MEASUREMENT OF COMPLEX COUPLING DRIVING TERM OF LINEAR DIFFERENCE RESONANCE USING TURN-BY-TURN BEAM POSITION MONITORS

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Abstract

We have developed a method of measuring a complex coupling driving term C of linear difference resonance using turn-by-turn beam position monitors (BPMs) on the basis of the perturbation theory with the single resonance approximation. Since both amplitude and phase of C are obtained, we can uniquely determine strengths of counter skew quadrupole magnets for correction of the linear resonance coupling. We measured the coupling driving term C using the developed method at the SPring-8 storage ring where small skew quadrupole components are distributed as error magnetic fields. The strengths of the counter skew quadrupole magnets derived from the measured C were consistent with the strengths adjusted to minimize vertical beam size near the linear difference resonance.

INTRODUCTION

Betatron coupling is one of the essentials for machine performance in circular accelerators such as an electron storage ring for high-brilliance light source and a particle collider of high luminosity. A strong coupling may induce optics distortion, shifts of betatron tunes and excitation of resonances. In case a certain equipment installed in the ring causes the betatron coupling, a local compensation of the error field near the equipment can be an effective mean. On the other hand, if the coupling comes from small error skew quadrupole components distributed along the ring, the local compensation is not a practical method. The small error fields can be made by complicated factors such as alignment errors and field errors of magnets of the ring. In that case, a global compensation of the betatron coupling can be a practical way. Each error skew field is very small, but if the small errors are distributed all over the ring, their integration can make a non-negligible contribution to the coupling driving term C. For the global coupling correction, two independent families of skew quadrupole magnets should be prepared to generate compensation terms to C in quadrature.

COUPLED BETATRON MOTION

We suppose small skew quadrupole magnetic components are distributed as error fields along the circumference of a circular accelerator. In such a case, the horizontal (x) and vertical (y) betatron beam motions can be coupled. The strong coupling resonance appears in the vicinity of a difference resonance $\nu_x - \nu_y \approx q(\text{integer})$. If the error skew quadrupole fields are sufficiently small in comparison with the fields of main magnets composing a lattice of the ring, we can treat the coupled betatron motions with the Hamilton perturbation theory [1] under the single resonance approximation. According to the perturbation theory, the x-y betatron beam motions are expressed by following formulas [1, 2],

$$\begin{aligned} x(s) &= \left(A_1 e^{\frac{-2\pi i \nu_1 s}{L}} + A_2 e^{\frac{-2\pi i \nu_2 s}{L}}\right)\sqrt{\frac{\beta_x(s)}{2}}e^{2i\pi\phi_x(s)} \\ &\quad + c.c, \\ y(s) &= \frac{C}{2}\left(\frac{A_1}{\nu_2} e^{\frac{2\pi i \nu_2 s}{L}} + \frac{A_2}{\nu_1} e^{\frac{2\pi i \nu_1 s}{L}}\right)\sqrt{\frac{\beta_y(s)}{2}}e^{2i\pi\phi_y(s)} \\ &\quad + c.c, \end{aligned} \tag{1}$$

where $c.c.$ denotes complex conjugate of the preceding term, $\beta_{x,y}(s), \phi_{x,y}(s), A_{1,2}, L$ and s are betatron functions, phase advances, integration constants, circumference of the ring and beam path length, respectively. The shifts $\nu_{1,2}$ of unperturbed betatron tunes $\nu_{x,y}$ are as follows,

$$\nu_{1,2} = \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2 + |C|^2}\right), \tag{2}$$

where $\Delta = \nu_x - \nu_y - q$. The complex number C in eq.(1) is referred to the coupling driving term and is given by the following formula,

$$\begin{aligned} C &= \frac{1}{2\pi}\int_0^L K_{err}(s)\sqrt{\beta_x(s)\beta_y(s)} \\ &\qquad \times e^{2i\pi\left[\phi_x(s)-\phi_y(s)-\frac{s}{L}\Delta\right]}ds \\ &\equiv |C|e^{2\pi i \phi_c} \end{aligned} \tag{3}$$

where $K_{err}(s)$ is the field strength of skew quadrupole errors.

