simulation Parameters and Methods

Simulations used the ORBIT code [3] to model 4D, transverse motion, with a detailed model of the ISIS lattice. This is effectively a coasting beam, with negligible momentum spread, at the injection energy of 70 MeV. Simulation PIC (particle in cell) parameters were selected for convergent results, as in [2] (e.g.  $5 \times 10^4$  macro particles and  $128 \times 128$  bins). A 4D waterbag (WB) beam distribution of  $\epsilon_{xRMS} = \epsilon_{yRMS} = 65 \pi$  mm mr was

transported around the ISIS lattice, over  $\sim 20$  turns. RMS Twiss parameters of the input beam were matched to reduce RMS width oscillations to  $< 1\%$ . A single quadrupole driving term corresponding to  $2Q_y=7$ , and a relative gradient strength of 0.002 was applied. Development of beam distributions over multiple turns was studied as a function of intensity. Holding bare tunes constant, and increasing intensity, depresses the beam toward envelope resonance. Previous work has shown that at high intensities ( $\sim 8 \times 10^{13}$  ppp), over  $\sim 100$  turns,  $\epsilon_{RMS}$  grows and the beam distribution changes as a significant fraction of core particles migrate into the halo [2]. Here, the early evolution is studied within the first  $\sim 20$  turns, whilst the core is mainly intact, and the halo a small fraction of the beam. This allows comparison with a particle core model.

Below we first look at the development of the *main beam* halo, that is macro particles that form part of the fully interacting WB beam. Then we study the dynamics using a subset of special diagnostic particles, or *test beam*.

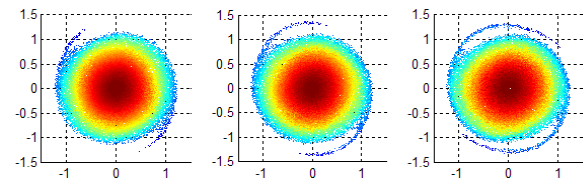


Figure 1: Halo evolution in main beam at  $7.5 \times 10^{13}$  ppp. (In all figures axes are *normalised transverse coordinates*)

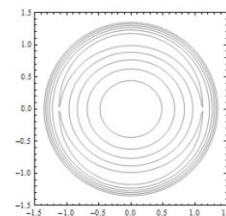


Figure 2: Expected structure for envelope driven parametric motion at  $7.5 \times 10^{13}$  ppp.

## MAIN BEAM STUDY

### Evolution of Halo Near Envelope Resonance

The evolution of beam and halo in transverse phase space for a beam of  $7.5 \times 10^{13}$  ppp, on turns 9, 12, 15, is shown in Figure 1: particles are coloured according to their emittance on turn 1. Figure 2 shows the expected contours from closely related theory ([4] described below). All plots are normalised particle phase space:  $(y/\sqrt{\beta}, y(\alpha/\sqrt{\beta}) + y'/\sqrt{\beta})$ , but scaled such that unit radius corresponds to the edge of the RMS equivalent Kapchinsky-Vladimirsky (KV) beam. The edge of the corresponding, simulated WB is at  $\sim 1.2$ . Here  $\alpha, \beta$  are Twiss parameters, and  $(y, y')$  particle position and angle.

In Figures 1 and 2 a simulated WB beam is compared with the idealised contours for the equivalent KV beam. In reality, the contours for these two cases will differ due to non-linear forces in the outer core of the WB distribution. However, away from the core, and for the relatively small tune shifts in a ring, we may expect halo structures to be similar. The simulation clearly shows features that can be identified with the expected structure at larger amplitudes: particle paths separate as they encounter island structures. The whole structure rotates slowly in turn-by-turn “snap shots” as determined by the fractional part of the envelope tune. This fairly crude comparison strongly suggests the presence of parametric resonance in its expected form. Detailed observations of other similar data over many turns clearly show particles trapped and oscillating within islands as expected.

### Behaviour with Non-Resonant Envelope

At low intensities ( $6 \times 10^{13}$  ppp), with beam far from envelope resonance, the dominant envelope harmonic is that from the driving term  $2Q_y=7$ , which is periodic with revolution. Figure 3 shows the corresponding Poincaré map of three selected particles, with a range of vertical ( $\epsilon_y$ ) but small horizontal ( $\epsilon_x$ ) emittances. This clearly shows the characteristics expected for 2:1 envelope driven parametric motion.

