TRANSVERSE SCHOTTKY NOISE AND BEAM TRANSFER FUNCTIONS WITH SPACE CHARGE*

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Abstract

Transverse beam transfer functions (BTFs) and Schottky bands of ion beams with high phase space density were measured in the heavy ion synchrotron SIS18 with coasting $^{40}\mathrm{Ar}^{18+}$ beams. A deformation of the BTFs and Schottky signals was found and compared to a linear space-charge model.

INTRODUCTION

The future facility for antiproton and ion research FAIR will demand beams of unprecedented quality and intensity from the existing heavy ion synchrotron SIS18 at GSI [1]. SIS18 will be upgraded to fulfill these requirements but with the increase of intensity collective effects gain weight for the beam dynamics and may cause instabilities. Particularly space charge and impedances are subject to investigation at GSI [2].

The beam intensity currently available at SIS18 allows the experimental investigation of space charge while the impact of impedances is usually negligible. A linear model for the description of space-charge affected BTFs and Schottky spectra is introduced in the next section followed by its application to the data from measurements in SIS18. Non-linear space charge effects in more intense beams are investigated theoretically [3, 4], but will not be discussed here.

BTF AND SCHOTTKY NOISE

BTF and Schottky noise are commonly used for non-destructive diagnostics in synchrotrons. In the transverse plane their positions indicate the revolution frequency f_0 and the fractional betatron tune $Q_{\mathrm{f},0}$. In addition their shape reflects the beam's momentum distribution. But at high intensity these information can not be read directly from the signals because collective effects deform BTFs and Schottky bands.

The space-charge force is incoherent since it acts with an individual strength on each ion, depending on its distance from the beam center [5]. This force has two effects on the beam. Firstly the incoherent bare tune Q_0 is shifted by ΔQ towards

$$Q = Q_0 + \Delta Q. \tag{1}$$

Secondly it acts comparably to a purely reactive impedance. This effect manifests itself in a shift of the stability diagram, given by the inverse of the BTF r, with respect to the low intensity BTF r_0^{-1} by virtue of [6]

$$\frac{1}{r(f)} = \frac{1}{r_0(z)} - \Delta U. \tag{2}$$

Here ΔU replaces the normalized impedance which is linked to ΔQ by f_0 and the rms width σ_f of the BTF or Schottky band by

$$\Delta U = \frac{f_0 \Delta Q}{\sigma_f}. \tag{3}$$

For convenience we introduced the normalized frequency

$$z(f) = \frac{f - f_0(m \pm Q_{\mathrm{f},0} \pm \Delta Q)}{\sigma_f}, \tag{4}$$

where m is the harmonic number of the adjacent longitudinal Schottky band. The upper sign corresponds to a lower side band or BTF and vice versus.

The transverse Schottky spectrum P of an intense beam is deformed according to [7, 8]

$$P(f) = \frac{P_0(z)}{|1 - \Delta U\, r_0(z)|^2}, \tag{5}$$

where P_0 is the low intensity spectrum.

Assuming an elliptical beam cross section, ΔU in the vertical plane can be estimated from the beam parameters using [9]

$$\Delta U_{\mathrm{est}} = -\frac{r_{\mathrm{p}} Z^2 N}{2\pi A \beta^2 \gamma^3} \frac{2}{\epsilon_{\mathrm{v}} + \sqrt{\epsilon_{\mathrm{v}}\epsilon_{\mathrm{h}} Q_{0,\mathrm{v}}/Q_{0,\mathrm{h}}}} \frac{f_0}{\sigma_f}, \tag{6}$$

with the classical proton radius r_{p}, the ion charge Z and mass A, the particle speed over the speed of light β, the Lorentz factor γ and the emittance ϵ. The index 'v' refers to the vertical plane and 'h' to the horizontal one.

EXPERIMENT AND RESULTS

Transverse BTF measurements in SIS18 were begun in 2006 [10]. Space-charge effects were observed for the first time in February and August 2007 with coasting $^{40}\mathrm{Ar}^{18+}$ beams. Table 1 lists the beam parameters corresponding to the results presented in the following subsections. f_0, Q_0 and the momentum spread $\frac{\Delta p}{p}$ were obtained from our Schottky or BTF measurements.

An ionization profile monitor [11] was used to measure the transverse beam profile and the rms beam width in both directions for the computation of ϵ. In Eq. (6) the emittance of the suppositionally homogeneous beam was approximated by the 2σ emittance $\epsilon_{2\sigma} = 4\,\epsilon_\sigma$ of the measured Gaussian profile.

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Table 1: Parameters of the beams used in our experiments and comparison of estimated and fitted ΔQ and ΔU (for both upper and lower Schottky band).

