ADIABATICITY AND REVERSIBILITY STUDIES FOR BEAM SPLITTING USING STABLE RESONANCES

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Abstract

At the CERN Proton Synchrotron, a series of beam experiments proved beam splitting by crossing the one-fourth resonance. Depending on the speed at which the horizontal resonance is crossed, the splitting process is more or less adiabatic, and a different fraction of the initial beam is trapped in the islands. Experiments prove that when the trapping process is reversed and the islands merged together, the final distribution features thick tails. The beam population in such tails is correlated to the speed of the resonance crossing and to the fraction of the beam trapped in the stable islands. Experiments and possible theoretical explanations are discussed.

INTRODUCTION

A new extraction scheme to eject the beam over five turns [1] is due to be commissioned at the CERN Proton Synchrotron (PS) this year [2]. During this new multi-turn extraction, the beam is split and trapped inside stable islands of the horizontal phase space, which are generated and separated by sweeping the horizontal tune through the one-fourth resonance, $Q_x = 6.25$. This occurs while non-linear magnets, such as sextupoles and octupoles, are powered. A loss-free beam splitting in five beamlets (four islands plus the beam core) was already proved [3].

Among other aspects, both the adiabaticity (i.e. the preservation of the integrals of motion) and the reversibility (i.e. the capability of retrieving the initial conditions after reversing the process) are key ingredients to guarantee a robust and efficient particle trapping around the fixed points created by the non-linear elements. Indeed, while the horizontal tune moves away from the resonance, the distance between the fixed points and the central region of the phase space increases. A too fast crossing would prevent particles from following the fixed points, thus reducing the amount of beam trapped. On the other hand, operational constraints (such as the accelerator magnetic cycle) do not allow an arbitrarily long flat-top, to be shared between the splitting and other beam manipulations. As far as the CERN PS is concerned, the time available for the beam splitting will be of 50-90 ms, corresponding to about $24 - 34 \times 10^3$ turns at 14 GeV/c. Hence, it is of interest to define the minimum time required to cross the resonance so to leave the final beam parameters unaffected.

It is worthwhile mentioning that a simplified 2D Hénon model [1] predicts a complete reversibility (and hence adiabaticity) of the process for a sufficiently large crossing time $T^* \sim 200$ ms ($\sim 10^5$ turns). This however does not include neither the small, albeit non negligible, non-linear coupling between the two transverse planes introduced by sextupoles and octupoles, nor the coupling with the synchrotron motion (the horizontal tune being modulated by the synchrotron oscillations via the natural chromaticity).

MEASUREMENT STRATEGY

Both the adiabaticity and the reversibility of the beam splitting were studied experimentally by setting up a long flat-top during which the resonance is crossed twice, in opposite directions, as sketched in Fig. 1. The beam profile is then measured with a flying wire scanner at three different moments: before the first crossing (initial condition), after the first crossing (beamlets are generated and measured), and after the second crossing (beamlets are merged together and the final profile is compared to the first one). This procedure is then repeated for different resonance crossing times T^*, which are the same for both crossings. The second profile is measured in the middle of a stage with constant tune. Results showed in this paper refer to a stage of constant tune of 20 ms. Measurements repeated with a longer stage (180 ms) did not show significantly different results.

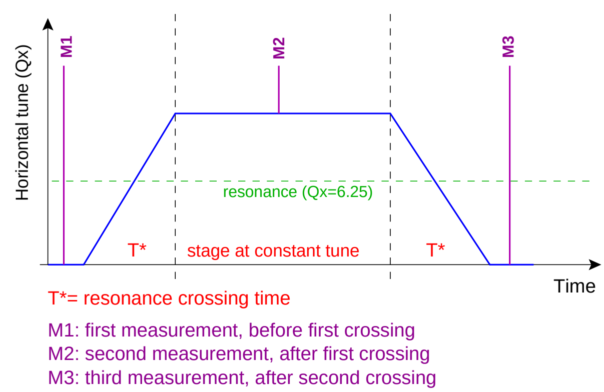

Figure 1: Sketch of the measurement procedure.