COUPLING DRIVING TERM MEASURED BY TURN-BY-TURN BPM

We measured the coupling driving term C using a turn-by-turn BPM system of the SPring-8 storage ring. The x-y betatron oscillations expressed by eq.(1) were induced by pinging a single-bunch beam stored in the storage ring

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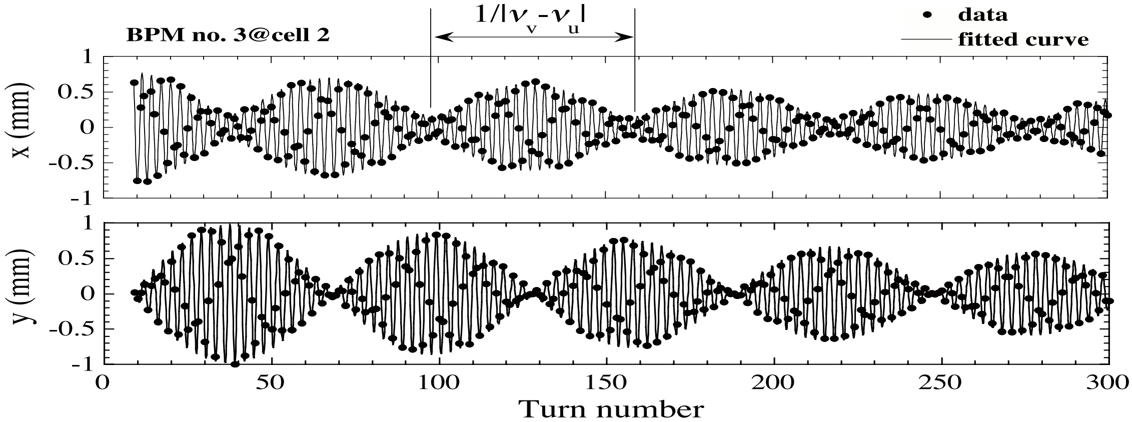

Figure 1: An example of the turn-by-turn BPM data (dots) measured at the operation point $(\nu_x, \nu_y) = (40.3509, 18.3477)$ near the linear difference resonance. The solid lines show the curves of eq.(4) fitted to the data.

in the horizontal direction with a pulsed bump magnet for beam injection. The betatron oscillations were then observed using the turn-by-turn BPMs located about every two cells of the storage ring. To increase sensitivity of measuring the vertical oscillation induced by the betatron coupling, we chose an operation point at the vicinity of the difference resonance. Equation (1) can be reduced to the following formulas suitable for fitting with measured turn-by-turn BPM data: if $\Delta > 0$,

$$\begin{aligned} x_n &= e^{-nD}\Big\{A_u \cos 2\pi\,(n\nu_u + \Phi_u) \\ &\qquad + A_v \cos 2\pi\,(n\nu_v + \Phi_v)\Big\}, \\ y_n &= e^{-nD}\Big\{B_u \cos 2\pi\,(n\nu_u + \Psi_u) \\ &\qquad + B_v \cos 2\pi\,(n\nu_v + \Psi_v)\Big\}, \end{aligned} \tag{4}$$

where the damping factor e^{-nD} was artificially added, ν_u and ν_v are tunes of the eigen-modes (u, v), respectively, and n is the turn number. If $\Delta < 0$, ν_u should be exchanged for ν_v in eq.(4). Free parameters in the fitting process are $A_u, A_v, B_u, B_v, \Phi_u, \Phi_v, \Psi_u, \Psi_v, \nu_u, \nu_v$ and D. Figure 1 shows an example of the turn-by-turn BPM data with the fitted curves. We can observe a modulation of oscillation amplitudes with frequency $|\nu_u - \nu_v|/T_{rev}$ in the x and y directions, where T_{rev} is the revolution time. The modulation in the x and y directions has a phase difference of 90 degree. This is a typical feature of the coupling resonance which exhibit the exchange of energy between the two planes.