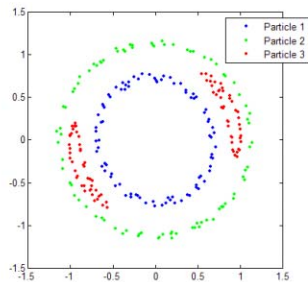


Figure 3: Single particle trajectories showing expected structure induced by envelope oscillations.

## TEST BEAM STUDY

### Simulation Study: Halo in One Plane

These studies made use of ORBIT test beams (“testHerds”) and Poincaré mapping routines [3]. Whilst the more direct observations of the previous section give clear indications of expected motion, it is only by locking observations to the envelope oscillation that single particle motion can be unequivocally associated with parametric resonance. The other limitations of studies based on the main beam are that it is the product of beam growth mechanisms, which may not sample all phase space, and can obscure structure with historic artefacts. ORBIT overcomes these problems by introducing a diagnostic test beam that “feels” but does not “push” the main beam: distributions of these test beams are then saved at a particular phase of the envelope oscillation (second moment), which yields the Poincaré map [3].

These simulations, with the  $2Q_y=7$  driving term, look at just the vertical plane, with Poincaré maps synchronized

to the vertical envelope (maxima). The test beam consists of a set of particles at 11 equally spaced, discrete values of  $\epsilon_y$  over the machine acceptance, but with zero  $\epsilon_x$ . Trajectories are plotted over  $\sim 10$  turns, on the left of Figure 4. The axes here are normalised, with unity radius corresponding to the equivalent KV beam edge (green). The WB edge is indicated by the red dotted line. These show the motion of the test beam (before *much* redistribution of the main beam occurs), for intensities of  $7.0, 8.0, 8.5 \times 10^{13}$  ppp from top to bottom.

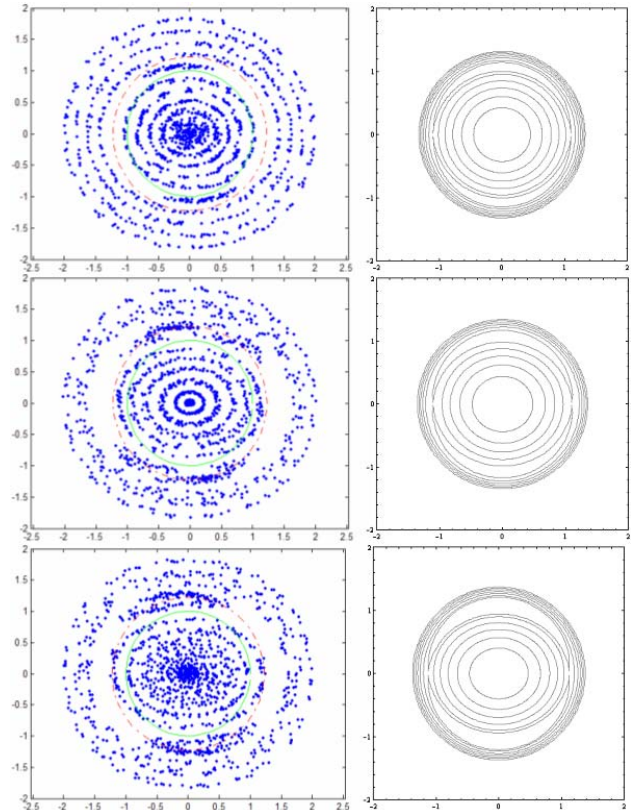


Figure 4: Halo structure from simulation (left) and theory (right). Intensity increases top to bottom.

The plots clearly show island structures locked into the envelope frequency, which extend and move towards the core with increasing intensity. The change of the core contours from circular to elliptical is also clear.

### Theoretical Comparison: Halo in One Plane

The behaviour of the test beam is now compared to a closely related theoretical model [4], which gives an analytical prediction for halo structure. The model treats a circular machine, with smooth focusing, in two transverse dimensions, with equal tunes and emittances, a matched KV distribution, driven by a quadrupole term. The plots, the right hand side of Figure 4, show contours of the invariant Hamiltonian (particle trajectories) derived from this model (equations 49-52 of [4]). The orientation corresponds to core envelope maxima, with the horizontal axis being the normalised width coordinate. Values of parameters used are: (bare tune)  $\nu_0=3.83$ , (relative gradient error)  $\epsilon=0.002$ , (azimuthal harmonic)  $m=7$ , (order)  $n=2$  and (depressed tune)  $\nu=Q_{RMS}$ .  $Q_{RMS}$  is the

depressed incoherent tune of the RMS equivalent KV beam. For the plots  $Q_{RMS}$  are calculated corresponding to simulated intensities of  $7.25, 7.50, 7.75 \times 10^{13}$  ppp, from top to bottom, chosen to show corresponding features in the simulation. The model assumes particles have zero emittance in the orthogonal (horizontal) plane.