Loss of adiabaticity and reversibility reveals in a final horizontal beam profile with tick tails. By measuring the properties of the latter against the resonance crossing time T^*, this loss can be quantified. For this purpose, it is hence necessary to define a model for the tails (and the underlying assumptions) to be used for fitting the measured profile. The non-Gaussian tails are interpreted as particles that during the second, reversed, crossing are no longer able to follow the fixed points that move towards the centre of the horizontal phase space. This may be due to: (i) the loss of adiabaticity when the fixed points approach the centre, the frequency being inversely proportional to the distance between them; (ii) close to the centre the dynamics is completely linear and the fixed points generated by the non-linear elements fade away. In both cases, the beamlets may survive around the beam core, even after the fixed points disappear. The absence of the latter makes the beamlets to spiral out in the horizontal phase space and hence to form a sort of annulus. This was already observed in the studies of an injection based on beam trapping in stable islands [4]. The final profile would then be the superposition of:

1. A Gaussian central core ρ_c containing all particles not captured during the first crossing plus some of the trapped ones that reached the core during the second crossing:
$$\rho_c(x,\sigma_c,\mu_c,A) = A\, e^{-\frac{(x-\mu_c)^2}{2\sigma_c}}\ , \tag{1}$$
where σ_c and μ_c are the RMS core size and centroid respectively, and A corresponds to the beam intensity in the core.

2. The projected annulus generated by the survived beamlets. Assuming the beamlets density profile is and remains Gaussian while spiraling out, the projection reads
$$\begin{aligned}\rho_a(x,\sigma_a,\mu_a,B) \;&=\; B\, e^{-\frac{x^2+\frac{\mu_a^2}{2}}{2\sigma_a^2}} \;\times \\ &2\pi\left[I_0\left(\frac{\mu_a^2}{2}\frac{1}{2\sigma_s^2}\right) I_0\left(\frac{2\mu_a x}{2\sigma_a^2}\right) + \right. \\ &\left. 2\sum_{k=1}^{\infty}(-1)^k I_k\left(\frac{\mu_a^2}{2}\frac{1}{2\sigma_a^2}\right) I_{2k}\left(\frac{2\mu_a x}{2\sigma_a^2}\right)\right],\end{aligned} \tag{2}$$
where I_k are the modified Bessel functions, σ_a is the RMS annulus thickness/size, μ_a corresponds to the distance at which the beamlets get de-trapped, and B is proportional to the fraction of particles forming the annulus. The analytical proof of Eq. (2) is out of the scope of this paper.

Six parameters can hence be found that best fit the final measured profile with the function $\rho = \rho_c + \rho_a$ (during the fit the summation in Eq. (2) is truncated at $k = 6$). The three parameters corresponding to the annulus can be eventually plotted against the crossing time T^*.

While the initial profile is well fitted by a single Gaussian, for the second one a superposition of a central Gaussian (the beam core) and of four Gaussians having the same area (i.e. intensity) is used. The third profile is eventually fitted as described above. Two examples of measurements are reported in Fig. 2. In the upper and lower pictures the three beam profiles are shown, and correspond to a crossing of 170 ms and 20 ms, respectively. While it is natural to have more populated islands when the crossing is slower, it is rather counterintuitive to observe in this case thicker tails, compared to the fast crossing in 20 ms. This is a sign that the process is not reversible. Hence, the slower the crossing, the least reversible the process is. It is worth mentioning that the different centroid of the third profile is due to a known offset introduced by the PS wire scanner when more than one measurement is taken during the same cycle.

