Electron Cooling Force Calculations for HESR*

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Abstract

The High energy storage ring HESR at FAIR is being realized by a consortium consisting of Forschungszentrum Jülich, GSI Darmstadt and Uppsala University. An important feature of this new facility is the combination of phase-space cooled beams and dense internal targets.

Charmonium spectroscopy, which is one of the main items in the experimental program, requires antiproton momentum up to 8.9 GeV/c with a resolution $\Delta p/p = 10^{-5}$. This can only be achieved with electron cooling.

The electron cooler proposed for HESR allows beam cooling for antiproton momentum between 1.5 GeV/c and 8.9 GeV/c. Along the 24 m interaction section the electrons are guided by a solenoid field of 0.2 T with a field straightness of 10^{-5} radians rms.

To predict the final momentum resolution of the antiproton beam in HESR, electron-cooling force calculations, simulations of electron cooling and comparison to experimental data are needed.

This paper focuses on the force calculations. The method is based on the theory by Derbenev and Skrinsky, (i.e. the Vlasov techique) and the electron cooling force is numerically calculated using adaptive Monte Carlo integration methods.

MOTIVATION

The electron cooling force in a magnetized electron beam with an anisotropic electron velocity distribution was derived by Derbenev and Skrinsky in 1978 [1]. The cooling force was expressed as a sum of contributions from fast and adiabatic collisions, referring to the interaction time between electrons and ions relative to the cyclotron period of the electrons. Within a logarithmic accuracy the fast and the adiabatic contributions to the cooling force could be derived analytically by approximating the magnetic field as $B = 0$ and $B = \infty$, respectively. The actual strength of the magnetic field appears in two Coulomb logarithms only.

One concern with this Coulomb logarithm approximation is that the argument of the Coulomb logarithm, ie, the ratio maximum to minimum impact parameter must be large. Otherwise the cooling force becomes sensitive to the limit between adiabatic and fast interactions. Another concern is that at many electron cooling facilities the method is not applicable. The disturbance of the electron plasma by the ions becomes too strong, especially for ion velocities close to the average electron velocity.

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Table 1: Comparison between the electron coolers at HESR and Fermilab

e-beam parameters	**HESR**	**FNAL**
Energy (MeV)	0.45-4.5	4.3
Current (Amp)	1	0.5
Solenoid field (T)	0.2	0.01
Rms straightness (rad)	10^{-5}	10^{-4}

Relating measured cooling forces to any theory is a complicated task because electron cooling depends on many parameters that must be known accurately. The electron cooling force that has been measured at various facilities tends to be much weaker compared to the cooling force by Derbenev and Skrinsky. A common solution is that a semi-empirical formula by Parkhomchuk [2] is used.

Most similar to the HESR-EC is the electron cooler in the recycler at Fermilab. However, the electron beam transport systems are very different as can be seen in table 1. This influences on both the electron beam quality and the interactions between electrons and antiprotons.

METHOD

To calculate the electron cooling force the Vlasov technique [3] is used. The Vlasov technique involves solving the collisionless Boltzmann (or Vlasov equation) and takes into account the collective response of charges in the plasma. The technique is a standard method in plasma physics, and was used by Derbenev and Skrinsky to derive the cooling force, but it is also referred to as the dielectric model [4].

By calculating the cooling force numerically rather than using approximations, the problem with cutoffs in the limit between fast and adiabatic interaction is overcome. The Vlasov technique also provides a tool to decide whether the method is applicable or not.

For the calculations the following assumptions are made:

- The electron plasma is large.
- Initially, the electron density n_e is uniform and given by the electron beam current.
- The electron velocities are Gaussian distributed with $\Delta_{\|}$ and $\Delta_{\perp}$ as the longitudinal and transverse rms velocities, respectively.
- Perturbations of the plasma are small.
- The influence of diffusion and friction within the electron plasma can be neglected.

- The ion is heavy, i.e., moves with a constant velocity $\mathbf{v}$ through the electron cooler
- The ion (or antiproton) intensity is low.
- The ion has been inside the plasma for a long time.

In table 2 parameters for the electron cooling force calculations are given.

