CONSTANTS AND PSEUDO-CONSTANTS OF COUPLED BEAM MOTION IN THE PEP-II RINGS*

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Abstract

Constants of beam motion help as cross checks to analyze beam diagnostics and the modeling procedure. Pseudo-constants, like the betatron mismatch parameter or the coupling parameter det C, are constant till certain elements in the beam line change them. This can be used to visually find the non-desired changes, pinpointing errors compared with the model.

BETATRON MISMATCH

The big betatron mismatch of a factor of 3 in the High Energy Ring (HER) was reduced using bumps of 1.3 mm in strong sextupoles (SF3/6) [1]. This is successful if the mismatch is in such a way that the beat is at a minimum (or maximum) at the location of the sextupole and the tune is close to the half integer. The mismatch could only be reduced down to about 1.4 (Fig. 1) with this method, since another source seems to be coming from the other phase (45° in phase advance).

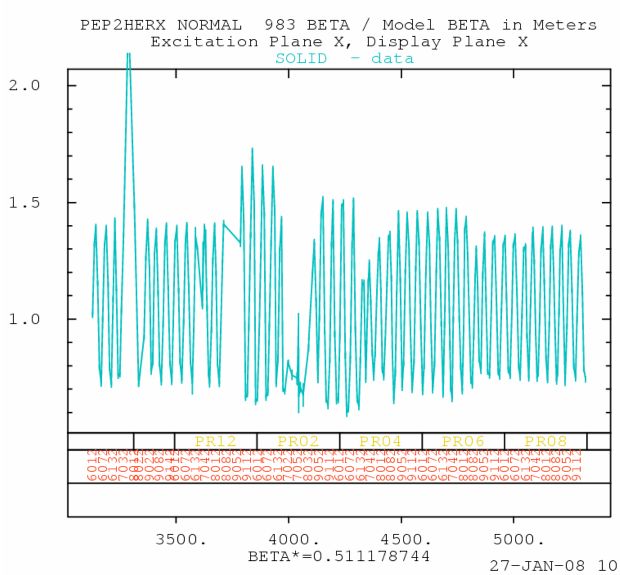

Figure 1: Online betatron ratio showing a mismatch.

Since the betatron function and the ratio of the measured betatron function to the model go up and down, it is difficult to see where the sources of the mismatch occur. Using a pseudo constant, the mismatch parameter *M* [2], which is roughly the peak of the betatron ratio or one over the minimum (where the alpha function is zero), the mismatch shows a lot of structure (see Fig. 2). There are some missing Beam Position Monitor (BPM) readings (29-40 and 52) and the interaction point is between BPM # 94 and 95.

*Work supported by Department of Energy contract DE-AC03-76SF00515.

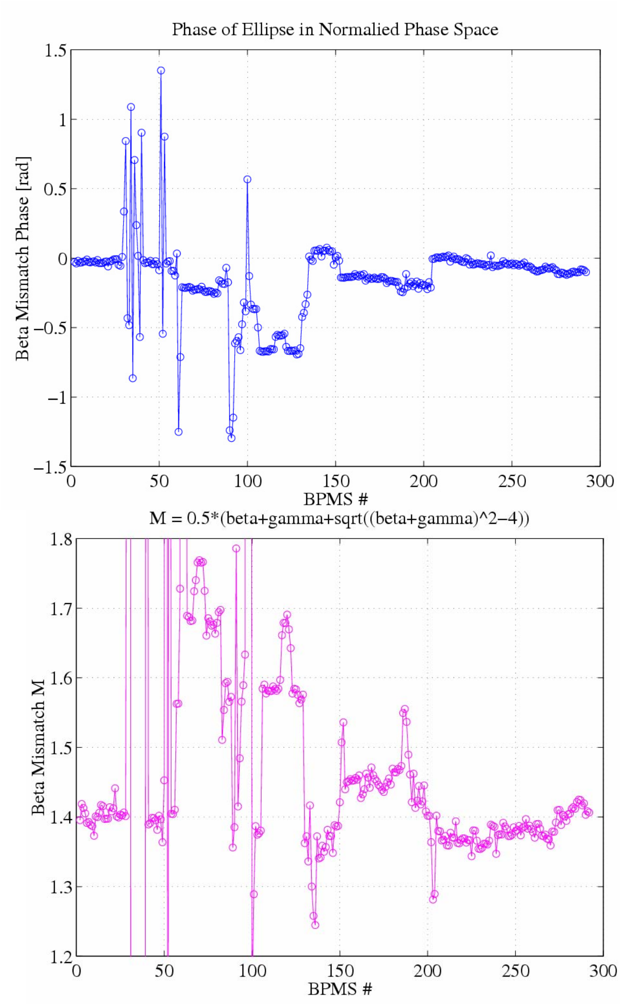

Figure 2: Betatron mismatch parameter *M* (bottom) and the mismatch phase (top) versus BPM #.

