VORPAL SIMULATIONS RELEVANT TO COHERENT ELECTRON COOLING*

George I. Bell†, David L. Bruhwiler, Andrey Sobol, Tech-X Corp., Boulder, CO 80303
Ilan Ben-Zvi, Vladimir Litvinenko, Brookhaven National Lab, Upton, NY 11973
Yaroslav Derbenev, Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606

Abstract

Coherent electron cooling (CEC) [1] combines the best features of electron cooling and stochastic cooling, via free-electron laser technology [2], to offer the possibility of cooling high-energy hadron beams with order-of-magnitude shorter cooling times. Many technical difficulties must be resolved via full-scale 3D simulations, before the CEC concept can be validated experimentally. VORPAL is the ideal code for simulating the modulator and kicker regions, where the electron and hadron beams will co-propagate as in a conventional electron cooling section. Unlike previous VORPAL simulations [3] of electron cooling physics, where dynamical friction on the ions was the key metric, it is the details of the electron density wake driven by each ion in the modulator section that must be understood, followed by strong amplification in the FEL. We present some initial simulation results.

COHERENT ELECTRON COOLING

The idea of Coherent electron cooling (CEC) was proposed by Derbenev in 1980 [4]. A novel FEL-based version of the idea was recently developed by Litvinenko and Derbenev [2].

CEC is initiated by a process nearly identical to conventional electron cooling. The beam of hadrons is co-propagated with a beam of electrons. Although these beams are highly relativistic in the laboratory frame, aIl velocities are non-relativistic in the "beam frame" drifting with the two beams.

The proposed CEC consists of three stages:

1. Modulator: Hardons induce a density perturbation or wake in the co-propagating electron distribution. In this paper we model this stage only.

2. FEL: The density perturbation is amplified by a high-gain Free Electron Laser (FEL). At the same time, hadrons experience longitudinal dispersion.

3. Kicker: The electric field induced by the amplified density perturbation reduces the longitudinal emittance of the hadron beam by accelerating less energetic hadrons and decelerating more energetic hadrons.

VORPAL [5, 6] is an ideal tool for investigating the modulator and kicker stages. These stages are similar to conventional electron cooling, and we can model the particle interaction using electrostatic PIC. In VORPAL there are also several specialized algorithms to accurately model small impact parameter collisions [7]. The FEL stage can be modeled using an FEL code, such as GENESIS [8].

SIMULATING THE MODULATOR

We simulate the modulator in VORPAL in the beam frame using a small portion of the beam in a 3D box a few Debye radii on a side. The electron density is initially uniform. A single gold ion of charge Ze is placed in the domain at time $t = 0$. We work in the reference frame of the ion in which the ion is initially at rest. Any velocity of the ion relative to the electron distribution is subtracted from the average electron velocity, giving the electrons a non-zero drift velocity.

Boundary conditions on fields are periodic. For the particles, the simplest boundary condition is also periodic, so particles leaving one side of the box appear on the opposite side. Alternatively, we consider a boundary condition where any particle leaving the box is removed from the simulation, and a thermal distribution of electrons with the proper drift speed enters the box from all sides.

Typical parameters for our runs are given in Table 1. The z coordinate is used to denote the direction parallel to the direction of beam propagation. We use $(v_{ix}, v_{iy} = 0, v_{iz})$ for the ion drift velocity with respect to the electrons and $(\sigma_{vx}, \sigma_{vy}, \sigma_{vz})$ as the RMS electron velocity. By non-dimensionalizing the equations of motion, the critical parameters are ζ, which measures the non-linearity of the plasma response, $R = \sigma_{vx}/\sigma_{vz}$, the ratio of transverse to longitudinal RMS electron velocity, and the normalized ion drift speeds $L = v_{iz}/\sigma_{vz}$ and $T = v_{ix}/\sigma_{vx}$. Electrons from a particle accelerator typically have a higher transverse RMS velocity σ_{vx} compared to the longitudinal velocity σ_{vz}. So we expect that $R > 1$, and take $R = 3$ as a typical value.