Experiment	BTF	Schottky
Energy [MeV]	11.4	11.4
$Q_{0,\mathrm{ver}}$	3.29	3.31
$Q_{0,\mathrm{hor}}$	4.31	4.21
$\epsilon_{\sigma,\mathrm{v}}$[mm mrad]	4.6	4.6
$\epsilon_{\sigma,\mathrm{h}}$[mm mrad]	7.8	5.7
N	$2 \cdot 10^9$	$7 \cdot 10^9$
$\frac{\Delta p}{p}$	$6.7 \cdot 10^{-4}$	$6.6 \cdot 10^{-4}$
m	98	75
ΔQ_{est}	-0.007	-0.03
ΔQ_{fit}	-0.004	-0.04
ΔU_{est} (low/up)	-0.13	-0.77 / -0.89
ΔU_{fit} (low/up)	-0.06	-1.03 / -1.28

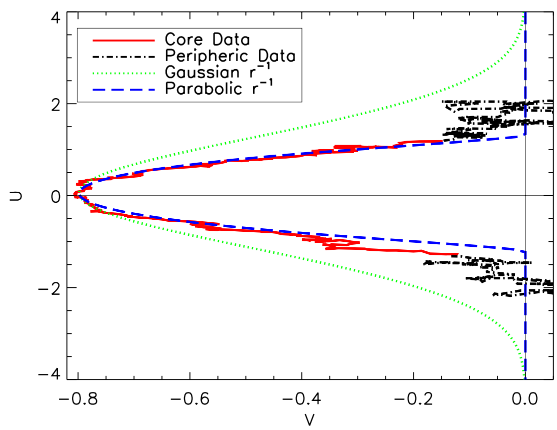

Figure 1: Stability diagram of the upper sideband of the 98^{th} harmonic in the vertical plane. For beam parameters see Tab. 1. Core data is called what defines almost the entire shape of the stability diagram but only the center of the BTF's amplitude in Fig. 2.

BTF Measurement

The stability diagram obtained from the measured amplitude and phase of a BTF is shown in Fig. 1 in comparison with the analytic computation for a Gaussian and a parabolic momentum distribution. The beam parameters for this measurement are given in Tab. 1. The stability diagram is matched quite well by the parabolic distribution as opposed to the Gaussian which is clearly excluded. However, the amplitude of r, shown in Fig. 2, implies a distribution with much wider tails. The data range highlighted in red in the two figures illustrates how this discrepancy can be explained: Only the core of the momentum distribution contributes remarkably to the stability diagram. The tails with a low particle density disappear in the noise at the edges of the stability diagram.

The best agreement of the parabolic BTF with the measured amplitude was found for $\Delta U_{\mathrm{fit}} = -0.06$ which differs considerably from the estimation $\Delta U_{\mathrm{est}} = -0.13$ using Eq. (6). Uncertainties of the emittance and the actual momentum distribution are hold responsible for this deviation. The shift of the stability diagram by ΔU_{fit} is hardly visible.

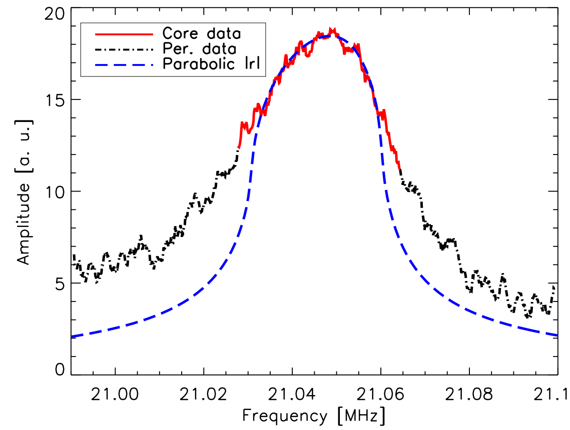

Figure 2: Amplitude of the BTF measured at the upper sideband of the 98^{th} harmonic in the vertical plane. The parabolic BTF matches the data only in the range that is relevant for the stability diagram (Fig. 1). The peak has a slight asymmetry that we refer to the space charge. The best agreement for the parabolic distribution was found for $\Delta U_{\mathrm{fit}} = -0.06$.

Measurements were also performed at the energy of 500 MeV/u. The results are qualitatively similar to those presented here for low energy. The stability diagram corresponds to a parabolic momentum distribution again while the amplitude spreads wider. The measured tune shifts differ from prediction by up to a factor of 2 in the worst case.