The absolute value $|C|$ is derived from the fitted eigentunes ν_u, ν_v and amplitude parameters A_u, A_v, B_u and B_v. We can obtain the expression for $|C|$:

$$|C| = |\nu_u - \nu_v| \sqrt{1 - \left(\frac{A_v B_u + A_u B_v}{A_v B_u - A_u B_v}\right)^2}. \tag{5}$$

The phase $2\pi\phi_c$ of C is derived from the fitted phase parameters Φ_u, Φ_v, Ψ_u and Ψ_v. We can obtain the expression for ϕ_c:

$$\phi_c = \left\{ \begin{array}{c} \Psi_u - \Phi_u + \phi_x - \phi_y - \Delta \cdot s_i/L \\ \text{or} \\ \Psi_v - \Phi_v + \phi_x - \phi_y - \Delta \cdot s_i/L \end{array} \right. , \tag{6}$$

where s_i denotes the beam path length at the i-th BPM. We use designed values for the phase advances ϕ_x, ϕ_y of the unperturbed system, which are defined as zero at the beam injection point $s = 0$.

The fitting was carried out for the data of each turn-by-turn BPM, and so we obtained as many experimentally derived C as the number of the BPMs. The results are shown in Figure 2, where the obtained $|C|$ and ϕ_c are plotted as a function of the path length s_i of the i-th BPM. We see that $|C|$ and ϕ_c in Figure 2 are almost independent of s_i, and we adopted averages of these values at all the BPMs as measured values of $|C|$ and ϕ_c. The averages were $|C| = 0.01628$ and $\phi_c = 0.492$. Consequently, the real and imaginary parts were $\big[\mathrm{Re}(C),\ \mathrm{Im}(C)\big]$=$\big[-0.01625,\ 0.00085\big]$.

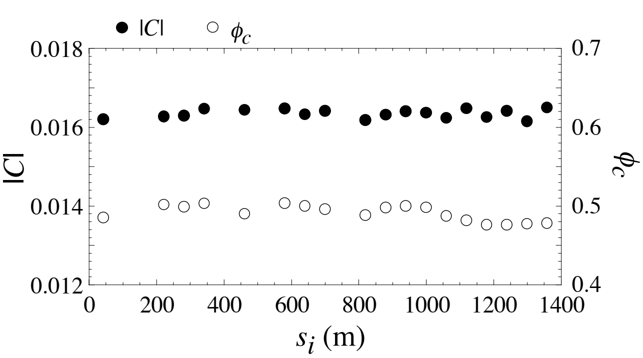

Figure 2: The absolute values (filled circles) and the phases (open circles) of C derived from the turn-by-turn BPM data measured at the operation point $(\nu_x, \nu_y) = (40.3509, 18.3477)$. The obtained $|C|$ and ϕ_c do not almost depend on the path length s_i of the i-th BPM.

THE STRENGTH OF THE COUNTER SKEW QUADRUPOLE MAGNETS

The SPring-8 storage ring is composed of the 36 normal Chasman-Green cells (CGCs) and the four maget-free long straight sections (LSSs) of 27m with matching sections [3]. Two-family counter skew quadrupole magnets for the coupling correction are installed at the achromatic arcs (ARCs) of the CGCs every two cells and at both ends of all the LSSs. Approximately, the phase $2\pi(\phi_x - \phi_y)$ in eq.(3) has a difference of $m\pi$ between the two ARCs or the two LSSs, and a difference of $(m + 1/2)\pi$ between the ARC and the LSS, where m is integer. A compensation term C_{SK} to the coupling driving term C is expressed by replacing K_{err} of eq.(3) with the strengths of the counter skew quadrupole magnets of the ARC and the LSS. The phases $2\pi(\phi_x - \phi_y)$ at the ARCs and the LSSs indicate that the compensation terms generated by the two-family correction magnets are in quadrature. Requiring C_{SK} to be equal to $-C$, we can uniquely determine the total strengths K_{ARC} and K_{LSS} of the counter skew quadrupole magnets of the ARCs and the LSSs, respectively. Using designed values for the betatron functions $\beta_{x,y}$, the phase advance $\phi_{x,y}$ and Δ in the unperturbed system, we obtained $K_{ARC} = -0.0129(\mathrm{m}^{-2})$ and $K_{LSS} = 0.0261(\mathrm{m}^{-2})$.

