

GLOBAL OPTIMIZATION OF THE MAGNETIC LATTICE USING GENETIC ALGORITHMS*

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Abstract

In this paper we explore the use of multi-objective genetic algorithms (MOGA) to locate globally optimized lattice settings. MOGA is an efficient and robust tool which can practically address problems with numerous parameters, constraints, and objectives. The MOGA technique complements the previously introduced GLASS (Global Analysis of all Stable Settings) analysis technique providing the lattice designer with powerful qualitative and quantitative tools for lattice studies. Using the Advanced Light Source (ALS) for illustration, two examples of MOGA are shown – (i) 3 parameters with 2 objectives case and (ii) a 6 parameter 3 objective case.

INTRODUCTION

The traditional process of designing and tuning a magnetic lattice of a particle storage ring lattice to produce certain desired properties is not straight forward. Often solutions are found through trial and error and it is not clear that the solutions are close to optimal. The goal of this study is to explore using genetic algorithms for global optimization of storage ring lattices. Optimizing a lattice is a complicated problem for a variety of reasons

- Very nonlinear – many local optimums
- Many parameters – most storage rings have 4 or more families of quadrupoles
- Multi objective – typically one wants to optimize more than one parameter simultaneously (such as beta-functions, emittance, ...) and would like to find the optimal tradeoffs

Multi-Objective Genetic Algorithms (MOGA) are ideally suited to this problem. They are

- Very efficient and robust – can vary many parameters and search for multi-objectives
- Well suited for nonlinear discretely continuous solutions

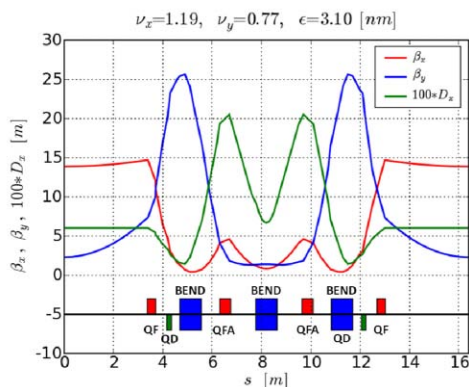


Figure 1: Twiss functions for one sector of the ALS adjusted to the present settings

*Supported by DOE BES contract DE-AC03-76SF00098.

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To illustrate the use of MOGA we chose to use the lattice of the ALS as an example. The ALS lattice consists of 12 sectors. Each sector is made up of a triple-bend achromat structure. In each sector there are 3 bends, six quadrupoles and 4 sextupoles sectors. In the nominal setting the lattice functions for one sector are mirror symmetric and shown in Fig. 1 for the nominal settings.

GLASS

Before introducing MOGA we begin with a GLASS analysis of the ALS lattice. GLASS (short for Global Analysis of all Stable Settings) [1] is a technique that gives a global view of the lattice.

For a full GLASS analysis one follows these steps

- Find all stable settings
- Compute properties of all stable settings
- Filter by property all settings that may be of interest

At the end of the process one has a database with all possible solutions and associated properties. Then querying the database against certain properties it is possible to find any and all lattice settings that satisfy the properties. For simple lattices this is not only possible but it also is very practical. Here we will only show the first step of a GLASS analysis for ALS. For a complete GLASS analysis see [1]. We assume mirror symmetry (just three quad families – QF, QD, QFA) and perform a grid scan locating all settings where the following constraints are satisfied

- Trace $|M_{x,y}| < 2$ and partition functions, $j_{x,z} > 0$
- $\beta_{x,y} < 1000\text{m}$, $\epsilon_x < 1\text{e-}4\text{mrad}$, $\sigma_x < 0.1\text{m}$.

The results are plotted in Fig. 2 with regions numbered.

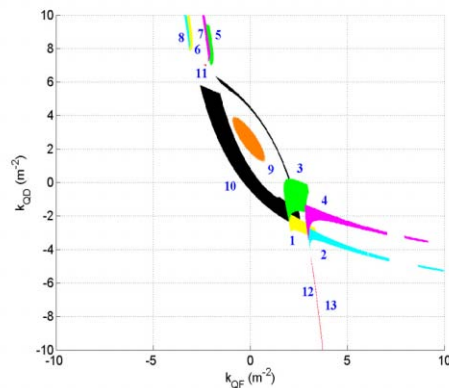


Figure 2: GLASS results for the symmetric ALS sector projected onto the QF/QD plane with 13 discretely continuous regions labeled.

Now we introduce genetic algorithms and show how one can find globally optimal sets of solutions where

multi-pole objectives (such as optimizing β -functions and emittances) are desired.

MULTI-OBJECTIVE GENETIC ALGORITHMS

The classical methods for multi-objective optimization are usually performed by applying different weight to each objective function, and then converting it into a single-objective optimization problem. The quality of solutions are depending on how the weight are chosen, and in some practical case, e.g. different unit and scale, it is not a trivial task. Genetic Algorithm (GA) together with Pareto optimality gives another approach to the global optimal of multi-objective problems.