Schottky Noise Measurement

Significantly distorted transverse Schottky bands were measured with the beam characterized by the parameters in Tab. 1. The fit of P [Eq. (5)] implied a Gaussian distribution P_0 while a parabolic distribution did not comply well. The amplitude, the frequency f_c for which holds $z(f_c) = 0$, σ_f and ΔU were free parameters in this fit. Note that f_c would be the center of the undistorted side band corresponding to the tune Q [Eq. (1)].

A pair of measured Schottky side bands is shown in Fig. 3 and Fig. 4 together with the fit. P_0 computed with the parameters from the fit but without tune shift is added to the graphs in order to demonstrate what the band would look like in the absence of space-charge effects. Finally the vertical line illustrates where the incoherent tune was shifted to by ΔU.

According to Eq.(3) ΔU is different for a pair of lower and upper side bands if they have a different σ_f because of the accelerators chromaticity, while ΔQ depends on beam

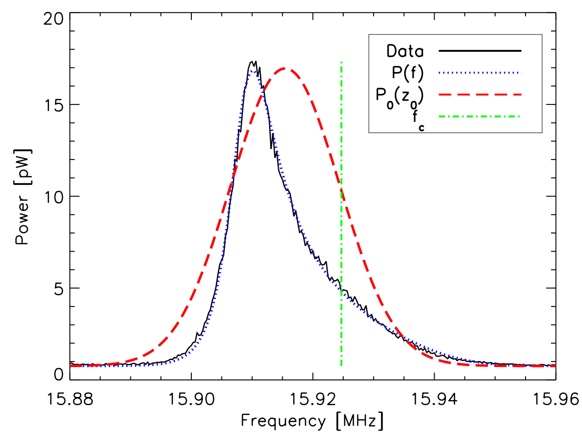

Figure 3: Vertical lower Schottky band of the $75^{\rm th}$ harmonic. For beam parameters see Tab. 1. P is the fit of Eq. (5) to the data. P_0 was computed with the parameters from the fit and shows what the band would look like in the absence of space charge. z_0 is equal to z [Eq.(4)] for $\Delta Q = 0$. The vertical straight line shows f_c obtained from the fit. It is shifted off the center of P_0 by the incoherent space-charge effect.

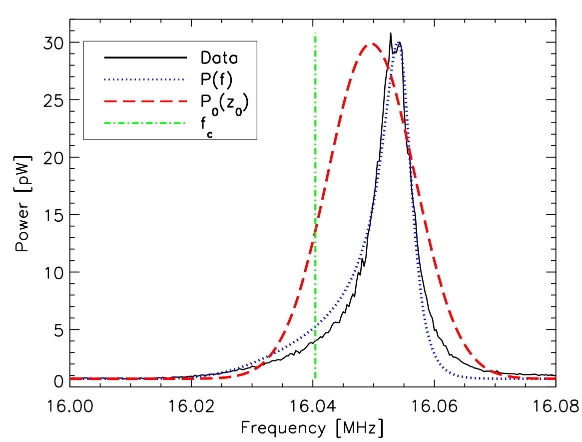

Figure 4: Vertical upper Schottky band of the $75^{\rm th}$ harmonic. The beam parameters are the same as in Fig. 3.

parameters only [Eq.(6)]. As presented in Tab. 1 the estimated and measured ΔQ and ΔU agree roughly. Both Schottky bands yield the same value for ΔQ. The bare betatron frequency obtained from the fit differs from the setting of the machine by about 1 kHz which lies within the expected uncertainty.

CONCLUSIONS

Space-charge induced deformations of transverse BTFs and Schottky spectra were observed in SIS18. A linear model was successfully used to describe the deformation of the signals and to determine the space charge parameter ΔU. The knowledge of ΔU is particularly important for beams at the edge of stability as it may inhibit Landau damping of transverse dipolar oscillations.

Also Q, Q_0 and σ_f which can not be read directly from the deformed signals could be evaluated. With respect to the operation of the machine this means that the linear model presented here allows the user to deduce important beam and machine parameters from Schottky bands and BTFs despite their deformation.

However, the mismatch between measurement and estimation is relatively large and the reason why different momentum distributions where found in similar experiments is not known.

OUTLOOK

The model of linear space charge was applied successfully to several measurements, but a systematic verification is still to be done. This includes the investigation of the dependence of the signal deformation on the beam intensity and the comparison of Schottky and BTF measurements with the same beam. The analysis of Schottky bands at higher energy demands further measurements as the model was not yet applied successfully under this condition. Finally we hope to reduce the discrepancy between the measured and the estimated tune shift. In addition to the experiments simulations will be run to verify our model and study beams under well defined and experimentally inaccessible conditions.

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