CONTINUOUSLY ADJUSTABLE PERMANENT MAGNET QUADRUPOLE FOR A FINAL FOCUS

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Abstract

A permanent magnet quadrupole with continuous strength adjustability has been fabricated. It has a five-ring-singlet structure, which was originally proposed by R.L.Gluckstern. Its small overall diameter allows an out-going beam-line to be installed pass close by the magnet. Since the permanent magnet pieces do not have any mechanical vibration source in themselves, this magnet could be suitable as a quadrupole in a final focus doublet. In this report, such a quadrupole system is presented.

INTRODUCTION

Since recent development for a Permanent Magnet Quadrupole (PMQ) enables high degree of field strength, a PMQ can be used as a focus magnet for a high-energy beam. However a focus magnet requires the tuning of field strength for the sake of practical beam energy and focal length. A five-discs-singlet configuration proposed by Gluckstern works as a PMQ, whose strength is continuously adjustable [1][2]. Each disc of a Gluckstern's PMQ comprises a PMQ, and the field strength in it is altered by rotating the discs with respect to each other (Figure 1). Though x-y coupling effect caused by a skew of each disc can be theoretically cancelled in this design, fabrication errors and rotation errors alter the situation. The effect of x-y coupling may prove fatal to a beam whose size in x-plane and y-plane are considerably different as in a case at Interaction Point (IP) of International Linear Collider (ILC).

We estimated an x-y coupling effect caused by a rotation error and fabrication errors, especially a length error of each disc and a shift in the magnetic centre of each disc at IP in ILC. We are fabricating it in practice. We constructed the prototype magnet and experimentally measured the field strength in each disc and analyzed the harmonics and a shift of a quadrupole component of magnetic fields. The harmonic analysis is discussed compared with the estimation above.

Now in a baseline design for ILC, a super-conducting magnet is supposed to be used as a Final Focus Quadrupole (FFQ) doublet. This may prove not to be good because it will have the mechanical vibrations due to liquid helium flow. ILC design also requires FFQ to be compact. This is because the crossing angle of ILC is very small (14 mrad), the out-going beam must pass by very close to the magnet. A Gluckstern's PMQ satisfies those requirement, so it may be replaceable with the super-conducting magnet. We are developing a Gluckstern's PMQ aiming for the Accelerator Test Facility (ATF2) at first.

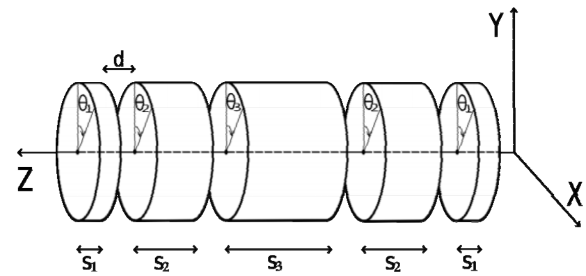

Figure 1: A Gluckstern's PMQ.

ESTIMATION OF ERRORS

We estimated an x-y coupling effect caused by three types of errors associated with each disc, namely a rotation error, a length error and a shift. This estimation includes the calculation of transfer matrices neglecting fringing field and multipole components.

We optimized the disc length such that an x-y coupling effect cancels out. In the transfer matrices calculation we set an absolute value of a rotation angle of a disc is equal to the others as in a following Eq. (1). This condition never prevented the x-y coupling cancellation.

$$\theta \equiv \theta_1 = -\theta_2 = \theta_3 \tag{1}$$

To simplify calculation, we introduced also the constraint of magnet length.

$$2s_1 - 2s_2 + s_3 = C, \tag{2}$$

where C is a constant representing the minimum of the width. In this estimation, we chose C as zero. This means that focusing force can be changed down to zero, and it should be noted that strength of focusing is approximated in proportion to length of the magnetic filed. Then we define new parameters, μ, λ and S, as follows

$$S \equiv 2s_1 + 2s_2 + s_3 \quad \lambda \equiv \frac{s_1}{S} \quad \mu \equiv kS, \tag{3}$$

where the constant, k^2 [L^{-2}] is the coefficient of the quadrupole component normalized by multiplying e/p, that is

$$k = \sqrt{\frac{B'}{B\rho}}. \tag{4}$$

Since Eq. (1), (2) include three equations independent from each other, six parameters in Gluckstern's PMQ, θ_1~θ_3, s_1~s_3 can be written down with three parameters, μ, λ and θ.