The derivation of the cooling force can be found in the work by Sørensen and Bonderup [4]. In the rest frame of the electron plasma the electron cooling force is given by

$$\mathbf{F}(\mathbf{v}) = -\frac{Z^2 e^2}{(2\pi)^3 \epsilon_0} \int d^3\mathbf{k} \, \frac{i\mathbf{k}}{k^2} \left(\frac{1}{\varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v})} - 1 \right) \quad (1)$$

where $\mathbf{v}$ is the ion velocity. Sørensen and Bonderup expressed the dielectric function as a sum over modified Bessel function, but it can also be represented by a dielectric integral:

$$\varepsilon(\mathbf{k}, \omega) = 1 + \omega_p^2 \int_0^\infty dt \, \frac{k_\parallel^2 t + k_\perp^2 \frac{\sin \omega_c t}{\omega_c}}{k^2} \times \exp\left(-\frac{k_\parallel^2 \Delta_\parallel^2 t^2 + k_\perp^2 \Delta_\perp^2 \left(\frac{\sin(\omega_c t/2)}{\omega_c/2} \right)^2}{2} + i(\omega + i\epsilon)t \right) \quad (2)$$

where indices $\perp$ and $\|$ refer to components perpendicular to and parallel with the solenoid field. A similar expression can be found in [5] by Nersisyan, Toepffer and Zwicknagel.

The cooling force derived by this method is based on the assumption that the collisionless Boltzmann equation can be linearized. That is, the perturbation of the phase space density is small compared to the initial unperturbed phase space density. The linearization implies that an upper cut in integration limit at $k = k_{\max}$ must be introduced in (1), otherwise the integral diverges. This cutoff is chosen as the reciprocal of the minimum impact parameter for fast collisions, which was also used by Sørensen and Bonderup. If this minimum impact parameter is much smaller that the Debye screening length the calculated cooling force will have a weak (logarithmic) dependence on the cutoff parameter $k_{\max}$. The criteria for the applicability of the method is that the electron density at the location of the ion n_e' (where the perturbation is assumed to be the strongest) should be similar to n_e. The density perturbation, defined as $\Lambda = n_e'/n_e - 1$, can be derived from the solution of the collisionless Boltzmann equation

$$\Lambda(\mathbf{v}) = -\frac{Z}{n_e (2\pi)^3} \int d^3\mathbf{k} \left(\frac{1}{\varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v})} - 1 \right) \quad (3)$$

and is calculated together with the electron cooling force components.

CALCULATIONS

To speed up the numerical calculations approximations of the dielectric function are used whenever possible. For $\max\left(|\omega|, k_\parallel \Delta_\parallel\right) \ll \omega_c$ and $\max\left(|\omega|, k_\parallel \Delta_\parallel\right) \gg \omega_c$ approximations for interactions that are adiabatic and fast are used, respectively.[1] Approximations for close interations take place for $k_\perp \Delta_\perp \gg \omega_c$. For all other cases the dielectric function are obtained as a sum over modified Bessel functions. The number of contributing terms scales with the parameter $k_\perp \Delta_\perp / \omega_c$ which is of unit order or smaller since the approximations for close interations are used.

To facilitate the integration over wave number $\mathbf{k}$ (eqs. 1 and 3) in three dimensions, where $1/\varepsilon$ can vary rapidly, the Monte Carlo integration routine VEGAS by Lepage [6] is used. Depending on variations, VEGAS rearranges the grid and provides error analysis of the result.

An important parameter for the numerical integration is the small and imaginary part $i\epsilon$ of the frequency in eq. 2, which has been introduced to smoothen out the integration over $1/\varepsilon$. If it is too small integration over $\mathbf{k}$ becomes difficult. If it is too large, it influences on the result. Careful optimization of this parameter and the limits for adiabatic, fast and close approximations has been undertaken.

Typically it takes 24 hours on a 2.4 GHz Intel dual core processor with 1.99GB of RAM to calculatate the transverse and longitudinal force components and the density perturbation for 20 × 20 different longitudinal and transverse ion velocities.