To demonstrate the validity of our method, we also carried out another approach to determine the strengths K_{ARC} and K_{LSS}, which were adjusted to minimize the vertical beam size σ_y expressed as the following formula [1, 2]: if the betatron coupling makes a dominant contribution to σ_y,

$$\sigma_y = \sqrt{\frac{\frac{1}{2}|C|^2}{\Delta^2 + |C|^2}\beta_y \epsilon_0}, \tag{7}$$

where ϵ_0 is a natural emittance in the unperturbed system. The vertical beam sizes measured by the x-ray beam imager (XBI) [4] and the two-dimensional interferometer (2D-interferometer) [5] using bending magnet radiation, and the absolute values of C measured by the turn-by-turn BPM are shown in Figure 3 as a function of K_{ARC} and K_{LSS}. We chose a operation point near the difference resonance in order to increase the response sensitivity of the vertical beam size to the skew fields. The strengths K_{ARC} and K_{LSS} giving the minimum vertical beam size were $-0.0115(\mathrm{m}^{-2})$ and $0.027(\mathrm{m}^{-2})$, respectively, where the measured $|C|$ also became minimum. The strengths K_{ARC} and K_{LSS} deduced from the measured C are almost consistent with that from the vertical beam size, but we can a see slight discrepancy between these two results. The possible explanations of the discrepancy are as follows; when the effective coupling driving term becomes very small with the counter skew quadrupole magnets, 1) eq.(3) is not applicable due to the higher order terms of C, 2) effect of the counter skew quadrupole fields on the vertical dispersion contributing to σ_y is not negligible.

SUMMARY

We have developed the method of measuring the complex coupling driving term C using the turn-by-turn BPM. C of the SPring-8 storage ring was measured by this newly-developed method and the validity was demonstrated experimentally. The field strengths of the two-family coupling correction magnets of the ARC and the LSS were deduced from C measured by our method. The determined strengths were almost consistent with that giving the minimum vertical beam size. The small discrepancy between the strengths obtained by those two approaches will not be important for the practical betatron coupling correction.

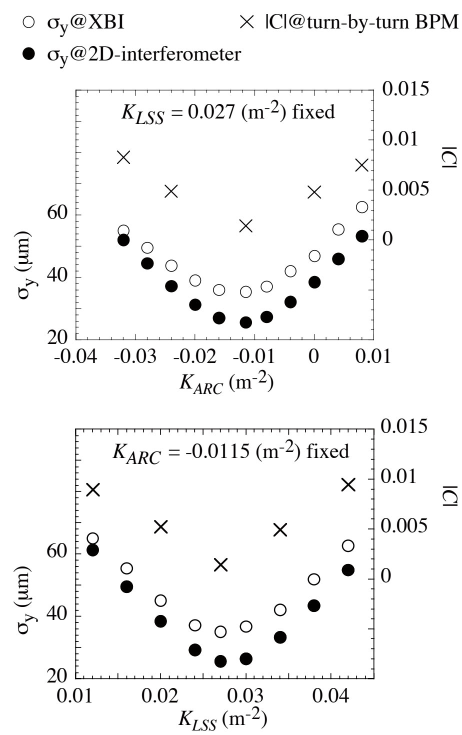

Figure 3: Vertical beam sizes measured by the XBI (open circles) and the 2D-interferometer (filled circles), $|C|$ (crosses) measured by the turn-by-turn BPM as a function of K_{ARC} and K_{LSS}. For both of the beam size monitors, the minimum vertical beam sizes are given when (K_{ARC}, K_{LSS}) are (-0.0115m^{-2}, 0.027m^{-2}), where the measured $|C|$ also becomes minimum. The operation point was chosen at $(\nu_x, \nu_y) = (40.3818, 18.3451)$ near the difference resonance.

REFERENCES

[1] G. Guignard, Phys. Rev. E**51**, 6104 (1995)

[2] M. Takao, Phys, Rev. ST-AB **9**, 084002 (2006)

[3] H. Tanaka et.al., Nucl. Instrum. Mehods Phys. Res. A **486**, 521 (2002)

[4] S. Takano et.al., Nucl. Instrum. Mehods Phys. Res. A **556**, 357 (2006)

[5] M. Masaki et.al., J. Synchrotron Rad. **10**, 295 (2003)