The transfer matrix of Gluckstern's PMQ, M_{GPMQ} is still complicated as follows.

$$M_{GPMQ} = M_{RQR}(s_1,k,\theta_1)\cdot M_{DS}(d)\cdot M_{RQR}(s_2,k,\theta_2)\cdot M_{DS}(d)\cdot M_{RQR}(s_3,k,\theta_3) \cdot M_{DS}(d)\cdot M_{RQR}(s_2,k,\theta_2)\cdot M_{DS}(d)\cdot M_{RQR}(s_1,k,\theta_1) \tag{5}$$

where $M_{DS}(d)$ is a transfer matrix representing Drift Section whose length is d, using a transfer matrix represents Quadrupole magnet whose length is s, and whose coefficient is k, $M_Q(s,k)$, $M_{RQR}(s,k,\theta)$ is given by

$$M_{RQR}(s,k,\theta) = M_R(\theta)\cdot M_Q(s,k)\cdot M_R(\theta), \quad (6)$$

where $M_R(\theta)$ is a rotation matrix with an angle θ. These transfer matrices are 4×4 matrices. The complication of calculations can be taken away under the above-mentioned assumption. In the calculations, μ is assumed to be small, and we neglected the third order or above terms. In practice the parameter, μ of the magnet under fabrication is about 0.090 (k is 0.41 [m^{-1}], S is 0.22 [m]).

The effect of an x-y coupling can be calculated with nominal parameters at IP in ILC. When nominal beam size and divergence at IP is represented as $\boldsymbol{X}$ (Table 1), practical beam size and divergence at IP can be represented as $\boldsymbol{X}^*$, nominal transfer matrix of FFQ is $\boldsymbol{M_Q}$ and the practical transfer matrix of a Gluckstern's PMQ as FFQ is $\boldsymbol{M_Q}^*$, and so on (Figure 2). Namely $\boldsymbol{X}$ and $\boldsymbol{X}^*$ are defined as follows,

$$X = M_{DS}M_QX_0$$
$$X^* = M_{DS}M_Q{}^*X_0. \quad (7)$$

The effect of an x-y coupling at IP, ΔX is

$$\Delta X = X - X^* = M_{DS}\left(E - M_Q{}^*M_Q{}^{-1}\right)M_{DS}{}^{-1}X. \quad (8)$$

We write down the elements of the matrix explicitly,

$$\Delta X = \begin{pmatrix} 0 & Lm_{12}m_{22}{}^{-1}L^{-1} \\ Lm_{21}m_{11}{}^{-1}L^{-1} & 0 \end{pmatrix} X\ , \quad (9)$$

where

$$M = \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix}\quad M^* = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\quad L = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}.$$

It should be noted that $\boldsymbol{M}$ and $\boldsymbol{M}^*$ are 4×4 matrices, but $\boldsymbol{L}$ and $\boldsymbol{m_{11}}$-$\boldsymbol{m_{22}}$ are 2×2 matrices.

Table 1: Nominal parameters of ILC

Parameter	Units	Value
Max Energy	GeV	250 (500)
Distance from IP to first Quad	m	3.5-(4.5)
Crossing Angle at IP	mrad	14
Beam size at IP, σ, x/y	nm	639/5.7
Beam divergence at IP, θ, x/y	µrad	32/14

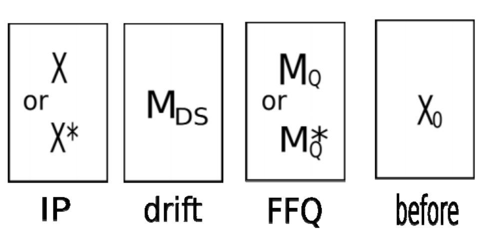

Figure 2: Sections by FFQ.

A Rotation Error

We calculated with ILC parameters ΔX when a rotation error $\delta\theta$ exists on each disc of a Gluckstern's PMQ with ILC parameters. Then we fixed length of a Gluckstern's PMQ to 220 mm, and used the optimum length as a length of each disc. For ΔX is to be less than 10 percents of X, $\delta\theta$ on each disc has to be less than the value shown in Table 2. We can say that the x-y coupling effect is in about proportion to the length of each disc.

Table 2: Nominal $\delta\theta$ on each disc

Disc	Optimum length [mm]	Nominal $\delta\theta$ [rad]
First	17.33	$< 2.2 * 10^{-4}$
Second	55.00	$< 7.0 * 10^{-5}$
Third	75.34	$< 5.3 * 10^{-5}$
Forth	55.00	$< 7.6 * 10^{-5}$
Fifth	17.33	$< 2.5 * 10^{-4}$

A Length Error

It was found that a length error of each disc has to be less than 100 µm for ΔX be within less than 10 percents of X. Since a fabrication error can be controlled within 100 µm, the x-y coupling caused by a length error isn't as bad as a rotation error. When fringing field in edges of each disc exists, since effective length of the field is changed a little, it is also shown the cause of the x-y coupling at IP never affected a change of the filed length derived from fringing filed.