It can be seen that the essential features of the model are present in the simulations: the presence of islands that enlarge with intensity, and increasingly elliptical core contours. It is also important to note that the simulation includes many more effects than the simplified theoretical model. The results show that the expected dynamics are the main identifiable process, but also indicate the presence of other effects. These are almost certainly associated with the non-stationary, non-KV beam starting to redistribute as it passes through the ISIS AG lattice. Resolution of structure is also limited by the number and placement of test particles used.

In fact, highly detailed comparison between the simulation and theory is not justified because of some important differences in assumptions. The theory assumes the simpler case of equal tunes in both planes ( $Q_x=Q_y$ ), and ISIS has non-equal tunes. Whilst envelope oscillations for the theory are closely coupled (small tune split), on ISIS they are effectively independent (large tune split [1]). However the results, using simply derived RMS equivalent parameters, do seem to describe the same underlying behaviour. It is hoped that future work will provide some analytical results for the important large tune split case. Note that accurate matching was essential in achieving the agreement in Figure 4.

#### Simulation Study: Halo in Two Planes (4D)

The simulations were repeated, as above, but with test particles having a range of finite  $\epsilon_x$ . Figure 5 shows trajectories of test particles, locked to vertical envelope motion, over  $\sim 10$  turns: left and right hand plots show normalised horizontal and vertical phase space respectively. The lowest plot corresponds to that in Figure 4. The  $\epsilon_x$  of particles increases from bottom to top: Intensity is  $8.5 \times 10^{13}$  ppp.

The results suggest that many features of the 2D case remain when particles are near the core in the orthogonal plane. However, the core region seems to shrink as larger horizontal emittances are allowed. This 4D motion is potentially very complicated and it seems likely that particles will have more routes to the halo in this case. Determining the importance of these effects requires more study. This is discussed in [5].

## CONCLUSIONS

Halo structures generated in simulations, near the half integer resonance, agree with expectations from theory [4], indicating they are an important aspect of beam evolution. Availability of closer theoretical models with non-equal tunes would allow more detailed comparisons.

Note that PIC predictions of the number of particles migrating to the halo are not tested here, and are a key

factor in predicting loss. Other important effects are those of images [6], momentum spread [5] and longitudinal motion. These are subjects for future work.

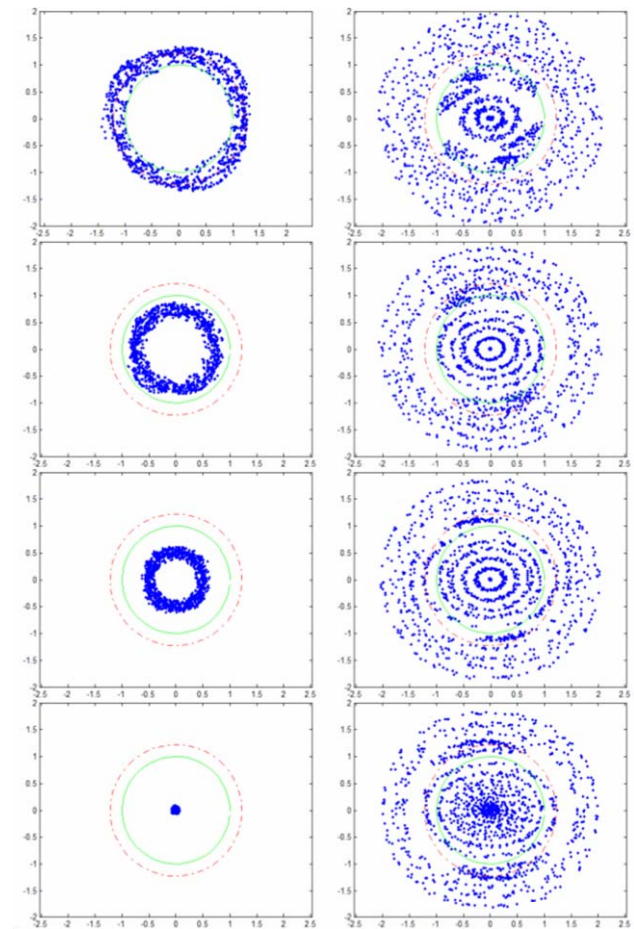


Figure 5: Effect of horizontal emittance (left) on vertical motion (right).

## ACKNOWLEDGEMENTS

The author acknowledges many useful discussions with S J Brooks of ASTeC Intense Beams Group.

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