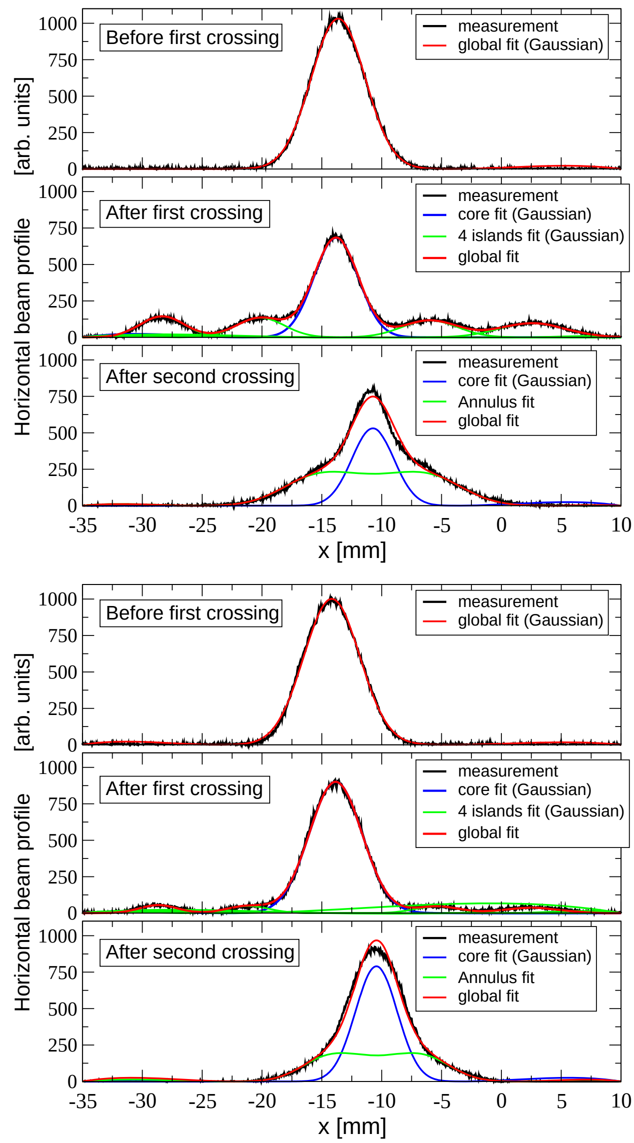

Figure 2: Examples of measured horizontal beam profile during the double resonance crossing. For each profile, the (multi-)fit is superimposed. The two sets of plots refer to a crossing in 170 ms (top) and 20 ms (bottom), respectively.

EXPERIMENTAL RESULTS

Data were acquired in 2004 by varying T^* from 10 ms to 170 ms, by steps of 10 ms. A single-bunch beam of 55×10^{10} protons per bunch, RMS momentum spread $\Delta p/p(2\sigma) = 1 \times 10^{-3}$, and normalized RMS horizontal emittance $\epsilon_x(2\sigma) = 8.3$ mm mrad was used. From a first analysis it turned out that core RMS size σ_c was independent from the crossing speed (σ_c =1.75 mm). It was also observed that the fit of ρ_a was not unique, different combinations of σ_a and μ_a providing equivalent global profile. Therefore, it was decided to fix μ_a and to perform the fit on the remaining four parameters. The above considerations (i) and (ii) induce indeed to consider μ_a as a geometrical parameter rather than a a dynamical one. After repeating the fit of all data for several values of μ_a, the one mini-

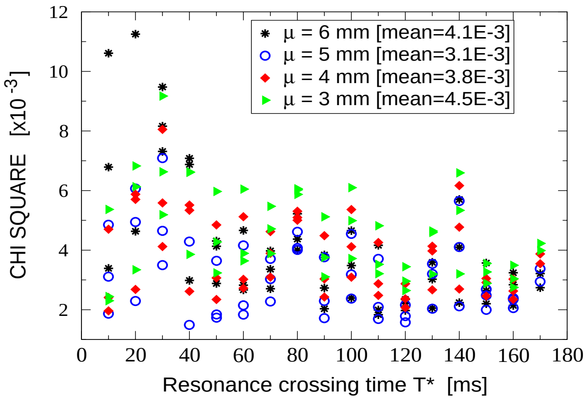

Figure 3: χ^2 from the four-parameter fit $\rho = \rho_c + \rho_a$ of the final horizontal beam profile against the resonance crossing time for different values of μ_a.

mizing the χ^2 value was chosen. As shown in Fig. 3 this is achieved by $\mu_a = 5$ mm, that gives an average χ^2 of 3.1×10^{-3}.