A Shift

A shift of a disc does hardly affect x-y coupling at IP in ILC, but shifts the position of interaction of beams instead. If the shift at IP less than 1 nm is wanted, a shift of each disc must be less than 1 µm. It is understood that a shift of each disc compressed less than 1µm is difficult to be realized but not impossible. As a first step, we measured the shift of a disc experimentally. It was found that the shift was in an order of 10 µm. This value is rather large and it attributable to fabrication errors. It can be improved so the shift less than 1 µm is not impossibly.

MEASUREMENT

We have fabricated only four discs yet not including the second disc (Figure 3). The second disc is going to be fabricated after the jig proposed now for making fabrication errors less will get to be ready.

We measured the field strength and analyzed harmonics (Table 3). Each value in Table 3 is the mean of values, which are obtained as the field strength at 36 points on a circle of radii, r = 4, 8, 12 mm. Then the filed strength was measured as we were rotating each disc put on the channel with a fixed probe. The harmonic analysis was done by the fitting on those data assuming the when we assumed field strength included dipole, quadrupole and sextupole componets. The higher multipole components were neglected. It is noted that the systematic error of the probe is still smaller than measurement errors.

Table 3: Harmonics analysis for each disc

Disc (length)	**Dipole (STD DV) [G]**	**Quadrupole (STD DV) [G/cm]**	**Shift (Error) [μm]**
First (20mm)	13.4 (7.45)	1740 (113)	76.8 (47.9)
Third (70mm)	5.87 (1.31)	2960 (27.7)	19.8 (4.62)
Forth (55mm)	99.0 (19.4)	2690 (97.3)	369 (85.4)
Fifth (20mm)	15.1 (2.70)	1670 (91.2)	90.1 (21.1)

Figure 3: The first disc fabricated.

When discs whose length are 20 mm or smaller, the magnetic fields tend to escapes out of discs because of the diameter is bigger than the length.[3] For instance at the centre of the magnet, the field strength depends on the magnet length as,

$$B(s) = \{F(z - s/2) - F(z + s/2)\} B_{\max}. \quad (10)$$

The variable s is the length of the magnet. The constant, B_{max} represents the field strength at the centre if the magnet length were infinite. *F(z)* is a factor of B/ B_{max}, where *B* is the field strength at the position of z on z-axis of magnet whose length is semi-infinite.

$$F(z) = \frac{1}{2}\left[1 - \frac{z}{8}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\left\{\frac{v_1^2 v_2^2 \left(v_1^2 + v_1 v_2 + v_2^2 + 4 + 8/v_1 v_2\right)}{v_1 + v_2}\right\}\right] \quad (11)$$

where

$$v_i = \frac{1}{\sqrt{1 + (z/r_i)^2}}.$$

The constant, r_1 is the inner radius of a quadrupole magnet, and r_2 is the outer radius of the magnet.[4] At the centre of the magnet, it is showed $B(z, s=20, 55, 70)/B_{max}$ is as Figure 4. It is found that the difference of the value of quadrupole component in Table 3 is roughly derivable from this effect.

It should be noted that the shift of forth disc is bigger than the others. That is caused by fabrication errors, for we failed to fabricate it well. We expect a shift of each disc to be less than 1 μm by making fabrication errors to be as small as possible in practice.

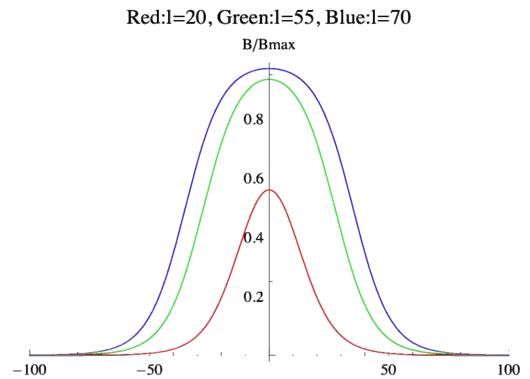

Figure 4: Field strength at the centre of discs.

CONCLUSION

Firstly, we estimated an x-y coupling effect in a Gluckstern's PMQ caused by fabrication errors. Then the effect of x-y coupling was estimated at IP with nominal parameters of ILC. As fabrication errors, we considered the effects of three errors: a rotation error, a length error and a shift of each disc. It is found out that a length error has negligible effect. We need to evaluate the effects including fringing field and higher multipole components by beam-tracking calculation using calculated magnetic fields.

Secondly, we fabricated four discs of a Gluckstern's PMQ, measured field strength, and analyzed harmonics, dipole component and quadrupole component. It is found out that a shift of the forth disc, which we failed to fabricate well, is still bigger than the others. We are fabricating a jig making fabrication errors smaller. We need to measure a rotation error by fabricating a Gluckstern's PMQ.

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