The sections, where the tune quadrupoles are located (# 133-153 and 185-205), show a more than expected variation. The reason is probably the implementation of matching solutions over the years, which focused on the global match instead of finding the localized root causes.

The main steps which cause a beta mismatch change are at BPM # 56 and 130 where there are no sextupoles and only dispersion suppression quadrupoles. It seemed that here is where an error sneaked in. The sextupole duplets SF3 and SF6 are at 67/73 and 116/122, where there is a clear mismatch bump of 0.1 visible in between.

Near the IP there are still some more rapid changes, but they might come from the fact that this is so far only a non-coupled treatment, although the coupled case is considered in [2].

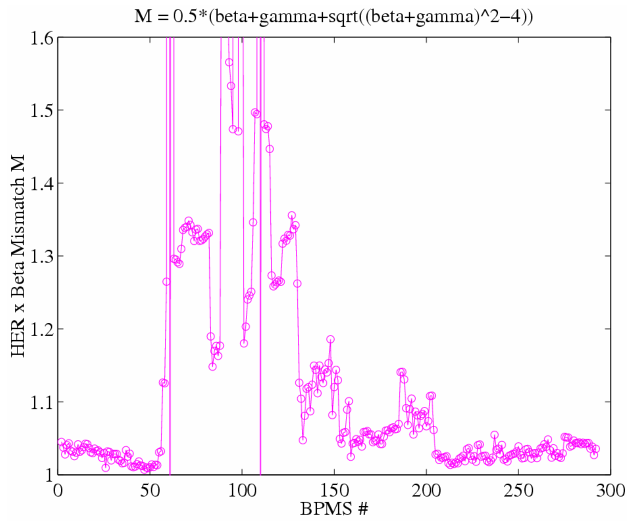

Figure 3: Betatron mismatch parameter M with lower mismatch versus BPM #.

A better matched lattice is show in Fig. 3. The opposite bumps are visible at the SF3 and SF6 regions. The phase picture is much noisier, since the phase of a nearly round ellipse is not well defined.

COUPLED PHASE ADVANCE

Instead of calculating the measured phase advance like

$$\text{Phi\_}x_1 = \text{atan2}\{\ \text{imag}(\ \text{fft}(x_1)\ ),\ \text{real}(\ \text{fft}(x_1)\ )\ \}, \qquad (1)$$

where x_1 is the horizontal reading of the mode 1 excitation, it is possible to calculate the "coupled phase advance" in a similar fashion, just replacing x_1 with y_1. The same is true for mode 2. Figure 4 shows the coupled phase advance Phi_y_1 and Phi_x_2 along the ring. These phase advances can go positive and negative, which is not immediately obvious to many people, but the measurements supports it.

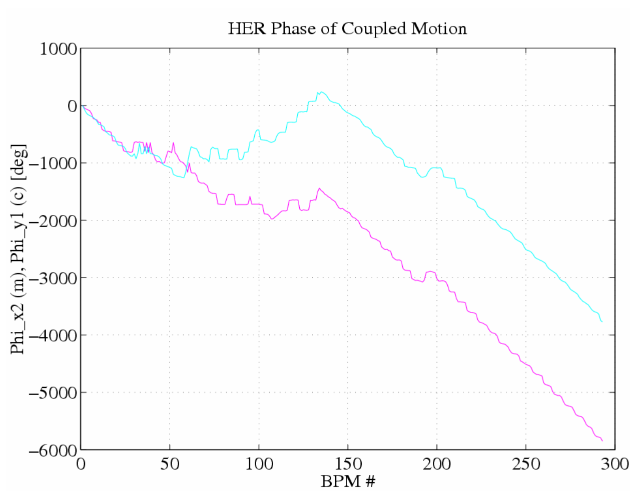

Figure 4: Phase advance of the coupled beam motion versus BPM #.

Where the direction changes, is normally a strong skew component located (skew quadrupole or vertical offset in sextupole). The coupled phase ellipse (e.g.: x_2 - x'_2) is inverted (flipped) there and detC changes sign. The phase advance shows stair case like behavior at places where the coupled ellipse is narrow (like a strong mismatch). At these places it is very tricky to adjust the 2π jump in the right direction, so the two phases should be closer on top of each other (only just the difference between the main x and y phase advance).