The problem is simulated in dimensional units. We start from the electron density n_e in Table 1, which fixes the plasma frequency and time scale. For a particular choice of the non-dimensional parameters ζ, R, L and T, we set σ_{vz} in order to give the desired ζ, then all other parameters

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† gibell@txcorp.com

Table 1: Typical simulation parameters and relevant dimensionless parameters

Parameter	Value	Description
n_e	$1.60 \times 10^{16}\ m^{-3}$	Electron density
$\omega_p = (2\pi)8.98\sqrt{n_e}$	7.14×10^9 rad/sec	Plasma frequency
$2\pi/\omega_p$	8.8×10^{-10} s	Plasma period
σ_{vx},σ_{vy}	1.6×10^5 m/s	Transverse RMS electron velocity
σ_{vz}	5.4×10^4 m/s	Longitudinal RMS electron velocity
$\lambda_D = \sigma_{vz}/\omega_p$	7.6 microns	Nominal longitudinal Debye radius
$R \equiv \sigma_{vx}/\sigma_{vz}$	~ 3	Ratio of transverse to longitudinal RMS velocity spread
$T \equiv v_{ix}/\sigma_{vx}$	0 to 1.8	Ratio of transverse ion velocity to RMS velocity spread
$L \equiv v_{iz}/\sigma_{vz}$	0 to 1.8	Ratio of longitudinal ion velocity to RMS velocity spread
$\zeta \equiv Z/(4\pi n_e R^2 \lambda_D^3)$	≤ 0.1	Plasma nonlinearity parameter

are fixed by the choice of R, L and T. Typically, we use $\zeta = 0.1$ or smaller to remain in the linear regime.

The particles are simulated using electrostatic PIC. At each time step the field due to the electrons and ions is calculated, which is then used to move the particles. In order to reduce noise, we split each electron into many particles with the same charge to mass ratio. We also use only half the electrons specified by the density n_e, but on top of each electron we place a positron of the same mass but opposite charge. This numerical trick reduces noise in the wake by ensuring that charge density deviations are due only to the differences between the trajectories of electrons and positrons.

An alternative binary collision algorithm [7] moves the electrons and ion by exactly solving the 2-body problem for each electron-ion pair. This has the advantage of being able to deal with very close collisions correctly, and is important to calculate the friction force accurately [7]. However, in this case we want to resolve the wake, and small impact parameter collisions have relatively little effect on the wake. We have seen only minor differences in the wake using electrostatic PIC versus binary collision algorithm. Thus, all results presented below use electrostatic PIC.

THE ISOTROPIC TEST CASE

When the electron distribution is isotropic ($R = 1$) and the ion at rest with respect to the electrons ($T = L = 0$), we have classical Debye shielding. In this case the steady-state solution is well-known.

Note that in the isotropic case, the typical speed of an electron is $\sqrt{3}\sigma_{vz}$. Thus, the Debye radius λ_D as defined in Table 1 is smaller by a factor of $\sqrt{3}$ than the definition used in some other works.

A useful metric to check the simulations is the electron charge perturbation $q(\alpha, t)$ within a sphere of radius $\alpha\lambda_D$, centered around the ion. For the steady-state Debye shielding solution, as $\alpha \to \infty$ this charge approaches $-Z$, exactly canceling out the charge of the ion. The steady-state charge distribution within a sphere of radius $\alpha\lambda_D$ is

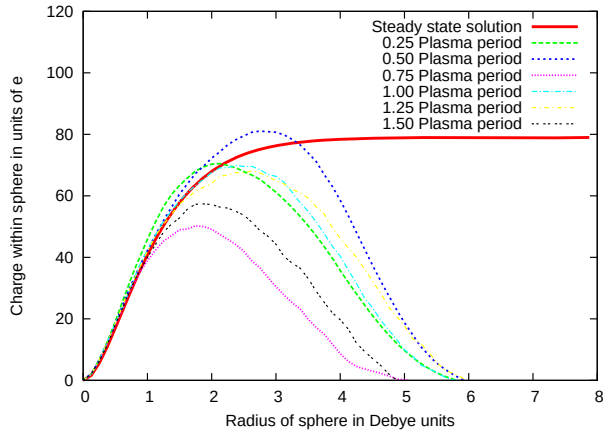

Figure 1: The charge within a sphere using periodic boundary conditions.

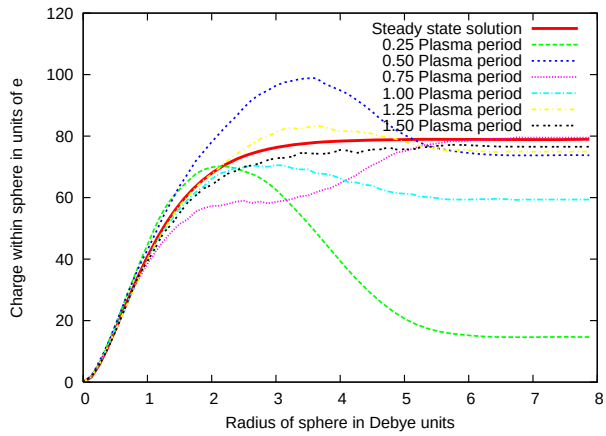

Figure 2: The charge within a sphere using emitting and absorbing boundary conditions.