Genetic Algorithms rely on the analogy with the law of natural selection. An initial population is generated randomly, ranked according to their value with respect to the optimization functions (dominance), and the best ones are chosen to create the next generation, and a random is then applied to these newly born children. This process is continued generation-by-generation until the stop condition is met, which can be either the number of generation, satisfaction of the result, or the stability of the population.

One can say that one solution dominates another solution when this solution is better in at least one optimization function, but no worse in the other functions. A second case would be one solution is better in some functions, but worse in others, and they are called non-dominated. A non-dominated set is composed of individuals where none of them is dominated by another one in this set. The set of non-dominated solutions is assigned rank 1. Once a non-dominated set are formed, a second non-dominated set can be formed in the rest of the population, and we assign a rank 2 to it. The natural selections are made from the first rank till the population size is full. The Optimal solution is the Pareto Set with rank 1 at the last generation.

The flow of GA is

- initialize the population
- naturally select the elite solutions and create the next generation.
- mutate the new generation.
- evaluate the objective functions of the new generation.
- repeat the selection (amongst the children and parents keeping the elitist from generation to generation), mutation until the stop conditions are met.

For more information on MOGA the following reference is suggested [2].

Example 1. 3 Parameters and 2 Objectives

In our first example of MOGA we find the globally optimize solution of a 3 parameter 2 objective search. The example is the following – the undulator brightness in a

light source depends upon the emittance and β -function in the undulator. For example for a 2 meter long ID the optimal β -function would be 1m and the optimal emittance is as low as possible. So for this example the following objective, constraints, and parameters are

- **Parameters** – k_{QF} , k_{QD} , k_{QFA}
- **Objectives** – Find the Pareto optimal curve where the objectives are to minimize emittance while keeping β (in our case we choose β_x) close to 1m in the insertion device straight.
- **Constraints** - $|\text{trace}(M_{x,y})| < 2.0$, $j_{x,y} > 0$, $\beta_{x,y} < 30\text{m}$, $\max|\eta_x| < 0.4\text{m}$

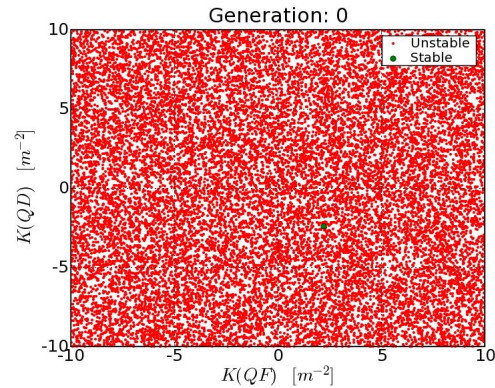


Figure 3: Initial distribution of quadrupole values projected onto the QF/QD plane. Values are between -10 and 10. (same as Fig. 2).

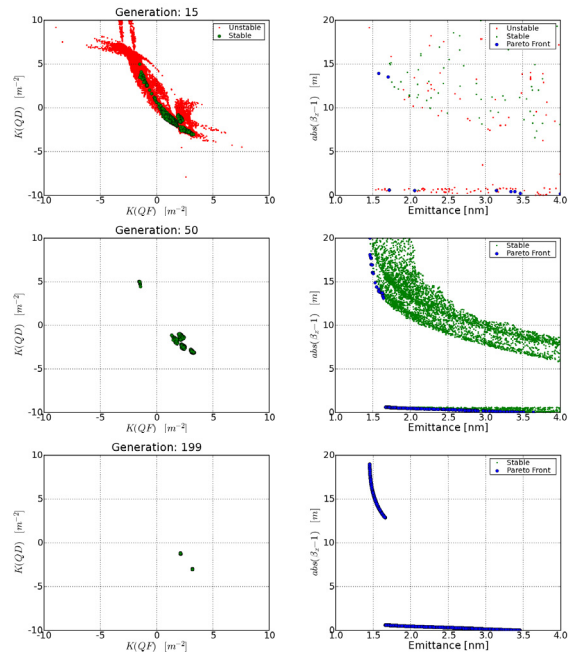


Figure 4: Distribution of generations 15 (Top), 50 (middle), and 199 (bottom) in parameter and objective space. The objective space axes are 1 to 4 nm hor. and deviation from 1 m vert. Red (not met all constraints), Green (mets all constraints), Blue (on Pareto optimum).

We begin by distributing an initial population. This is a random distribution of quadrupole (k_{QF} , k_{QD} , k_{QFA})

strengths. The population size we chose was 20,000 and the initial distribution is shown in Fig. 3. Then using MOGA, a new generation is created and evaluated. The process continues until convergence. For this example 200 generations were created. Computational time roughly 0.5 hours on a Pentium 4 duo core processor.