COUPLING PARAMETER C12BAR

The coupling C12_bar [3] is defined as how much of the mode 1 oscillation (mainly in x) goes into y at 90°:

C12bar = sqrt { beta_y1) / beta_y2 } * sin(+phy1 – phx1)

or (2)

C12bar = sqrt { beta_x2) / beta_x1 } * sin(–phy2 + phx2)

where beta_y1 for instance is the betatron function of mode 1 going into y and phy1 is the phase of that motion defined by Eq. 1. The disadvantage is the mixture of mode 1 and 2 in one expression. By multiplying both sides of Eq. 2 with sqrt { beta_x1 * beta_y2 } we get:

C12b_beta_1 = sqrt{ b_y1 * b_x1 } * sin(+phy1 – phx1)

and (3)

C12b_beta_2 = sqrt{ b_x2 * b_y2 } * sin(–phy2 + phx2)

for the two modes. Figure 5 shows these variables versus z for both modes and the design. Since there are now two independent measurements for each mode and not a combination of the two, any systematic problems with the BPMs should become visible.

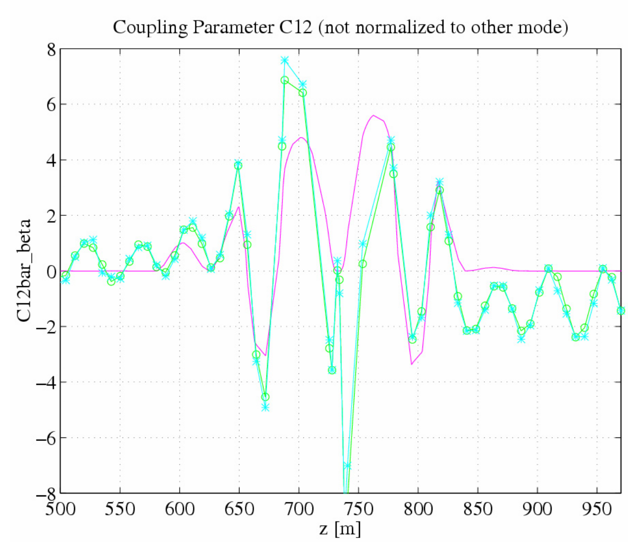

Figure 5: Coupling parameter C12bar_beta versus BPM #. Approaching the interaction point (730 m) from the opposite side creates something like a "coupled" hourglass effect for x' going into an additional y-size. This makes the HER beam very sensitive to LER y changes.

COUPLED CONSTANTS OF MOTION

By calculating the ratio of mode 1 with the x_1 reading at 0° and y_2 at 90° minus x_2 at 90° and y_1 at 0° divided by a similar expression for mode 2 [4]

$$Q12 / Q34 = -(x1y2 - x2y1) / (x3y4 - x4y3) \tag{4}$$

we get a constant of motion for a coupled case.

Figure 6 shows this "constant" for the HER beam indicating quite some variations. The problem with this constant is that there has to be some coupling present, otherwise the ratio can become infinity.