$$q(\alpha) = -Z\left[1 - (1+\alpha)e^{-\alpha}\right] \quad (1)$$

Fig. 1 shows the evolution of $q(\alpha, t)$ for a VORPAL simulation compared to the steady state solution (1), where $Z = 79$ for a fully-stripped gold ion. In this case we use

VORPAL with periodic boundary conditions on the fields and particles. Total charge in the simulation domain is conserved, so while the charge distribution is correct near the ion it is incorrect near the edges of the box. Each side of the box in this case has length $8\lambda_D$, and the farthest an electron can be from the ion is $4\sqrt{3}\lambda_D$.

Better results are obtained by destroying particles that leave the simulation domain. These are replaced by a constant thermal distribution which comes into the domain with the correct temperature and drift speed. As shown in Fig. 2, this gives a much better match to the steady-state, Debye shielding solution.

As the simulation proceeds, electron-positron pairs become separated, and the simulation noise increases. The technique in Fig. 2 has an additional advantage because the particles entering the domain begin as correlated electron-positron pairs (on top of one another).

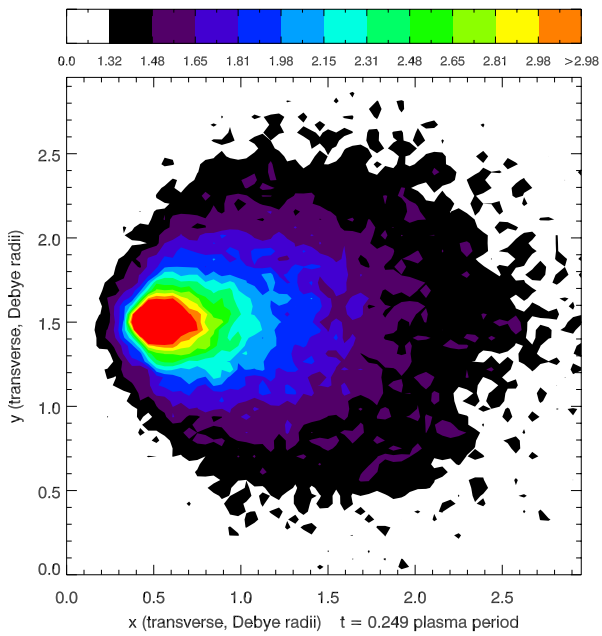

Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

SIMULATIONS OF FINITE ION VELOCITY

We now consider electrons with $R = 3$, with a traverse ion speed $T = 1.8$ and $L = 0$. Because we work in the ion frame, the electrons are drifting. The moving ion leaves a wake in the electron distribution, shown as a color contour plot in density in Fig. 3. Fig. 4 shows the longitudinal profile at the same time, about a quarter of a plasma period.

One parameter of interest is the time when the integrated wake is maximum, or has the highest peak. This information could be used to optimize the length of the modulator

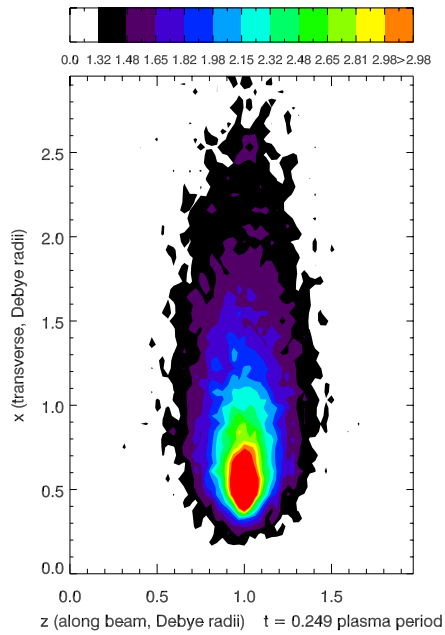

Figure 4: A longitudinal cross section of the wake behind a gold ion, with the color denoting density enhancement.

section. For a perfectly cold electron distribution this occurs at a quarter of a plasma period. For the simulations in Figs. 3 and 4, the integrated wake reaches a maximum at about half a plasma period, but is at 86% of this maximum at a quarter of a plasma period.

FUTURE WORK

We plan to run additional test cases to explore how the density perturbations change with ion velocity (T, L). We will explore the time evolution of the density perturbations. The density perturbations will then be used as the input to an FEL code to study the wake amplification. Finally, we will use the amplified density in the kicker to calculate the effect back on the hadrons.

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