The population evolves in a way that they first tend to satisfy the constraints and later to converge upon optimal objective values. Let's look a few generations to better understand the process. In Fig 4., the generations 15, 50, and 199 are plotted in both parameter space and objective space. We see that at generation 15 (top plots) the population has converged upon the stable region that was shown in Fig.2. There are still many points at generation 15 and this is due to the tighter constraints than for the GLASS analysis. However in objective space one is far from convergence. In generation 50 (middle plots) one clearly sees the Pareto set starting to form (blue dots) and there are two discrete regions (Regions 3 and 4 in Fig. 2). Finally after 199 generations (bottom plots) the entire population has converged on the Pareto optimal set. In Fig. 5 we plot 3 lattices on the Pareto optimal.

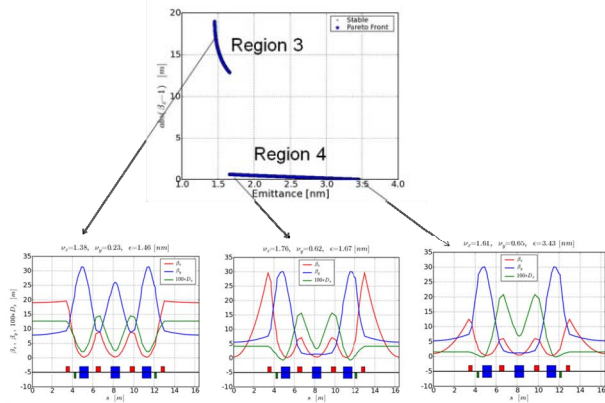


Figure 5: Twiss functions for three lattices on the Pareto optimal set. See the trade-offs between β_x and ϵ_x .

Example 2. 6 Parameters and 3 Objectives

In this example we use MOGA to explore finding global optimums with 6 parameters and 3 objectives. This is interesting because most accelerators have more than 3 “knobs” to optimize. In the case of larger number of parameters GLASS analysis rapidly becomes impractical. However MOGA is still very efficient and can handle more parameters. Here we search for optimal high/low beta structure using two sectors and imposing mirror symmetry. This is an interesting example because high beta straights are important for certain regions such as injection. The constraints are the same as before except with a 50 m β -max and the parameters and objectives are

- **Parameters** – k_{QF} , k_{QD} , k_{QFA} (left and right)
- **Objectives** – Find the Pareto optimal curve where the objectives are to minimize emittance while keeping β (in our case we choose β_x)

close to 1 m in the insertion device straight and 10 m in the other straight.

Again we chose to use a population of 20,000 but doubled the number of generations (200 – 400). Computational time roughly 4 hours on a Pentium 4 duo core processor. The Pareto optimum is a discretely continuous surface in objective space which is not plotted here. Fig. 6 is a plot of 2 of the Pareto optimum solutions. One solution has a large vertical β -function in the straight and the other doesn't. All of the solutions have similar lattice functions to one of these two.

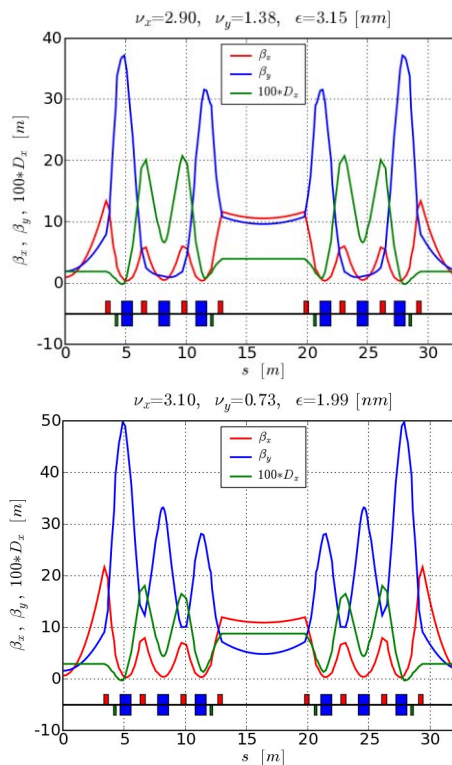


Figure 6: Twiss functions for two lattices on the Pareto optimum – one with high and other low β_y in center bend.

SUMMARY

We demonstrated the effectiveness of MOGA for locating Pareto optimal sets in a lattice optimization process. The MOGA technique is complementary to GLASS and we believe the combination of the two is extremely beneficial in the understanding and optimization of storage ring magnetic lattices. It is practical for lattices with a large number of parameters.

REFERENCES

[1] D. Robin et al. Phys. Rev. ST-AB **11**, 024002 (2008)
 [2] Kalyanmoy Deb, Multi-Objective Optimization using Evolutionary Algorithms, John Wiley & Sons Ltd, 2001