In figure 1 and 2 the calculated longitudinal and transverse cooling force are plotted as functions of the ion velocity. The Input parameters are given in table 2. The non-straight solenoid field along the interaction section was taken into account by averaging the calculated cooling force over a transverse Gaussian velocity distribution

Table 2:

Input Parameters	symbol	value
Solenoid field	B	0.2 T
Straightness	$\theta_{\rm rms}$	1×10^{-5} rad
Interaction length	L	22 m
Relative HVPS ripple	$\Delta U/U$	2×10^{-5}
Electron beam radius	r_0	5 mm
Electron beam current	I_e	1 A
Temperature	kT	1 eV
Ion momentum	P	8.9 GeV/c
Charge number	Z	1
Mass number	A	1
relativistic γ	γ	9.5
electron density	n_e	2.8×10^{13} m^{-3}
Rms electron velocities:		
transverse	$\Delta_\perp$	4.2×10^5 m/s
longitudinal	$\Delta_\parallel$	6.7×10^3 m/s
straightness	σ	2.8×10^4 m/s
Cyclotron frequency	ω_c	3.5×10^{10} s^{-1}
Plasma frequency	ω_p	3.0×10^8 s^{-1}
Interaction time	τ	8.4×10^{-7} s

[1] The approximations can easily be derived from (2) but were taken out of the paper to save space.

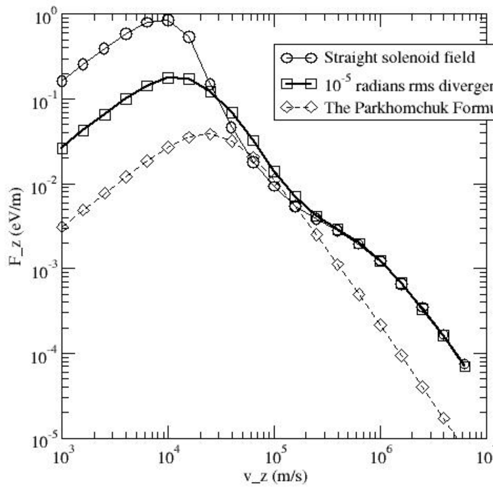

Figure 1: The longitudinal electron cooling force vs longitudinal ion velocity. The transverse ion velocity is zero

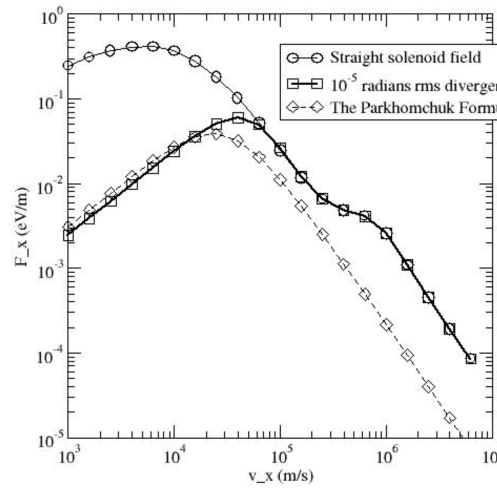

Figure 2: The transverse electron cooling force vs transverse ion velocity. The longitudinal ion velocity is zero

with rms $\sigma = \beta\gamma c\theta_{\rm rms}$. Indicated is also the Parkhomchuk formula [2] with the effective velocity given by σ.

CONCLUSIONS

From the ongoing investigations on electron cooling, which is undertaken within the design study project of the HESR electron cooler [8], the following conclusions have been made:

1. The Vlasov technique can be used for the electron cooling force calculations. The density disturbance is indeed slightly too strong for ion velocities close to the longitudinal rms velocity of the electrons, but this becomes insignificant after that the angular deviations of the magnetic field has been taken into account. Alternatively, by reducing the solenoid field to 0.1 T the method becomes adequate.

2. The electron cooling force is not particularly sensitive to the solenoid field strength. This does not necessarily imply that the magnetic field strength should be reduced. Then intra-beam scattering in the electron beam could increase the longitudinal rms velocity of the electron beam, which would influence on the force and worsen the collector efficiency. The electron beam transport would also be more difficult to control.

3. It is very important to maintain a straight magnetic field along the interaction region.

4. Compared to the Parkhomchuk formula the calculated longitudinal component of the electron cooling force is ten times stronger for the low antiproton velocities. Electron cooling simulations based on the Parkomchuk formula have shown that a 90 % antiproton momentum spread of 4.4×10^{-5} can be expected [7]. This momentum resolution could be an underestimate, but more investigations are needed [9].

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