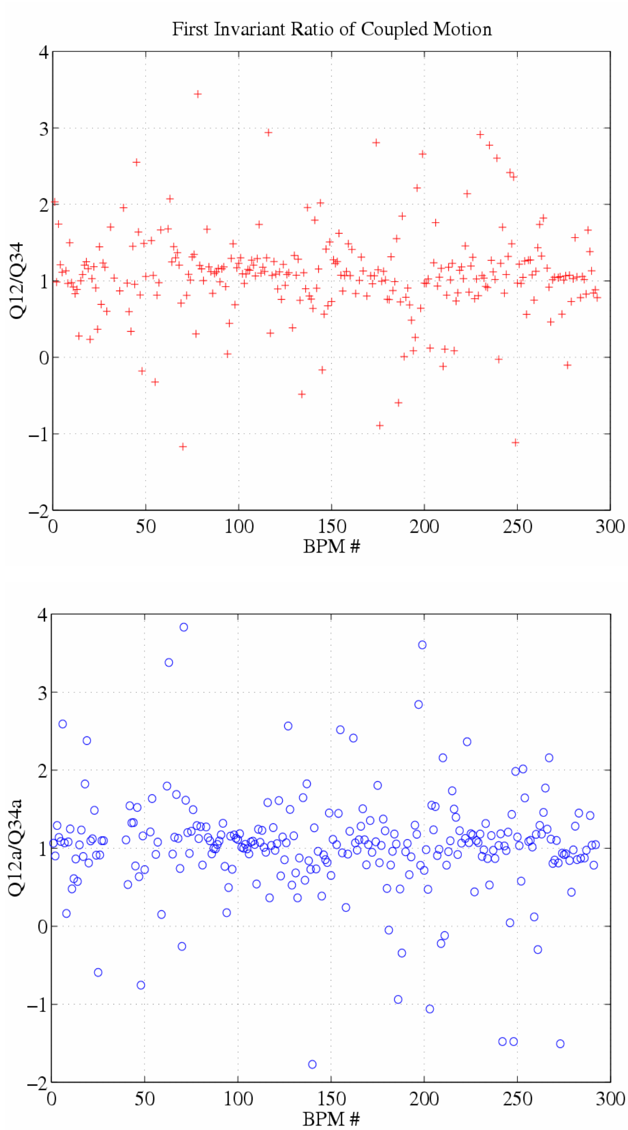

Figure 6: Coupled constants of motion versus BPM #. The top shows the ratio of mode 1 and mode 2 at one location (beta-like) while the bottom shows the ratio with the next BPM and is therefore more "alpha"-like.

COMPARISON WITH COUPLED BETATRON FUNCTIONS

Let's define the coupled betatron functions α, β, γ for mode1 with a subscript 1x for the normal part of mode of 1 and 1y for the coupled part, and the same for mode 2. Also some imaginary parameters are very handy (with a tilde) giving a "size-like" and "angle-like" interpretation of the phase space ellipse:

$$\begin{aligned} \sqrt{\tilde{\beta}_{1y}} &= \sqrt{\beta_{1y}} \cdot (\cos\nu_1 - i\sin\nu_1) \\ \sqrt{\tilde{\gamma}_{1y}} &= \frac{1}{\sqrt{\beta_{1y}}} \cdot \big(\alpha_{1y}\cos\nu_1 - u\sin\nu_1 \\ &\qquad - i(\alpha_{1y}\sin\nu_1 + \cos\nu_1)\big) \end{aligned} \tag{5}$$

where u =det C = $\beta_{1y}\, \gamma_{1y} + \alpha_{1y}^2$.

With these the following expressions are the same for mode 1 and mode 2

$$\begin{aligned} \mathrm{Im}\left(\sqrt{\beta_{1x}} \cdot \sqrt{\tilde{\beta}_{1y}}\right) &= \mathrm{Im}\left(\sqrt{\tilde{\beta}_{2x}} \cdot \sqrt{\beta_{2y}}\right) \\ \mathrm{Im}\left(\sqrt{\beta_{1x}} \cdot \sqrt{\tilde{\gamma}_{1y}}\right) &= \mathrm{Im}\left(\sqrt{\tilde{\beta}_{2x}} \cdot \sqrt{\tilde{\gamma}_{2y}}\right) \\ \mathrm{Im}\left(\sqrt{\tilde{\beta}_{1y}} \cdot \sqrt{\tilde{\gamma}_{1x}}\right) &= \mathrm{Im}\left(\sqrt{\beta_{2y}} \cdot \sqrt{\tilde{\gamma}_{2x}}\right) \\ \mathrm{Im}\left(\sqrt{\tilde{\gamma}_{1x}} \cdot \sqrt{\tilde{\gamma}_{1y}}\right) &= \mathrm{Im}\left(\sqrt{\tilde{\gamma}_{2x}} \cdot \sqrt{\tilde{\gamma}_{2y}}\right) \end{aligned} \tag{6}$$

This implies that beside the first invariant Q12/Q34 which is effectively the same like Eq. 6 (a), there are three more constants, all together four (size-size, size-angle, angle-size, angle-angle).

SUMMARY

Besides the pseudo-constant of the beta mismatch, coupled constants of motions are discussed. The coupling parameter C12bar, the constant of motion Q12/Q34 and Eq. 6 are expressing the same coupling behaviour.

REFERENCES

[1] G. Yocky, "Beta-Beat Correction Using Strong Setupole Bumps in PEP-II", EPAC 2006, Edinburgh, Scotland.

[2] M. Sands, "A Beta Mismatch Parameter", SLAC-AP-85, April 1991.

[3] M.H.R. Donald, T.M. Himel, M.S. Zelazny, "Diagnosis of Coupling And Beta Function Errors in the PEP-II B-Factory", EPAC 2004, July 2004, Lucerne, Switzerland.

[4] Y. T. Yan, Y. Cai, W. Colocho, F.-J. Decker, "Validation of PEP-II Resonantly Excited Turn-by-Turn BPM Data", PAC07, Albuquerque, NM, USA.