In Fig. 4 the fit results are illustrated against the resonance crossing time T^*. In the upper plot it can be clearly seen how the larger T^*, the more populated the annulus. For comparison the fraction of particles trapped in the four islands is shown. The particle sharing is inferred from the two fit parameters A and B: the particle fraction in the core reads $A/(A + B)$, while the one in the annulus is equal to $B/(A + B)$. It is worth mentioning that whenever the resonance is crossed in a time $T^* > 10$ ms no particle loss is observed during the double crossing. The particle share in the annulus seems to have the same dependence on T^* as the one in the four islands, thus indicating that more populated islands are more difficult to merge down into the core. For completeness it has to be mentioned that for large T^* the islands are not only more populated, but also larger: In

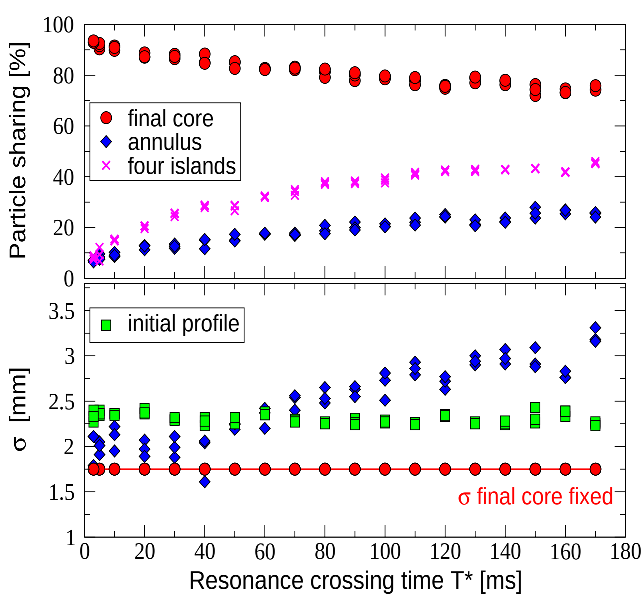

Figure 4: Dependence of the particle sharing (top) and of the fit parameter σ (bottom) on the resonance crossing time.

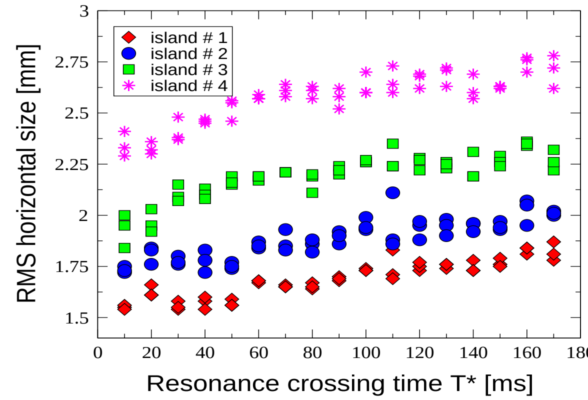

Figure 5: Evolution of the islands' RMS sizes against the resonance crossing time T^* .

Fig. 5 the evolution of the islands' sizes is plotted again T^*, showing a clear growth. The fact that the RMS sizes are different, while the areas (and the emittances) are the same, is a consequence of the non-linear magnetic fields: The relation $\epsilon_x = \sigma_x^2/\beta_x$ does not hold for the islands. Thicker tails might hence be the consequence of having more particles at large distance from the fixed points that spiral out when the latter collapse onto the centre. The lower plot of Fig. 4 shows how the annulus size σ_a increases for large T^*. This clearly shows how a longer crossing time improves adiabaticity of the capture process at the expense of reversibility.

CONCLUSION AND OUTLOOK

Measurements at the CERN PS with a bunched beam showed that the beam splitting in stable islands of the horizontal phase space is an adiabatic process (the longer the resonance crossing time, the larger the number of particles trapped in the islands) but not reversible: Even if the islands merge together towards the centre during the inverse resonance crossing, the horizontal beam profile shows thick tails whose population and size increase when the crossing speed is reduced. This feature is not explained by the 2D Hamiltonian model of the splitting, which is invariant for time reversal. Coupling with the vertical plane might induce the preservation of invariant different from the horizontal RMS emittance (i.e. beam size). Numerical studies with a more realistic 5D model, taking into account also chromatic effects and tune modulation induced by the synchrotron motion via the natural chromaticity are ongoing.

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